UC-NRLF
$B 79 477

UNIVERSITY OF CALIFORNIA
ANDREW
SMITH
HALLIDIE:
REGENT
1868 1901

Halliday

Practical Alternating Currents
and
Alternating Current Testing.

A COMPANION VOLUME.

By the same Author.

Practical Testing of Dynamos & Motors.

CONTAINS CHAPTERS ON

General purpose of tests; measurement of armature and field resistances; production of electromotive force in a dynamo; the magnetic circuit; armature reactions; shunt-wound dynamo; series dynamo; compound dynamo; effect of current in the motor armature; efficiency tests of a motor; efficiency tests of a dynamo; miscellaneous tests; motor generators and boosters.

PRESS OPINIONS.

"For completeness, conciseness, directness, accuracy, clearness of exposition, and intelligent selection of methods, there is nothing in print in the English language that compares with this little book in the slightest degree. It is worth ten times its price."—*American Electrician.*

"It is seldom the reviewer has had through his hands a book he can more cordially recommend for all-round excellence. The treatment of the subject is thorough and lucid, and the diagrams, and especially the numerous curves got from actual tests, are all excellently reproduced in a very clear manner."—*Electrical Review.*

"This is an excellent and practical little book. Mathematics are very little employed, the author instead using graphic methods of explanation. It contains a considerable amount of valuable information."—*The Engineer.*

PRICE 5s. 4d., POST FREE.

Scientific Publishing Co.,

53, New Bailey Street, Manchester

Practical Alternating Currents

AND

Alternating Current Testing.

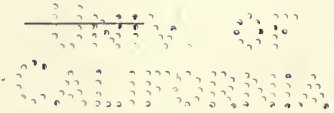
BY

CHARLES F. SMITH,

Assoc. M. Inst. C.E., A.M.I.E.E., Whit. Schol.

HALLIDIE LIBRARY
OF THE
UNIVERSITY
OF
CALIFORNIA

PRICE 6s. NET.



MANCHESTER:
THE SCIENTIFIC PUBLISHING COMPANY.
[ALL RIGHTS RESERVED.]

TK2711
S6

HALLID

PREFACE.

THIS volume deals in an elementary and practical manner with the principles of alternating currents and alternating-current machinery. It is hoped that it may prove of special use to students in technical schools and colleges in connection with their laboratory work, and to young engineers engaged in handling or testing alternating-current machinery.

The writer hopes that the book may also appeal to a wider range of readers who wish to gain some practical insight into alternating-current working. He believes that the experimental method of treatment is for this purpose the one best fitted to convey exact and concrete ideas, even to the student who is unable himself to carry out the measurements described. He has thus aimed at producing a text-book on the principles of alternating-current working, as well as a companion to the laboratory or test house. The experiments are described in great detail, so as to enable the conclusions derived from them to be followed by the reader who has not the means for obtaining experimental results of his own.

It will be seen that the use of the higher mathematics has been avoided, except in the final chapter on curve analysis, and, as far as possible, a treatment based on actual results and examples has been employed.

With regard to the selection of the experiments, some of the earlier ones have been given mainly for the sake of the principles involved, or as exercises in calculation. In the later part of the book, while the most general

144880

practical measurements are given in full, many other experiments are only alluded to, and left to the student to carry out with such modifications as his experience may suggest.

The experiments may be taken as a general outline for a course of practical work in the laboratory; but as the student makes progress he should learn to depart more and more from the exact form given in the book, and endeavour by different ways to familiarise himself with the behaviour of the machines he is handling. He must not forget that a large part of the "experiment" is actually to be done on paper after the readings have been taken.

The experimental results given in the various tables and curves were, with few exceptions, obtained expressly for the purpose of this book. The greater part of this arduous work was actually carried out by the writer's friend, Mr. U. A. Oschwald, B.A., without whose assistance the book could hardly have appeared in its present form.

The author wishes to record his indebtedness to Mr. Oschwald for this help and for assistance in the final revision.

He has also to thank his friend, Mr. W. H. F. Murdoch, B.Sc., for very kindly reading through the proofs, and for making numerous valuable suggestions. Mr. S. Irwin Crookes has kindly pointed out several errors, and suggested improvements in certain details.

TABLE OF CONTENTS.

CHAPTER I. PAGE.

ALTERNATING ELECTROMOTIVE FORCE AND CURRENT.. .. 1

General laws; production of electromotive force; relation between current and magnetic field; nature of electromotive force produced by an alternator; voltage and frequency of an alternator; electromotive force of self-induction; coefficient of self-induction: resultant electromotive force in a circuit; angle of lag; vector diagrams; resultant of two vectors; resultant voltage of two alternators; determination of angle of lag by three voltmeters.

CHAPTER II.

IMPEDANCE.. 34

Impedance diagram for circuit; determination of dependence of impedance upon resistance, frequency, air-gap, and strength of current; graphic calculation of impedance; impedance of resistance coils; determination of impedance of an armature.

CHAPTER III.

POWER AND POWER-FACTOR 56

Curves of current, volts, and watts; power-factor; determination of power-factor; energy and idle voltage and their calculation; function of choking coil, determination of power saved by its use; determination of voltage of choking coil required for an arc lamp; resultant of two currents not in phase, experimental determination; determination of idle and energy currents; measurement of power by 3-voltmeter and 3-ammeter methods.

CHAPTER IV.

PAGE.
VIRTUAL VALUE OF AN ALTERNATING CURRENT 84

Curve tracing by contact-maker; determination of wave form of an alternator; determination of simultaneous curves of current and voltage; comparison of values of alternating and direct current; R.M.S.value; graduation of instruments; example of Cardew voltmeter; average power.

CHAPTER V.

EFFECT OF CAPACITY 97

General explanation of effect of capacity in a circuit; relation between voltage and current; impedance of circuit containing capacity; determination of effect of condenser upon phase relations in a circuit; determination of variation of current with capacity, frequency, and voltage; measurement of power in circuit with capacity; circuit containing resistance, self-induction, and capacity; determination of power and phase relations in such a circuit; resonance; capacity and inductance in series and in parallel.

CHAPTER VI.

THE TRANSFORMER 125

General principles; ratio of voltages; effect of current in secondary winding; voltage drop in windings; determination of regulation; direct-reading method; measurement of ratio of transformation with direct current; no-load current of a transformer; determination of magnetising current; losses due to defective insulation; losses at various frequencies; iron losses; their separation; separate determination of copper losses; magnetic leakage; effect of power-factor of secondary circuit; efficiency test on non-inductive and inductive loads; indirect methods of measuring efficiency; graphic determination of drop at any power-factor; efficiency by double conversion; split dynamometer method; Sumpner's method of testing; makers' tests; the auto-transformer; transformer diagrams for various loads and power-factors; curve tracing of primary and secondary volts.

CHAPTER VII.

PAGE.
ALTERNATORS 191

Generation of electromotive force in the armature; voltage formula; the magnetic circuit; determination of magnetisation curve at no-load and under load; calculation of armature self-induction; magnetisation curves on inductive loads; armature reactions; characteristic curve; cause of voltage drop; effect of armature currents on field; determination of variation of field current; characteristics on inductive and capacity load circuits; calculation of characteristics; phase difference between total volts and current; determination of excitation to maintain constant volts; short-circuit characteristic; graphic determination of voltage at any power-factor; relation between speed and voltage, on no-load, at full load, and on short-circuit; losses in an alternator; their determination; retardation method of measuring iron losses; alternators in parallel; synchronisers; method of synchronising.

CHAPTER VIII.

SYNCHRONOUS MOTORS 266

Principle of operation; method of starting; determination of effect of variation of excitation; discussion of experimental curves; variation of load with constant excitation; graphic representation of conditions.

CHAPTER IX.

THE POLYPHASE CIRCUIT.. 294

Generation of 2 and 3-phase currents; 3-phase transmission by three wires; star and mesh connection; current and volt relations; determination of power in non-inductive and inductive 3-phase circuits by various methods; measurement in unbalanced circuits.

CHAPTER X.

PAGE.
THE ROTARY CONVERTER 313

General principles; losses in a converter; ratio of conversion; polyphase converters; open circuit characteristic; load characteristic when run from direct-current side; calculation of voltage drop; determination of effect of variation of excitation; characteristic when driven from alternating-current side; effect of variation in excitation; compounding of rotary converters.

CHAPTER XI.

THE INDUCTION MOTOR 341

Production of a rotating field; production of current in the rotor winding; magnetic reaction of rotor current; heating of rotor and turning moment; value of torque; relation between slip and torque in ideal case; effect of magnetic leakage; measurement of slip; measurement of resistance of windings; speed regulation; braking and return of current to line; static and running no-load curves; ratio of primary to secondary current; load tests; variation of primary current, power absorbed, efficiency, power-factor, and slip; effect of variation of rotor resistance; determination of copper and iron losses; efficiency from no-load readings; single-phase induction motors; starting devices for single-phase motors; induction motor diagrams; the Heyland diagram; experimental determination of Heyland diagram.

CHAPTER XII.

THE ANALYSIS OF ALTERNATOR CURVES 420

Composition of curves; form of the wave; analysis by summation: outline of proof; example of calculation; analysis by superposition: outline of proof; example of calculation.

INDEX .. .. AT END

LIST OF EXPERIMENTS.

		PAGE.
1.	Determination of the Resultant Electromotive Force of Two Alternators Coupled Together	25
2.	Determination of Angle of Lag in an Alternating Circuit by Three Voltmeters	29
3.	Determination of Dependence of Impedance upon Resistance	36
4.	Dependence of Impedance upon Frequency	39
5.	Determination of Dependence of Impedance upon Self-induction	42
6.	Determination of Dependence of Impedance upon Self-Induction	44
7.	Determination of the Impedance of the Armature of an Alternator while Stationary	52
8.	Determination of the Power-Factor of a Circuit	61
9.	Determination of Power Lost in a Choking Coil and Power Saved by Its Use	66
10.	Determination of Voltage of Choking Coil Required by an Arc Lamp	68
11.	Determination of Relation Between Currents in Branch and Main Circuits	72
12.	Determination of Idle and Energy Current in a Circuit	75
13.	Measurement of Power by the 3-Voltmeter Method	78
14.	Measurement of Power by the 3-Ammeter Method	81
15.	Determination of Wave-form of an Alternator	85
16.	Determination of Simultaneous Curves of Current and Voltage in a Circuit	87
17.	Determination of the Effect of a Condenser upon the Phase Relations in a Circuit	103
18.	Determination of Dependence of Current in a Circuit upon Capacity	106
19.	Determination of Dependence of Charging Current upon Frequency	109
20.	Determination of Effect of Terminal Voltage upon Charging Current of a Condenser	111

PAGE.

21. Determination of Power and Phase Relations in a Circuit Containing Resistance, Self-Induction, and Capacity 116
22. Determination of Regulation of a Transformer 130
23. Determination of Drop in a Transformer (Direct-Reading Method) 135
24. Determination of Ratio of Windings of a Transformer (Without Alternating Current) 138
25. Determination of Magnetising Current of a Transformer at Various Voltages 141
26. Determination of No-Load Current and Watts with Varying Frequency 146
27. Separation of Losses in a Transformer 149
28. Determination of Copper Losses in a Transformer .. 152
29. Efficiency Test of a Transformer on Non-Inductive Load 157
30. Efficiency Test of a Transformer on Inductive Load .. 160
31. Determination of Efficiency of a Transformer by Split Dynamometer 169
32. Determination of Efficiency of a Pair of Transformers by Sumpner's Method 172
33. Test of Uniformity of Ratio of Transformation in a Series of Similar Transformers 178
34. Determination of Magnetisation Curve of an Alternator at No-Load 196
35. Determination of Magnetisation Curve of an Alternator at Full Non-Inductive Load 201
36. Determination of the Characteristic of an Alternator .. 214
37. Determination of Excitation Required to Maintain Constant Voltage 237
38. Determination of Short-Circuit Magnetisation Curve, or Short-Circuit Characteristic 241
39. Determination of Relation Between Speed and Voltage of an Alternator at No-Load, at Full Load, and on Short-Circuit 248
40. To Synchronise an Alternator with Another Already Running 263
41. To Start a Synchronous Motor and Ascertain the Limits of Excitation for which it will start 269
42. Determination of the Effect of Variation of the Excitation of a Synchronous Motor 271

PAGE.

43. Determination of Effect of Variation of Load upon a Synchronous Motor Having Constant Excitation 283

44. Determination of the Power Transmitted by a 3-Phase Circuit (Load Non-Inductive) 303

45. Determination of Power and Power-Factor in a Balanced 3-Phase Line 309

46. Determination of Characteristic and Efficiency of a Rotary Converter Run from the Direct-Current Side 321

47. Determination of Effect of Variation of Excitation upon Ratio of Transformation of a Rotary Converter Driven from Direct-Current Side 327

48. Determination of Characteristic and Efficiency of a Rotary Converter Run from Alternating-Current Side 330

49. Determination of Effect of Variation of Excitation upon Ratio of Transformation of a Rotary Converter Driven from Alternating-Current Side 335

50. No-Load Tests of Induction Motor at Varying Voltage 370

51. Load Test of an Induction Motor 378

52. Determination of Effect of Variation of Rotor Resistance 393

53. Experimental Derivation of the Heyland Diagram .. 411

HALLIDIE LIBRARY OF THE UNIVERSITY OF CALIFORNIA

PRACTICAL ALTERNATING CURRENTS AND ALTERNATING CURRENT TESTING.

Chapter I.

Alternating Electromotive Force and Current.

General Laws.—Two experimental laws may be said to form the foundation of the actions which take place in electrical generators and motors, whether for alternating or continuous currents.

Law of the Generator.—When a conductor moves in such a way as to cut magnetic lines of force, there will be an electromotive force generated in the conductor.

Law of the Motor.—When a current flows along a conductor, it produces a magnetic field round the conductor which is capable of acting upon, or being acted upon by, other magnetic fields.

In order to make clear the application of these laws to the case of alternating-current machinery it will be advisable to consider them more fully.

Production of Electromotive Force.—The strength of a magnetic field is expressed numerically by the pull, measured in dynes*, which would be experienced by a unit magnetic pole † if situated in the field. A magnetic field is usually spoken of as consisting of a number of lines of force. The density of these mathematical lines is, for convenience, so

* A dyne is the unit of force in the absolute or C. G. S. system of units. It is the force necessary to give unit acceleration (1 cm. per second per second) to unit mass (1 gramme). A force of 1 dyne is roughly equal to the weight of 1 milligramme.

† Unit magnetic pole is defined to be a pole which, if placed 1 cm. from an equal and similar pole, would be repelled with a force of 1 dyne, the medium being air.

chosen that the number of lines which pass through a square centimeter of a surface which is at right angles to their direction is also the number representing the strength of the field. Thus the number of lines of force per square centimeter gives the force which would act on a unit magnetic pole at the point considered.

Whenever an electric conductor moves in such a magnetic field so as to cut the lines of force, an electromotive force, or E.M.F., is produced‡ in the conductor. The numerical value (in C.G.S., or " absolute " units) of this electromotive force is the number of lines cut through by the conductor in 1 sec.

Thus

E.M.F. = Rate of cutting magnetic lines.

Expressed in volts (the " practical " unit of electromotive force), the electromotive force will be the above number of units divided by 10^8 (since one volt is equal to 10^8 absolute units of electromotive force), so that

E.M.F. (in volts) = Rate of cutting magnetic lines $\times 10^{-8}$.

If the conductor moves in a direction parallel to the lines of force it cannot be said to cut them. In such a case no electromotive force results.

If there are a number of conductors, instead of one only, in each of which is induced an electromotive force, these electromotive forces will produce a resultant electromotive force if the conductors are connected together. This will be equal to their sum, if the conductors are connected together in such a manner that the electromotive forces are all directed towards the same end of the composite conductor, which they then form.

The direction in which this electromotive force acts depends upon the relative direction of the lines of force and the movement of the conductor. If the conductor moves at right angles to the lines of force there will be three directions mutually perpendicular to each other, viz. : (1) The direction of the lines of force (from the north to the south pole), (2) the direction of motion of the conductor, and (3) the direction in which the electromotive force is induced, *i.e.*, the direction along the conductor in which it tends to produce a current

‡ This electromotive force must be considered to be an experimental fact. The relation existing between the value of the electromotive force and the rate at which the conductor cuts the lines is a result of the definition given to the unit of electromotive force.

(from + to −). The relation between these three quantities is most conveniently remembered by Fleming's rule, as illustrated in the annexed Fig. 1.

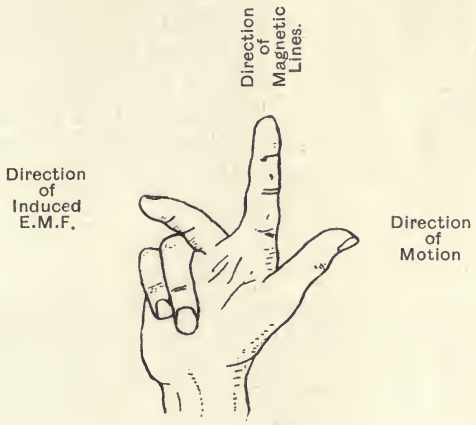


FIG. 1.—FLEMING'S RULE FOR A GENERATOR.

Using the right hand, and holding the thumb and first two fingers so that each is perpendicular to the plane containing the other two, the three directions are given as follows :—

Thumb—Direction of motion of conductor.
First finger—Direction of lines of force.
Second finger—Direction of electromotive force* induced.

The Magnetic Effects of a Current. — A current flowing in a straight conductor gives rise to a field composed of circular lines of force, the plane of the circles being perpendicular to the conductor, and their common centre being the centre of the conductor. The direction of these lines of force is related to the direction of the current in the manner indicated in the following rule.

Place the thumb of the right hand along the conductor so as to point in the direction of flow of the current; if the fingers

* The following mnemonics may help the student in remembering Fleming's rule :—

Thu *M* b.....*M* otion.
F irst finger .....*F* orce.
Se *C* ond finger.....*C* urrent.

then grasp the conductor they will point in the direction in which a north pole would be urged by the magnetic field round the conductor.

In machines, conductors are generally grouped together in windings forming coils or solenoids. In such cases the same relative direction of current and magnetic field must exist, but usually in these cases it is the field formed *within* the coil which is the most important, and the rule for easily determining this direction may be modified as follows :—

Place the fingers of the right hand so as to follow the direction of the current round the coil, and the extended thumb will indicate the end of the solenoid which will be a north pole.

Action of a Current on a Magnetic Field.—As already stated, a conductor moved across lines of force has an electromotive force induced in it tending to produce a current. Conversely, if a conductor situated in a magnetic field carries a current it will tend to move across the lines of force. In other words, the field will exert a force upon a conductor carrying a current. The direction of the force is such that if the conductor is free to move, it will do so in a direction perpendicular to the lines of force, so as to cut them in the way necessary to produce an electromotive force opposing the current. Thus a " back electromotive force " is set up in any conductor moving under the mutual action of its current and a magnetic field, opposing the current which causes the motion. In this case, which is that of the conductors in the armature of a motor, the relation between the direction of motion, lines of force, and current is given by Fleming's rule, as previously stated (see Fig. 1), except that the *left* hand must be used instead of the right, the fingers indicating the direction of the same quantities as before.

The magnitude of the force between a conductor carrying current and the field in which it is situated is given by the following formula, absolute C.G.S. units being employed

$$\underset{\text{(in dynes)}}{\text{Force acting on conductor}} = \underset{\text{(in absolute units)}}{\text{Current}} \times \underset{\text{(in cm.)}}{\text{Length of conductor}} \times \underset{\text{(in lines per cm.}^2\text{)}}{\text{Strength of field.}}$$

In this form the statement is only true when the conductor is at right angles to the lines of force, and the direction of

the force is then perpendicular to the plane containing the conductor and the lines forming the field.

If the conductor is inclined at an angle θ to the lines of force, the pull = length × current × field strength × $\sin \theta$.

When converted into more convenient units, the force is given by the following equations :—

$$f \text{ (kilogrammes)} = \frac{\text{C. l. H.}}{9{,}810{,}000}$$

C = current in amps.
l = length of conductor in cm.
H = lines per sq. cm.

$$f \text{ (lbs.)} = \frac{C.\, l''.\, H.}{72{,}904{,}350}$$

l'' in inches.
H in lines per sq. cm.

$$f \text{ (lbs.)} = \frac{C.\, l''.\, H''.}{11{,}303{,}000}$$

l'' in inches.
H'' in lines per sq. inch.

Production of an Alternating Electromotive Force.—An electrical generator is a machine in which conductors are made to cut lines of force in order that they may have an electromotive force generated in them. The most convenient way of causing conductors to cut lines of force is to make either the conductors or the magnetic field rotate. The conductors usually pass alternately across poles of north and south polarity, and in doing so the direction of the electromotive force induced is reversed as the conductor passes from pole to pole. Consequently, in both direct and alternating generators the electromotive force produced in the armature conductors (and also the current) is an *alternating* one. The difference between a direct and alternating current generator lies in the method of collecting the current from the armature conductors, rather than in the nature of the currents induced. In the direct-current machine the current is rectified at the commutator, while in the alternator the current in the external circuit has the same direction and variations as it has in the armature conductors.

Nature of Alternating Electromotive Force.—In order to study the manner in which the alternating electromotive force in a generator varies, we may take the case of a single conductor travelling in a circular path at a uniform speed in a uniform field between a pair of opposite poles. (See Fig. 2.)

The electromotive force generated in the conductor is numerically equal to the rate at which it cuts the lines of force divided by 10^8. Evidently this rate varies with the position of the conductor. Referring to Fig. 2, the electro-

motive force generated in the conductor P will be zero at the moment when the conductor moves horizontally at the top and bottom of its path, since in this position it moves *along* and not across the lines. The maximum rate of cutting lines occurs when the conductor moves vertically, *i.e.*, perpendicularly to the lines. Any movement horizontally or parallel with the lines has no effect in producing electromotive force, whereas the vertical velocity determines the

VARIATION IN E.M.F. INDUCED IN ROTATING CONDUCTOR

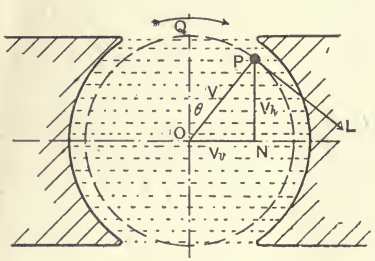


FIG. 2.

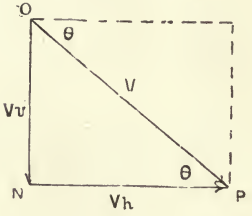


FIG. 3.

amount of electromotive force generated. It therefore becomes important to consider separately the vertical velocity of the conductor, since this is the factor affecting the voltage generated.

Now a velocity, when represented graphically in magnitude and direction by a straight line, may be resolved into two components in any direction represented by the sides of a parallelogram of which the line representing the velocity forms the diagonal. Thus if $O\ P$ (Fig. 3) represents to scale the velocity of the conductor, $N\ P$ and $O\ N$ will represent to the same scale the horizontal and vertical velocities of the conductor when travelling in the direction $O\ P$.

Thus the sides of the triangle $O\ P\ N$ represent to the same scale the velocity of the conductor P and its vertical and horizontal components respectively,

$O\ P$ representing the actual velocity,
$O\ N$ representing the vertical velocity.
$N\ P$ representing the horizontal velocity.

Now the triangle $O\ P\ N$ in Fig. 2 is similar to the triangle similarly lettered in Fig. 3, but is turned through a right angle. Thus, if in Fig. 2 the line $O\ P$ is taken to represent in magnitude

the velocity of the conductor in a direction at right angles to $O\ P$, $O\ N$ will represent the vertical velocity, and $N\ P$ will represent its horizontal velocity in magnitude. This will be true whatever position the conductor P may be in.

If it is known that when the conductor is moving vertically the electromotive force induced in it is V volts, then the voltage induced when the conductor is moving in any other direction, such as the direction $P\ L$ (Fig. 2), is less than V, since the vertical velocity of the conductor is less, and its actual value will be given by

$$\text{Volts} = V \times \frac{\text{vertical velocity at time considered}}{\text{vertical velocity when conductor moves vertically}}$$

$$= V \times \frac{O\ N}{O\ P} = V \sin \theta,$$

if θ is the angle $P\ O\ Q$ through which the conductor has moved from the vertical axis.

Thus the variations of the induced electromotive force are the same as those of the sine of an angle which passes through all values from 0° to 360°.

The values of the electromotive force may be readily plotted graphically from Fig. 2, since the length of $O\ N$ may always be taken to represent the electromotive force generated on the same scale of volts upon which the radius $O\ P$ represents the maximum voltage. This has been done in Fig. 4, where the path of the conductor P is indicated by a circle, and for each value of the angle $Q\ O\ P$, measured horizontally in degrees, the length $O\ N$ is set up vertically to represent the value of the electromotive force for that position of the conductor, a curve being drawn through the points thus obtained.

It will be seen that the curve obtained in this way is a sine curve. The change of direction of the electromotive force, which occurs when the conductor begins to cut the lines in the reverse direction, is shown by the curve crossing the zero line. Ordinates above the line indicate electromotive forces in one direction along the conductor. Ordinates below the line represent those in an opposite direction. For the sake of uniformity it is usual to call electromotive forces measured below the line *Negative*.

The curve shown in Fig. 4 may be taken to represent the variations of voltage given by an alternator. For example, the maximum voltage on the vertical scale of the diagram is 25. It would be possible to obtain from the curve the exact

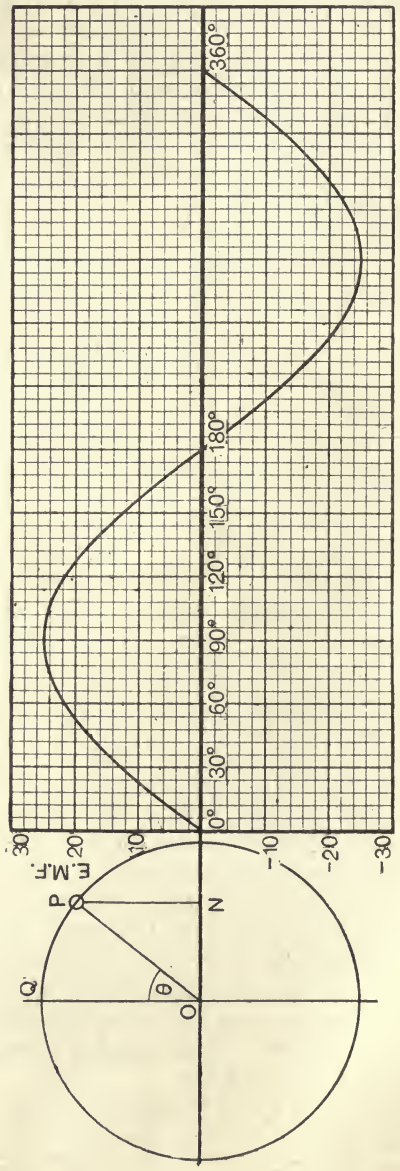


FIG. 4.—GRAPHIC REPRESENTATION OF VARIATION OF VOLTAGE.

voltage given out by the machine for any position of the armature if the position of maximum or minimum voltage is known.

The complete curve shown for a variation from 0° to 360° corresponds to a rotation of the armature or field of the alternator equal to the angular distance between the centre of one N pole and the centre of the next N pole. In a 2-pole machine this is one complete revolution.

Definitions.—The time taken by the electromotive force to pass through the complete series of changes represented in Fig. 4 is called a *Period*. The *Periodicity* of an alternating circuit is the number of periods in one second. Thus an alternator giving a periodicity of 50 would impress on the circuit the series of changes shown in the figure, 50 times per second. *Frequency* is another term for periodicity.

The *Amplitude* is the maximum value of the variable quantity which passes through the changes shown above. Thus the length $O\ P$ is the amplitude in Fig. 4, corresponding to 25 volts.

The angle measured from the starting point Q to any particular point P on the curve is called the *phase* of this point; thus in Fig. 4 the phase of P is measured by the angle $Q\ O\ P$.

Example.—An alternator gives a maximum of 100 volts. If its frequency is 50, what will be the phase and value of its voltage after $3\frac{2}{3}$ seconds starting from the point of zero electromotive force? In three seconds it will have passed through 360° 150 times, and in $\frac{2}{3}$ sec. it will further pass through $\frac{2}{3} \times 50$ of 360°, *i.e.*, $33\frac{1}{3}$ times 360°. Hence at the end of $3\frac{2}{3}$ secs. it will have passed through the complete cycle of changes 183 times, and its phase will be $\frac{360}{3}$ or 120°. The value of the electromotive force will then be $V \sin \theta$, or

$$100 \sin 120° = 100 \times \cdot 866 = 86 \cdot 6 \text{ volts.}$$

Effect of Increase in Number of Poles.—If instead of two poles only, the machine has a greater number, the rotating armature might be provided with as many conductors as the alternator has poles. By coupling them together suitably the electromotive forces generated in them may be directed along each successive conductor in the same sense from end to end of the winding. The conductors must all be situated symmetrically with respect to the poles, in order that they may

go through the cycle of changes simultaneously. If the conductors are thus connected in *series*, the resultant voltage due to the whole winding will vary in the same way as the voltage in each single conductor. Consequently the value of the total voltage will be the sum of the individual voltages at any instant.

In the case of a multipolar machine a period will be the time during which a conductor moves from one pole to a similar position under the next pole of the same polarity. Thus in a 12-pole alternator there will be six periods during each revolution of the armature. In general, if

r = revs. per minute of armature, or field,
n = periodicity,
P = number of pairs of poles,

$$n = r \times P \div 60.$$

Hence large machines running at comparatively low speeds invariably have a large number of poles in order to supply current of the requisite frequency.

In an electrical distribution system the periodicity is a fixed quantity, and the number of poles and speed of the generator have to be adjusted to suit this condition.

Increase in Number of Conductors.—In order to increase the voltage generated in the armature, the number of conductors actually employed is much greater than was assumed above. The total voltage generated in an armature composed of a great number of conductors will only be equal to the sum of the voltages induced in the individual conductors if the following conditions are fulfilled: (1) Conductors under opposite poles must be joined together alternately at the front and back of the armature, so that the electromotive force which is induced in successive conductors in opposite directions is made to act in the same sequence in the complete winding. (2) The conductors passing under a single pole at any time must be grouped together sufficiently closely to enter and leave the pole nearly simultaneously.*

The connections of such a winding are shown diagramatically in Fig. 5.

Production of Current by an Alternator.—If the armature winding of an alternator is connected to an external circuit,

* The effect of a distributed winding is discussed later.

the electromotive force generated in the armature will send a current through the circuit. The strength of this current will depend on the ratio between the armature voltage and

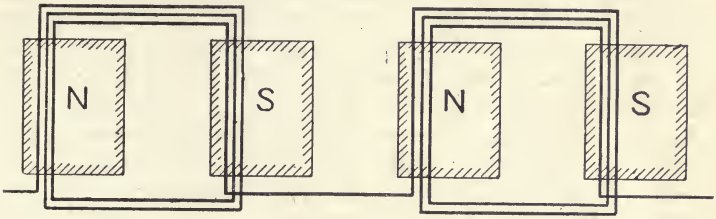


FIG. 5.—CONNECTIONS OF CONDUCTORS IN MULTIPOLAR ARMATURE.

the resistance of the circuit. Since the armature voltage is a constantly varying quantity, the current in the circuit will undergo the same variations.

Thus at any instant † $C = \frac{E}{R}$

Where C value of current,
E ,, ,, voltage,
R resistance of circuit.

Since the current will go through similar variations to the voltage, it may be represented by a curve in the same way as the voltage in Fig. 4.

E.M.F. of Self-induction in a Winding.—As in the case of a continuous current flowing through a winding, a magnetic field is produced by an alternating current. In the latter case, however, an important difference is introduced owing to the fact that the field must vary as rapidly as the current. Consequently, an alternating current is always accompanied by a rapidly *changing* field, instead of the steady one which is produced by a direct current. The effect of the varying field upon the circuit by which it is produced must next be considered.

Imagine a coil of wire through which an alternating current is flowing.

The number of lines of force which are produced within the coil will be proportional to the amount of current flowing; consequently every variation in current produces a proportionate change in the number of lines passing through the coil. The formation of lines of force passing through

† The effect of self-induction is, of course, not considered in this example.

a coil is equivalent to the movement of that number of lines from the outside to the inside of the coil. That is to say, the formation of 1,000 lines of force passing through the coil will produce the same effect as if the movement of a magnet caused 1,000 lines of force to be cut by the conductor.

As explained on page 2, an electromotive force is produced in a conductor when it is made to cut lines of force. Hence the change of field in a coil produced by a change of the current in it will also produce an electromotive force. This electromotive force is called the *electromotive force of self-induction.* The magnitude of the electromotive force is governed by the same rule as already given on page 2 for the electromotive force induced by motion in a magnetic field. The voltage induced in each conductor of the coil is numerically equal to the number of lines cut per second divided by 10^8. The direction of the electromotive force thus produced is always such as to oppose the change of the current to which the electromotive force is due.

Thus a current started in a conductor will produce a magnetic field, which will in turn produce an electromotive force opposite to the direction of the current. Similarly, the stoppage of a current will cause an electromotive force, owing to the disappearance of magnetic lines, in the direction which tends to maintain the flow of current. Since an alternating current is constantly changing both its magnitude and direction, there will be electromotive forces constantly induced, and always tending to oppose the change of current occurring at the time. Because the induced electromotive force always opposes the changes occurring in the current, it is often called the " back electromotive force " or " counter electromotive force " due to self-induction. These electromotive forces will be proportional to the rate at which the magnetic lines change, and hence proportional also to the rate at which the current changes, and not directly to the magnitude of the current or to the number of lines of force in the circuit.

In order to make clear this relation between the electromotive force of self-induction and the current in a circuit, it will be best to draw two curves representing respectively the successive values of the current and the corresponding values of the rate at which the current changes. This is shown in Fig. 6 where curve I. shows the values of a current whose maximum value is 12·5 amperes, and the dotted

curve No. II. shows the change of current measured in amperes per cycle.

The second curve is obtained from the first by drawing at a number of points tangents to curve No. I., as shown at P, a horizontal line P N corresponding in length to $\frac{1}{4}$ cycle, and a vertical line through N to meet the tangent at M. The length N M then shows the change in current which would actually occur in $\frac{1}{4}$ cycle if the rate of increase or decrease remained the same as it actually is at P.

Four times the length of M N will consequently give the height of the "rate of change" curve at the point corresponding to the same angle as P. The scale for the dotted curve (Fig. 6) is taken four times as great as for the original curve, so that when the length N M is plotted vertically it corresponds to four times the change of current in $\frac{1}{4}$-cycle, *i.e.*, to the rate of change of current for a whole period.

When N M is drawn downwards, it is plotted below the base-line in curve II., and vice versâ.

It will be noticed that the curves I. and II. in Fig. 6 are similar in shape, but that the second curve is displaced by one-quarter of a period to the left of the curve I. From this it is apparent that the *change* of current is most rapid at the point where the current curve crosses the axis and changes its direction. In other words, the *rate of change* is greatest when the *current* is zero. The rate of change is zero, on the other hand, at the moment when the current has attained its maximum value in either direction.

The maximum value of the rate of change of current curve is seen to correspond to a change of 78·5 amperes per cycle, *i.e.*, the rate of change of current per cycle is 6·3 times the maximum value of the current. More accurately, the rate of change per cycle is 2π times the maximum current. Since the current passes through n cycles per second the maximum rate of change of current per second is $2\pi n \times$ the maximum value of the current.*

* The reason for this exact relation between current and rate of change of current is that the rate of change of the sine of an angle is equal to its cosine when the angle is measured in units of circular measure or radians. Since there are 2π radians in 360°, this corresponds to a rate of change of $2\pi \times$ the cosine, measured in rate of change per cycle. The maximum value of the cosine is equal to the maximum value of the sine, and the fact that $\cos\theta = \sin(\theta + 90°)$ is shown by the relative displacement of the curves by 90°. If points on curve I. are represented by $C \sin\theta$, the corresponding points on curve II. will be given by the expression $2\pi C \cos\theta$.

Consequently a curve showing rate of change of current *per second* would be a curve similar to curve II., but having each ordinate n times its value.

Since the electromotive force of self-induction is proportional to the rate of change of lines caused by the current, and

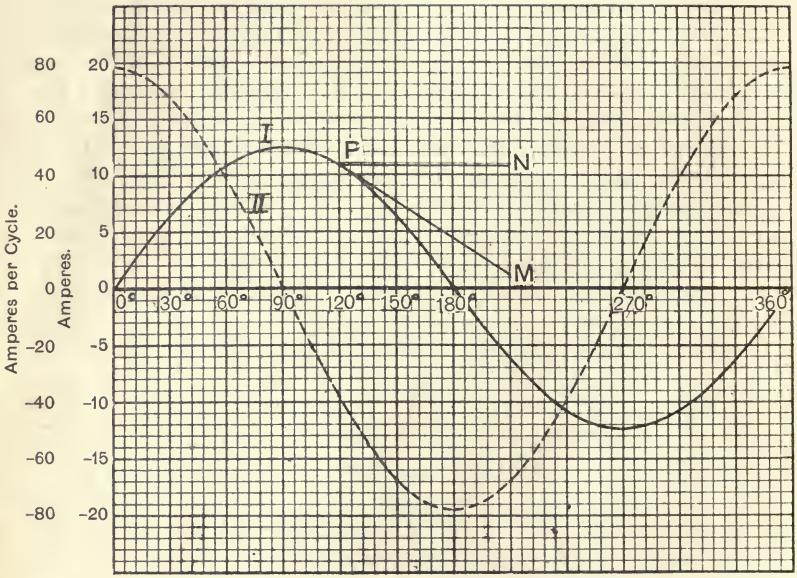


FIG. 6.—CURVES OF CURRENT AND RATE OF CHANGE OF CURRENT.

since these lines are necessarily proportional to the current itself, curve I. will show the manner in which the number of lines in the circuit fluctuates, and curve II. will represent the variations undergone by the electromotive force.

The actual value of this electromotive force must now be investigated.

Coefficient of Self-induction.—Let c = value of current in the coil at any moment.

N = number of turns composing the coil.

Z = number of lines of force produced in the coil when c = 1 ampere.

Then, when the current changes at the rate of 1 ampere per second, the change produced in the number of lines

entering the coil will be Z per second. The total voltage induced in the coil by this change will be

$$\frac{\text{lines cut per second by each coil} \times \text{number of coils}}{10^8}$$

or in symbols—

$$\text{Voltage} = E = \frac{Z.N.}{10^8}$$

The fraction $\frac{Z.N.}{10^8}$ is called the coefficient of self-induction of the coil, and is generally denoted by L.

Definition.—The coefficient of self-induction of a coil is numerically equal to the electromotive force induced in it by a change of current of 1 ampere per second.

or

The coefficient of self-induction is numerically equal to the number of magnetic lines formed in a coil by a current of 1 ampere multiplied by the number of turns of the coil through which these lines are threaded, divided by 10^8.

This coefficient is termed the *henry*.

The self-induction of a circuit is frequently called the *inductance* of the circuit.

A milli-henry $= \frac{1}{1{,}000}$ henry, and is generally used in stating the properties of a circuit, as it is unusual for a circuit to have an inductance greater than a small part of one henry.

We may thus write the expression for the electromotive force of self-induction when the rate of change of current is 1 ampere per second—

$$E = L.$$

If the current changes at the rate of $\dot{c}$ amperes per second, then the induced voltage will be

$$E = \dot{c}\, L.$$

Since the dotted curve in Fig. 6 shows the rate of change of current per cycle, it would give the rate of change per second if its ordinates were multiplied by n. It would then be the curve of $\dot{c}$.

Hence we might obtain a curve showing the fluctuations of the electromotive force of self-induction by drawing a curve in which each ordinate of Curve II. (Fig. 6) is multiplied by nL. This curve must, of course, be plotted

to a scale of volts vertically, the horizontal scale being the same as for the other curves.

Since the maximum rate of change of current per second = maximum value of $\dot{c} = 2\pi n C$, where C = maximum value of current, we may write as the maximum value of the electromotive force of self-induction

$$E = 2\pi n C L.$$

From what has just been stated, it appears that when an alternating current flows in a circuit, it will give rise to an alternating electromotive force opposing the change of current at every moment. Also, if the curve I in Fig. 6 is taken to represent the values of such a current, a curve similar to curve II. (but drawn to a scale n times as great) will show the values of the voltage which must be applied to the circuit in order to overcome the back electromotive force of self-induction caused by the changing field which the change of current produces.

The curve of voltage is $\frac{1}{4}$ period *in advance* of the current curve in phase, as shown by the fact that the voltage curve reaches its maximum and minimum values always 90° earlier than the curve of current.

We are now in a position to apply the results shown in the curves in Fig. 6 to explain the conditions governing the production of a current.

According to Ohm's Law for an electric circuit, in order to maintain a current, an electromotive force must be applied to the circuit whose value at any instant is equal to the product of the current multiplied by the resistance of the circuit through which it flows. This will evidently be an electromotive force which varies with the current, and is *in phase* with it, *i.e.*, passes through its maximum and minimum values at the same time as the current.

In Fig. 7 is represented a current having a maximum value of 25 amperes (Curve C). This current is supposed to be flowing in a circuit having a resistance of 1·2 ohm. The dotted curve E_r represents the electromotive force which has to be overcome owing to this resistance, and is obtained by multiplying each ordinate of the current curve by 1·2. The maximum value of this electromotive force is $25 \times 1{\cdot}2 = 30$ volts. In this case current and volts can be represented to the same scale, although this would not be convenient if

the number of volts and the number of amperes were very different.

The circuit in which the current flows has a self-induction of 5 milli-henries = ·005 henries.

The change of current will produce an electromotive force of self-induction in the circuit which must also be overcome by the alternator supplying the circuit in order that the current c may be maintained. This electromotive force is an alternating electromotive force $\frac{1}{4}$ period in advance of the current. The curve representing it might be obtained by drawing a curve similar to Curve II. in Fig. 6, and then

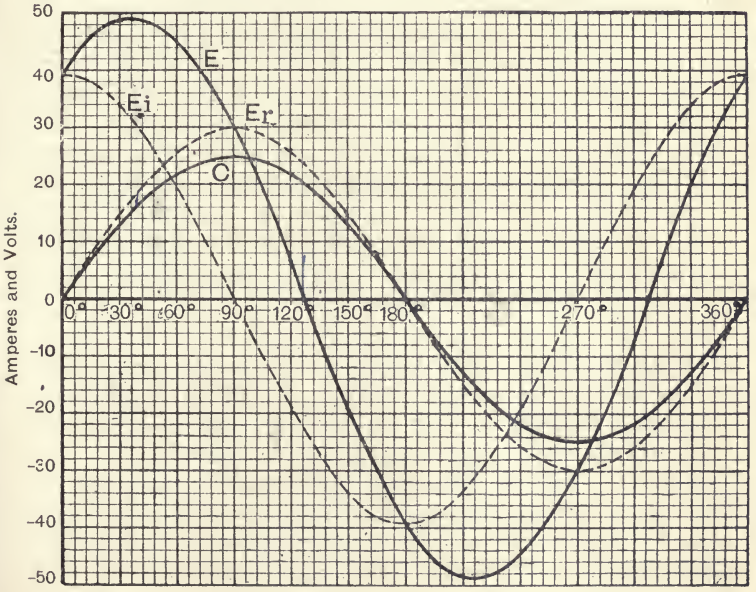


FIG. 7.—CURVES OF CURRENT AND COMPONENT AND TOTAL E.M.F.

multiplying each ordinate by $n\ L$, a more direct way is the following. It has already been pointed out that the maximum value of the electromotive force of self-induction $= 2\pi n C L$, when C is the maximum value of the current, and that the curve representing this electromotive force is $\frac{1}{4}$ period in advance of the current in phase. Consequently, we may directly draw a sine curve having a maximum value of $2\pi n L C$ and $\frac{1}{4}$ period in advance of the curve of current.

This has been done in Fig. 7, and is shown as the dotted curve E_i whose maximum value is $2\pi nLC = 2\pi \times 50 \times \cdot 005 \times 25 = 39 \cdot 3$ volts, where n = periodicity = 50 cycles per second.

The curves drawn in Fig. 7 represent completely the conditions which usually exist in an alternating circuit. These may be briefly recapitulated as follows :—

A current c (Curve C) to produce which requires the application of an electromotive force varying in the same manner as, and simultaneously with, the current in order to overcome the resistance of the circuit. This electromotive force (Curve E_r) is always numerically equal to the product of $c \times R$ where R is the resistance of the circuit.

Further, the variation of the current calls forth an electromotive force opposing this variation (Curve *Ei*). This electromotive force is numerically equal to $\dot{c}L$, where $\dot{c}$ is the rate at which the current varies (measured in amperes per second), and L is the coefficient of self-induction of the circuit. The curve representing voltage overcoming the latter electromotive force is similar in character to the curve of current, but a quarter of a period earlier in phase.

The ratio $\frac{L}{R}$ is sometimes called the *time constant* of the circuit.

Resultant Electromotive Force.—In the previous paragraph it was explained that two electromotive forces are required to maintain an alternating current, viz., one electromotive force in phase with the current to produce the *passage* of the current. This may be called the resistance electromotive force, and is the electromotive force which would be required in a continuous-current circuit to maintain the flow of the current.

A second electromotive force produces the *variation* of the current, and may be called the inductance electromotive force. This is only required in a circuit in which the current changes, and constitutes the main difference between alternating and continuous current problems. The inductance electromotive force is a quarter of a period earlier in phase than the resistance electromotive force, that is, the curve is displaced to the left in the diagrams.

An alternator must supply to a circuit sufficient voltage to be equivalent to both of these electromotive forces if

an alternating current is to be maintained. The actual voltage supplied by the alternator at any instant must be equal to the sum of the two individual voltages. The variation of the voltage of the alternator may consequently be plotted as a curve, the height of which at each point is equal to the sum of the ordinates of the voltages overcoming the two opposing electromotive forces. This curve has been obtained in Fig. 7 from the resistance electromotive force and inductance electromotive force, and the resultant single voltage of the alternator (Curve E) has at each point a height equal to the sum of the heights of these curves.

It is to be observed that (1) this curve is again similar in character to the previous curves; (2) its maximum value (49 volts) is greater than that of either of the component electromotive forces, but is less than the sum of the two; (3) that the resultant electromotive forces is intermediate in phase between the two component curves.

Angle of Lag.—A further most important result is to be noted from an inspection of the curves just given, viz., that the curves representing the current and the resultant voltage (*i.e.*, the current and total voltage in a circuit) are not in phase with each other. In the curves given in Fig. 7, it will be seen that the curves of current and total voltage pass through zero at points situated ·145 period or $52\frac{1}{2}°$ apart.† The angle between these curves, which is most conveniently measured as the angle on the horizontal scale of the curve between the points at which they pass through their zero value, is the *angle of phase difference* between current and voltage. This angle is called the *angle of lag* of the circuit if the current passes through zero *after* the voltage, or the *angle of lead* if the current passes through zero *before* the voltage. The current always lags behind the voltage, except in cases where the circuit possesses electrostatic capacity or supplies over-excited synchronous motors—conditions discussed later on.

The angle of lag of a circuit is of great practical importance, and depends upon the nature of the resistances and other apparatus forming the circuit.

The student cannot be too careful to familiarise himself with the reasons for the fact that in all alternating circuits possessing self-induction there is a difference of phase between current and voltage.

† A complete period is always taken as corresponding to 360°, or an angle of 2π in circular measure.

He should carefully work the following examples, and, if necessary, read again the foregoing pages.

Example.—A circuit has a resistance of ·5 ohm., and a self-induction of 20 milli-henries. It carries an alternating current whose maximum value is 24 amperes. Plot curves on squared paper to represent the following quantities :—

(*a*) The current. (*b*) The rate of change of current. On a separate sheet plot the following electromotive forces making use of the curves (*a*) and (*b*) : (*c*) Electromotive force necessary to overcome resistance. (*d*) Electromotive force necessary to overcome self-induction. From (*c*) and (*d*) obtain the curve (*e*) representing the total electromotive force necessary to maintain the current.

Cautions.—It may not be out of place to mention one or two points which frequently cause confusion at first in connection with curves similar to those just discussed.

(1) A curve displaced towards the *right* is *later* in phase. This is the contrary of what one might expect on considering the matter carelessly, but is at once evident from the construction given at first for obtaining the curve.

(2) In commencing the drawing of a curve a little consideration is required to ascertain whether its maximum value should be measured above or below the horizontal. If the quantity upon which the curve is dependent is decreasing in value, the maximum value of the curve must be *above* the line, so that succeeding points on it may be lower, and vice versâ.

(3) The voltage supplied to a circuit is opposite in direction to the voltages set up in the circuit due to resistance or self-induction. This may appear obvious, but neglect of this distinction will cause confusion. In diagrams Figs. 6 and 7 the electromotive forces shown are those supplied from the external source to the circuit, and not those set up in the circuit, since they are on the same side of the horizontal axis, and therefore in the *same* direction, as the current, whereas the opposing voltages must obviously be in the opposite direction, *i.e.*, on the opposite side of the zero line.

Thus the curve marked E_r in Fig. 7 is the voltage overcoming (and, therefore, oppositely directed to) the voltage C R of the circuit opposing the flow of current. The latter might be represented by a similar curve to E_r at equal distances on the opposite side of the base line.

The distinction here alluded to has been carefully observed in the text, but the difference between the electromotive force *due* to resistance and *overcoming* resistance is sometimes confusing to the student, on account of the loose use of the terms often applied to them.

Vector Diagrams.—The method of representing variable quantities such as currents and electromotive forces by curves is exceedingly laborious, and consequently a more rapid but equally complete method of representation is usually employed.

On page 7 was explained the method of obtaining points on the curve of electromotive force. A point on the curve was found corresponding to each position of the radius $O\ P$ in Fig. 2 by marking off a height equal to the horizontal projection $O\ N$ of $O\ P$. The complete curve was determined by imagining that P made a complete revolution of the circle, and thus gave points forming one complete period of the curve.

By drawing the line $O\ P$ of such a length that it represents some number of volts to a fixed scale, and imagining it to rotate about the end O, it may be taken to represent the varying electromotive force, without the curve obtained from its successive positions being actually drawn. It must then be remembered that the actual value of the variable quantity represented by $O\ P$ may always be obtained by measuring the length of the horizontal line drawn from O to meet a vertical through P. The length of this horizontal line is usually called the horizontal projection of $O\ P$. The length of $O\ P$ itself will be the *maximum* value of the variable quantity represented by it. Its phase is represented by the angle between $O\ P$ and the vertical through O, while the actual value of the quantity when in this phase is represented by the horizontal projection of $O\ P$ in the position in which $O\ P$ is drawn.

When drawing such a line to represent a varying quantity, an arrow-head is shown at one end to indicate that this end is to be taken as rotating about the other. Further, a convention must be adopted with regard to the direction in which rotation occurs. In the figures employed in this book, the rotation will always be taken in a clockwise direction. The rate of the imaginary revolution of the line will be equal to the frequency of the current or electromotive

force represented. On a complete diagram this should be indicated as in Fig. 8, thus ∽ = 50, indicating a frequency of 50 cycles per second, so that the vector *O C* must be taken to rotate 50 times per second.

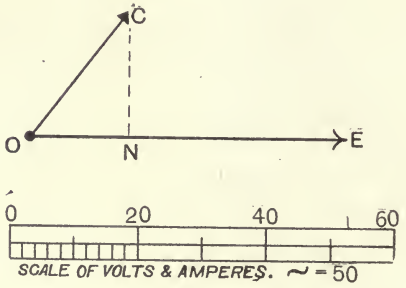


FIG. 8.—VECTOR DIAGRAM OF VOLTAGE AND CURRENT.

By means of the convention just described, a single line is sufficient to indicate the maximum value and manner of variation of a periodic quantity, and also to indicate its phase and instantaneous value at any moment. This method further enables us to show the relation between two such quantities which have the same periodicity, but which may have any relative magnitude and phase. The quantities need not necessarily be of the same nature. Thus, for instance the relation between a current and a voltage may easily be shown on the same diagram by two different lines.

The scale to which the lines have been drawn should be indicated whenever use is made of vector diagrams.

The difference in phase between the various quantities is measured directly as degrees of angle between the lines representing them in the vector diagram, 360° being one complete period.

By way of illustration, Fig. 8 has been drawn to represent the current and voltage of the circuit given by the curves in Fig. 7. Consequently the vector *O C* represents 25 amperes on the scale of amperes, this being the maximum value of the current. Similarly *O E* represents 49 volts, the maximum value of the voltage. The angle *C O E* between the vectors is $52\frac{1}{4}°$, which was found to be the angle of lag of the circuit.

In order to distinguish between lengths representing electromotive forces and those representing currents, the following distinction will be observed in all figures subsequently

given, where the meaning of the vectors is not otherwise evident. Vectors indicating electromotive forces are shown with thin arrow-heads. Vectors representing currents have thick black arrow-heads. Also vectors referring to magnetic flux have arrow-heads with double barb.

The diagram Fig. 8 represents the instantaneous condition of current and voltage at the instant when the voltage has its maximum positive value, since the line *O E* is shown horizontal and directed towards the right; it will thus have its maximum horizontal projection at this moment. The line *O C*, representing the current is drawn $52\frac{1}{4}°$ later in phase, the angle *C O E* being $52\frac{1}{4}°$, and the direction of rotation being clockwise. The instantaneous value of the current is obtained by drawing a vertical line *C N* and measuring the length of *O N* on the scale. It is seen to be 15·3 amperes, which corresponds with the value measured on the curve in Fig. 7 for the same phase.

Resultant of Two Vectors.—Fig. 9 shows by means of vectors the component and resultant electromotive forces of Fig. 7; the instant for which the diagram is drawn is not the same as in the former figure. The two voltages E_r and E^i are drawn at right angles, since they differ in phase by 90°. The lettering will make obvious the identity of the vectors. The scale is the same as for Fig. 8.

As before, the instantaneous values could be found by drawing verticals through the rotating end of each line. The student should identify the phase and instantaneous values of Fig. 9 in Fig. 7.

In this diagram it will be found that if the two component voltages E_r E_l are taken to be two sides of a parallelogram the completed parallelogram would have the vector E as its diagonal. That is, the resultant electromotive force would be represented in magnitude and phase by the diagonal of a parallelogram of which the two components formed the sides.

That this must always be true follows from the following consideration. Since the instantaneous value of the resultant electromotive force is the sum of the values at each instant of the component electromotive forces, the horizontal projection of the vector representing the resultant must always equal the sum of the projections of the components, because the horizontal projections are instantaneous values of the quantities.

The easiest way to obtain the sum of the projections of two such lines as E_r E_i is to draw one of them (say E_r) from the end of the other (E_i) (see Fig. 10), so that the arrows of the two lines point in the same direction. The horizontal projection of E_i is then $O e_1$, and the horizontal projection of E_r is e_1 e_2. Since the projection of E_r is measured from right to left, instead of from left to right, it must be considered negative, and thus the sum of the two projections is Oe_2. But Oe_2 is the same as the projection of the line O E joining the

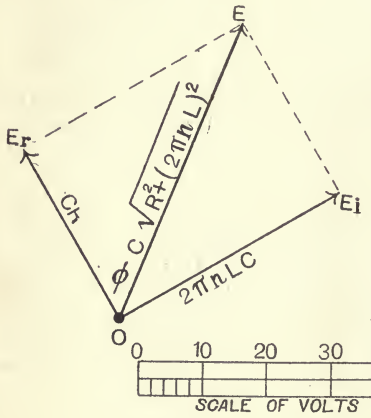


FIG. 9.

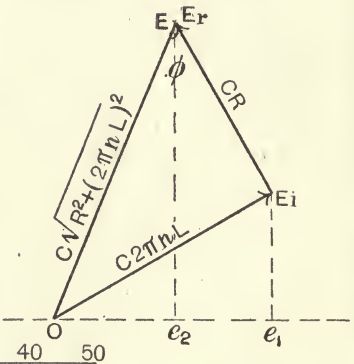


FIG. 10.

RESULTANT AND COMPONENT ELECTROMOTIVE FORCES.

free ends of E_r and E_i. O E is, however, the diagonal of the parallelogram whose sides are E_r and E_i, the line O E, Fig. 10, is, in fact, the same as O E in Fig. 9. Hence the instantaneous value of the resultant of the two varying quantities is the instantaneous value of the diagonal of the parallelogram of which the quantities form the sides.

The result just obtained is of the greatest practical importance, and shows that, when represented by vectors, two electromotive forces or two currents may be combined together in exactly the same way as may the forces or velocities illustrated in mechanics by straight lines, which are combined to form single resultant forces or velocities by the law of the parallelogram of forces.

A similar rule to this may therefore be given in the following terms. The resultant of two currents or electromotive

forces in a circuit may be obtained by representing the two quantities in magnitude and phase by two vectors drawn from a common point and both directed away from this point. The vector of the resultant quantity is the diagonal of a parallelogram of which the two vectors form the sides. Its direction will also be away from the common point. This form of the rule is illustrated by Fig. 9. The method shown in Fig. 10 for obtaining the resultant may be put into words as follows :—

Draw the vectors of the two quantities with the fixed end of one upon the rotating end of the other, *i.e.*, with the arrows of both directed towards the same end of the completed bent line. The vector of the resultant is the line joining the free ends of the bent line thus obtained. Its direction will be such that its arrow points in the opposite direction round the closed figure formed by the vectors to the arrows of the component quantities.

This construction may be extended to any number of component quantities. The resultant of them all is always the line joining the free ends of the figure obtained by drawing all the quantities in regular sequence, the fixed end of one vector coinciding with the rotating end of the previous one. The angle which the resultant line makes with the components represents its phase relative to them.

The application of the construction just given may be illustrated by the following experiment :—

EXPERIMENT I.—DETERMINATION OF THE RESULTANT ELECTROMOTIVE FORCE OF TWO ALTERNATORS COUPLED TOGETHER.

DIAGRAM OF CONNECTIONS.

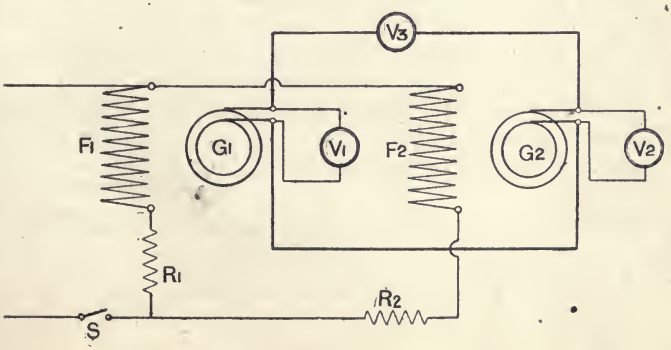


FIG. 11.

G_1, G_2. Two Alternators coupled mechanically.
F_1, F_2. Alternator field windings.
V_1, V_2. Voltmeters for reading voltage of each machine separately.
V_3. Voltmeter for reading resultant voltage.
R_1, R_2. Field regulating resistances.
S. Switch in field circuit.

Note.—In the case of this and a number of other experiments it is not necessary to use the full number of voltmeters shown in the diagram. A single voltmeter with throw-over switch may be used to take the readings of the two alternators. This is often preferable, not only in order to save instruments, but also in order to save errors due to uncertain constants of the instruments, &c.

Instructions.—Two alternators have their shafts mechanically coupled together by a coupling which can be adjusted to alter the relative angle of the rotating parts of the machines.

Excite the field windings from a supply of continuous current.

Insert in the field of each machine a field regulator, so that the fields can be independently regulated. Connect a voltmeter to the terminals of each alternator. Connect the two armatures together, putting a third voltmeter in this circuit, in order to read the resultant voltage of the two alternators.

Run the alternators at a constant speed, and take simultaneous readings on the three voltmeters, entering the readings in the proper columns of the table given below. Make the same observations for a series of relative positions of the machine coupling, which should be altered from the point where the machines are in series to the position when they are in parallel, or opposition. The readings should be entered as shown in the form below.

From the readings of the voltmeters the angle of phase difference between the machines can be calculated, without reference to any graduation of the coupling.

Volts. Machine A.	Volts. Machine B.	Resultant Volts. A and B.	Angle between Machines.

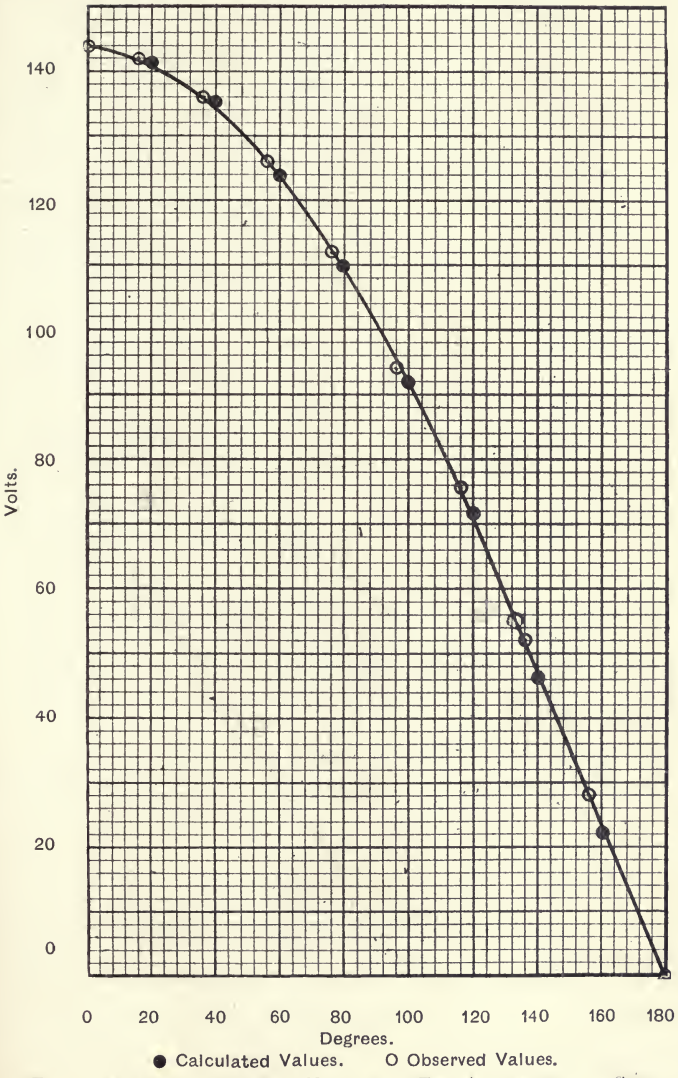


FIG. 12.—CURVE OF RESULTANT VOLTAGE OF TWO ALTERNATORS IN SERIES.

HALLIDIE LIBRARY OF THE UNIVERSITY OF

In order to determine from the readings taken the angle of phase difference between the machines, construct for each set of readings a triangle of which the sides represent to some convenient scale the three voltmeter readings. In doing this it will be convenient to draw a horizontal line representing to scale the resultant voltage. From one end of this line draw a circle whose radius is equal on the same scale to the voltage of one alternator. From the other end draw a circle with a radius equal to the voltage of the other alternator. Join the intersection of these circles to the ends of the horizontal line. The triangle so obtained shows graphically the relations between the voltages of the two machines and the voltage which would be supplied to a circuit connected to the machines in series. The angle between the inclined sides is the angle of phase difference between the machines. A curve should be plotted showing the resultant voltage at various angles of the coupling.

The resultant voltage will vary between a maximum when the voltage of both machines is added, and a minimum when the machines are in opposition, when the resultant voltage is the difference between the two. When working in parallel, alternators are excited, so as to give the same voltage, and since they are connected together like terminals to like, the resultant voltage in a circuit connected to the terminals of any one machine is then equal to that of any other machine. Under these conditions a voltmeter connected between the machines at no load in the manner of V_3 in the diagram Fig. 11, would read zero.

In carrying out the experiment just described, in order to determine the angle between the machines, it will be found convenient to excite the alternators to give *equal* voltages.

As an illustration of this experiment, the curve shown in Fig. 12 has been drawn. Two similar alternators with an adjustable coupling were excited to give 72 volts each. From the graduation of the coupling the resultant value of the voltage was calculated for a series of relative positions of the machines. These values are shown on the curve together with a set of observed values read direct on a voltmeter in series with the machines.*

---

* The resultant value was calculated as follows :—

$E_1 = E_2 = 72$ volts = voltage of each alternator.

$e_1' = E \sin \theta$ $\qquad e_2 = E \sin (\theta + \phi)$

The preceding experiment is not generally capable of any direct useful application, and is given here chiefly as illustrating the way in which two voltages may be applied to a circuit and the phase angle between them obtained by direct measurement of the resultant.

It should be noted, however, that two alternators which can be coupled at any angle form a valuable addition to the equipment of a test-house where meters, &c., have to be tested with currents of varying phase difference. In such cases current may be taken from one machine and voltage from the other, with any desired angle between them. It might, therefore, easily occur that this experiment might be turned to practical purposes.

The geometrical construction made use of in the last experiment is employed in the following measurement, which has many important applications.

EXPERIMENT II.—DETERMINATION OF ANGLE OF LAG IN AN ALTERNATING CIRCUIT BY THREE VOLTMETERS.

DIAGRAM OF CONNECTIONS.

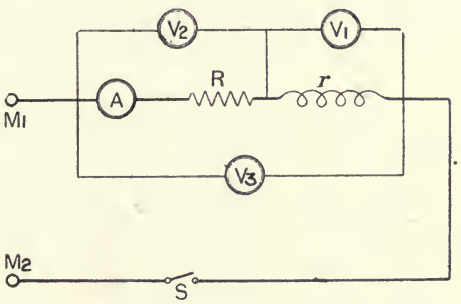


FIG. 13.

$\therefore e_1 + e_2 = E\,[\sin\theta + \sin(\theta+\varphi)] = 2\,E \sin\left(\theta + \frac{\varphi}{2}\right)\cos\frac{\varphi}{2}$

Where θ = phase angle of machine I.

φ = angle between machines.

Hence voltmeter reading $= 2\cos\frac{\varphi}{2} \times$ resultant voltage.

R.M.S. value of $\sin\left(\theta + \frac{\varphi}{2}\right) = \frac{1}{\sqrt{2}}$

,, ,, ,, $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore$ R.M.S. value of $(E_1 + E_2) = 2E\cos\frac{\varphi}{2}$

The meaning of "R.M.S." value is explained later.

M_1, M_2. Source of alternating current.
r. Resistance partly inductive and partly non-inductive.*
R. Non-inductive resistance.
V_1. Voltmeter reading voltage of inductive resistance r.
V_2. Voltmeter reading voltage of non-inductive resistance R.
V_3. Voltmeter reading total voltage of $R + r$.
A. Ammeter.

Instructions.—The portion of the circuit in which the angle of lag is to be determined will consist partly of inductive and partly non-inductive resistances; this is represented by r in the diagram.

Insert in the circuit a non-inductive resistance, R, and an ammeter.

Connect three voltmeters† to points in the circuit so as to measure respectively (1) the voltage at terminals of non-inductive resistance (V_2), (2) voltage of r (V_1), (3) total voltage of circuit (V_3).

Take simultaneous readings on the ammeter and three voltmeters, and enter them in columns headed as given below:—

Current.	Voltage of Non-inductive Resistance V_2.	Voltage of Remainder of Circuit V_1.	Total Voltage of Circuit V_3.	Angle of Lag between V_1 and $V_3 = \phi$
9·0	84·0	44·0	105·5	23°
11·2	75·0	53·0	104·9	29°
12·14	71·3	56·0	103·5	31°
14·67	64·0	62·0	100·7	36·6°

The readings should, if possible, be repeated for several values of the current, obtained by altering the voltage of supply or by a resistance in series with R and r.

The angle of lag of the circuit must then be determined for each set of readings by construction similar to that described in the previous experiment.

Draw a horizontal line to represent to a convenient scale the total voltage of the circuit. Draw from opposite ends

* All so-called "inductive resistances" are partly non-inductive, since they necessarily have some ohmic resistance.

† A single voltmeter and multiple-way switch may be used instead of the three voltmeters.

of this line circles of radius equal respectively to the two other voltages. By joining the intersection of the circles to each end of the horizontal line complete the triangle. This will then be a triangle of electromotive forces for the circuit. (See Fig. 14.)

The electromotive force measured at the terminals of the non-inductive resistance will be in phase with the current in the resistance, since there is no self-induction. The electromotive force V_2 is, therefore, always equal to $C \times R$, the product of the current, and the constant resistance.

Consequently the phase difference between this electromotive force and the electromotive force of the remaining part of the circuit will be the same as the phase difference between the current and the voltage of the remainder of the circuit. Thus in the triangle (Fig. 14) the angle φ_1 will be the angle of lag in the portion r of the circuit.

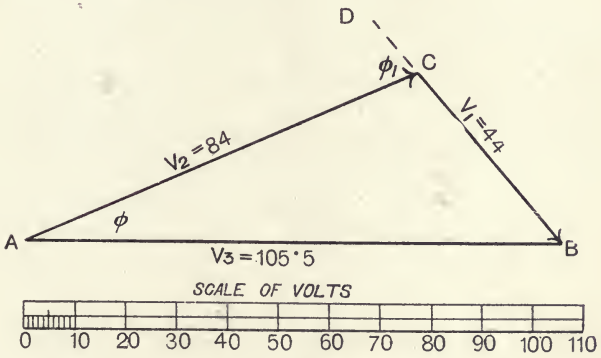


FIG. 14.—TRIANGLE OF E.M.F.

The angle φ between V_2 and V_3 will be the angle of phase difference between the current and total voltage of the circuit *including* the added non-inductive resistance.

The readings entered on the table are those taken from an actual experiment, the corresponding vector diagram for the first set of readings being drawn in Fig. 14.

Example.—(1) An alternator giving a voltage of 200 supplies an alternating circuit in which is inserted a non-inductive resistance of 0·2 ohm. The current is found to be 32 amperes, and the voltage of the part of the circuit not including the ·2 ohm resistance is 198 volts. Draw a diagram of the various voltages in the circuit, and ascertain

therefrom the angle of lag between the current and voltage given by the alternator.

(*Note.*—In solving this problem remember that the value of the voltage in the non-inductive resistance is the product of current and resistance.)

(2) An arc lamp is connected to a 100 volt alternating supply in series with an inductive resistance. The arc is found to take 30 volts and the inductive resistance 90 volts. Find by diagram the angle of lag of the circuit.

(*Note.*—The arc lamp behaves as a non-inductive resistance.)

Magnitude of Resultant Voltage.—We are now in a position to ascertain the relation between the resultant voltage in a circuit and the current, resistance, and self-induction.

Fig. 9. (page 24) shows the method of obtaining graphically the magnitude and phase of the resultant voltage when the component voltages are known. Since the two component voltages spent in overcoming respectively the resistance and inductance of the circuit are known to be always $\frac{1}{4}$ period, or 90°, apart in phase (one being in phase with the current and the other $\frac{1}{4}$ period in advance of it), the angle $E_r\ O\ E_i$ will always be a right angle. Consequently the length of the diagonal representing the resultant = $\sqrt{\text{sum of squares of 2 sides}}$, or

$$O\ E^2 = O\ E_r^2 + O\ E_i^2$$

or (resultant voltage)^2 = (resistance voltage)^2 + (inductance voltage)^2.

We have seen that

resistance voltage = $C\ R$.

inductance voltage = $2\ \pi\ n\ L\ C$. (See page 16).

Hence if V = resultant voltage of the circuit

$$V^2 = C^2\ R^2 + (2\ \pi\ n\ L\ C)^2$$

$$\text{or } V = C\sqrt{R^2 + (2\ \pi\ n\ L)^2}.$$

This result is of the greatest importance.

If the circuit has no self-induction, *i.e.*, $L = 0$,

We have $V = C\sqrt{R^2} = C\ R$, as in the case of a direct-current circuit.

A further important result follows from Fig. 9.

The voltage $O\ E_r$ is in phase with the current, while the relative phase of the resultant voltage of the circuit is represented by the line $O\ E$.

Hence the angle $E_r\ O\ E$ is the angle of phase difference between current and voltage in the circuit. This angle is usually denoted by the symbol φ, and will be frequently referred to hereafter.

Chapter II.

Impedance.

Impedance.—A most important consequence of the back electromotive force of self-induction in a circuit is that the current produced by an applied electromotive force is no longer numerically equal to the voltage divided by the resistance, and Ohm's law apparently ceases to apply. Whenever an alternating voltage is applied to a circuit possessing self-induction the current will be less than the value of the fraction $\frac{\text{voltage}}{\text{resistance}}$.

The quotient $\frac{\text{voltage}}{\text{current}}$ is often called the *apparent resistance* of the circuit. This quantity is the *impedance* of the circuit.

Thus we have the relation

$$\text{current} = \frac{\text{voltage}}{\text{impedance}}$$

$$\text{or impedance} = \frac{\text{voltage}}{\text{current}}$$

The impedance depends on two distinct properties of the circuit, viz., its resistance and its self-induction, the former opposing the flow of current, the latter opposing the change of current. The resistance of the circuit is independent of frequency of current, the shape of the conductor, or its magnetic surroundings. The part of the impedance dependent on self-induction varies with the frequency, and also with the form and surroundings of the circuit if these affect the magnetic field set up by the current.

It has been shown above that the electromotive force to be applied to a circuit in order to maintain a current of C amperes in it is

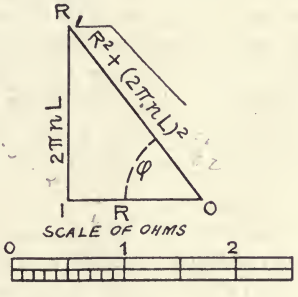
$$E = C\sqrt{R^2 + (2\pi n L)^2}$$

Consequently the value of the impedance of a circuit is given by

$$\text{Impedance} = \frac{\text{voltage}}{\text{current}} = \frac{C\sqrt{R^2 + (2\pi n L)^2}}{C} = \sqrt{R^2 + (2\pi n L)^2}$$

In the triangle of electromotive force, Fig. 10, page 24, each side is proportional to the current multiplied by some

function of the circuit, since the three sides represent the quantities $C \times R, C \times 2\pi n L, C \times \sqrt{R^2 + (2\pi n L)^2}$. Suppose the triangle is re-drawn to a scale such that the side $E_i E_r$ instead of representing $C R$ volts, represents R ohms. Then, if the quantities representing the remaining sides are also drawn to this scale, after dividing their value by C, we shall get a triangle similar to the previous one, but drawn to a scale of



IG. 15.—TRIANGLE OF RESISTANCE AND IMPEDANCE.

ohms instead of volts. This has been done in Fig. 15, where each side of the traingle $O E_i E_r$, Fig. 10, has been divided by 25 (the current of the circuit), and the whole drawn to the scale of ohms shown below the triangle. For convenience the triangle has been drawn with the side representing the resistance horizontal. This direction is the one usually adopted where other circumstances do not afford any reason to the contrary.

If a circuit has a resistance which is exceedingly small, the value of the impedance becomes practically $2\pi n$ L, *i.e.*, equal to the *reactance*. If the self-induction is very small, the impedance approximates to the resistance of the circuit.

It is to be noted that the new triangle has no arrow heads, as the quantities represented by it are no longer variable, and the sides of the triangle do not now represent rotating vectors, but fixed quantities, or *scalars*.

The three sides represent respectively

$O I$ = resistance = R.

$I R$ = reactance = $2\pi n L$.

$O R$ = impedance = $\sqrt{R^2 + (2\pi n L)^2}$,

the angle of lag of the circuit still being represented by the angle $R O I = \phi$.

Hence from the triangle $\cos \phi = \frac{IO}{OR} = \frac{R}{\sqrt{R^2 + (2\pi n L)^2}}$

$$\sin \phi = \frac{IR}{OR} = \frac{2\pi n L}{\sqrt{R^2 + (2\pi n L)^2}}$$

$$\tan \phi = \frac{RI}{IO} = \frac{2\pi n L}{R}$$

The impedance of a circuit or part of a circuit is of very great importance, and the following experiments illustrate the various factors upon which the impedance depends.

As a practical and convenient example, an ordinary arc lamp choking coil may be taken for the purpose of the following measurements. The methods may, however, equally well be applied to any other form of inductive resistance, *e.g.*, an armature coil of an alternator, magnet winding, &c.

The three variable quantities in the expression $\sqrt{R^2 + (2\pi n L)^2}$ for the impedance are: (1) The resistance, R; (2) the periodicity, n; (3) the self-induction L.

In order to determine the influence of each variable separately, two of them must be kept constant, while the third is varied.

EXPERIMENT III.—DETERMINATION OF DEPENDENCE OF IMPEDANCE UPON RESISTANCE.

If the coil on which the measurement is made contains iron, variation in current will produce variations in the permeability of the iron, and consequently variations in the self-induction of the circuit.

Therefore it is necessary in this experiment to maintain the current approximately constant.

DIAGRAM OF CONNECTIONS.

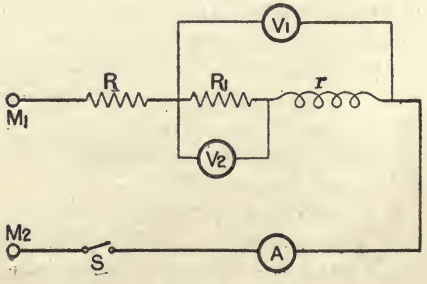


FIG. 16.

M_1, M_2. Source of alternating current.
A. Ammeter for reading current in circuit.
V_1. Voltmeter, reading voltage across impedance.
V_2. Voltmeter, reading voltage across non-inductive portion of impedance.
R. Variable resistance.
R_1. Non-inductive variable resistance forming part of impedance to be measured.
r. Inductive coil.
S. Switch.

Instructions.—Connect in series two variable non-inductive resistances, the inductive coil and an ammeter. One variable non-inductive resistance R_1 and the coil r together make the impedance to be measured. Connect voltmeters so as to measure the volts across the non-inductive resistance R_1 and across the impedance $R_1 + r$ respectively.

A single resistance may be used instead of the two R, R_1 shown in the diagram. This is then kept constant throughout the experiment, and the point of contact of the voltmeter is moved along it between each reading, thus changing the amount of the resistance included in the impedance measured, while leaving the total resistance in the circuit constant.

Supply the circuit thus formed with alternating current of constant periodicity.

By varying R and R_1 together alter the value of R_1 while keeping the sum of R and R_1 about constant, so as to maintain the current at an approximately constant value, and take readings on the instruments for a series of values of R_1.

If the resistance of the coil r is not known make a measurement of this by sending a measured direct current through it, and measuring the drop of volts across its terminals. Its resistance is then given by —resistance $= \frac{\text{volts drop.}}{\text{current.}}$

For each series of readings taken with the alternating current the value of the impedance should be calculated thus : Impedance $= \frac{V_1}{C}$. Also the value of the non-inductive part R_1 of the impedance should be calculated $R_1 = \frac{V_2}{C}$, the resistance of the coil r (which if not known must be measured by direct current as stated above) should be added to the value of R_1.

Readings should be tabulated as follows :—

DETERMINATION OF DEPENDENCE OF IMPEDANCE ON RESISTANCE.

Description of coil used .....

Resistance of coil r = ...ohm

Current C.	Voltage across Impedance V_1.	Voltage across Non-inductive Resistance V_2.	Impedance $\frac{V_1}{C}$.	Non-induction Resistance $R_1 = \frac{V_2}{C_1}$.	Total Non-inductive Resistance $R_1 + r$.

The last column is the sum of the non-inductive resistance R_1 and the resistance of the coil.

The results should then be shown graphically by drawing a curve with values of $R_1 + r$ plotted horizontally, and corresponding values of impedance plotted vertically.

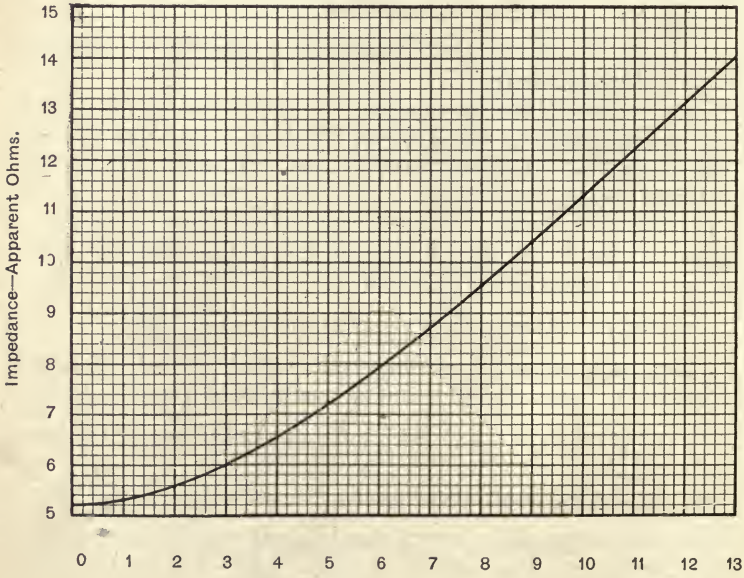


FIG. 17.—CURVE SHOWING DEPENDENCE OF IMPEDANCE UPON RESISTANCE.

A curve obtained in this way is shown in Fig. 17. The readings here plotted were taken on a 10-ampere arc lamp choking coil having a laminated core and cast-iron base, completing the magnetic circuit. The resistance of the coil was ·15 ohm, and the frequency of the current 45 cycles.

It is to be noticed that the curve bends sharply at first, but becomes practically a straight line after a short distance. This is owing to the relatively great importance of the reactance compared with the resistance at first, where the resistance is small, so that the curve at first is nearly horizontal, since the resistance has hardly any effect in increasing the impedance. As the resistance becomes considerably greater than the reactance, the impedance increases more rapidly for a given increase of resistance, and finally increases practically in the same proportion as the resistance, as shown by the line becoming straight.

From the curve it is seen that the value of the impedance when the resistance is zero is 5·2 apparent ohms. This is consequently the value of the reactance of the coil $= 2 \pi n$ L. From this value L, the coefficient of self-induction of the coil could be calculated from the known frequency.

The upper part of the curve approaches, and would ultimately become tangential to a straight line drawn through zero at an angle of 45°, *i.e.*, passing through the points having ordinates and abscissæ equal.

The next experiment shows that the impedance of a given circuit or coil does not depend only on the coil or resistance itself, but on the frequency of the current sent through it.

EXPERIMENT IV.—DEPENDENCE OF IMPEDANCE UPON FREQUENCY.

DIAGRAM OF CONNECTIONS.

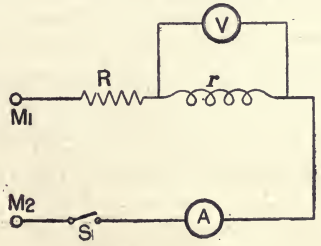


FIG. 18.

M_1 M_2. Source of alternating current.
r. Inductive resistance or coil of which impedance is to be measured.
R. Adjustable resistance.
A. Ammeter reading current in circuit.
V. Voltmeter reading voltage across inductive resistance.

Instructions.—Connect to the terminals of the alternator an inductive resistance or coil in series with an adjustable resistance and ammeter.

Connect a voltmeter to the terminals of the coil.

Take readings on ammeter and voltmeter for a number of different speeds of the alternator. At each speed before taking readings, adjust the current approximately to a fixed value, which should be maintained throughout the experiment. Note the speed of the alternator in each case, and calculate the frequency thus :—

Frequency = revs. per minute × No. of pairs of poles of alternator ÷ 60.

The readings should be tabulated thus:—

Determination of Dependence of Impedance on Frequency.

Description of coil .....Cast iron base, movable core.
Resistance..... ·15 ohms.
Current ..... 3 amps.

Revolutions per Minute of Alternator.	Current in Circuit = C.	Voltage across Impedance = V.	Frequency = n.	Value of Impedance = $\frac{V}{C}$
350	3·0	4·3	11·7	1·43
490	3·0	6·0	16·3	2·00
820	3·0	10·0	27·3	3·33
1500	3·0	18·3	50·0	6·1

A curve should be plotted with frequency measured horizontally and impedance vertically.

Fig. 19 shows two curves obtained in the manner just described. The lower curve gives the values obtained for the impedance of an arc lamp choking coil. The ohmic resistance of the coil was ·15 ohm, so that nearly all the impedance was due to its self-induction. That is, the

impedance $\sqrt{R^2 + (2\pi n L)^2}$ was very nearly equal to the reactance $2\pi n L$, because the value of R was so small. Evidently the reactance increases in direct proportion to n, the frequency. This is shown to be the case by the straightness of the curve, which only bends very slightly at the bottom when the frequency becomes so small that the reactance becomes comparable with the resistance.

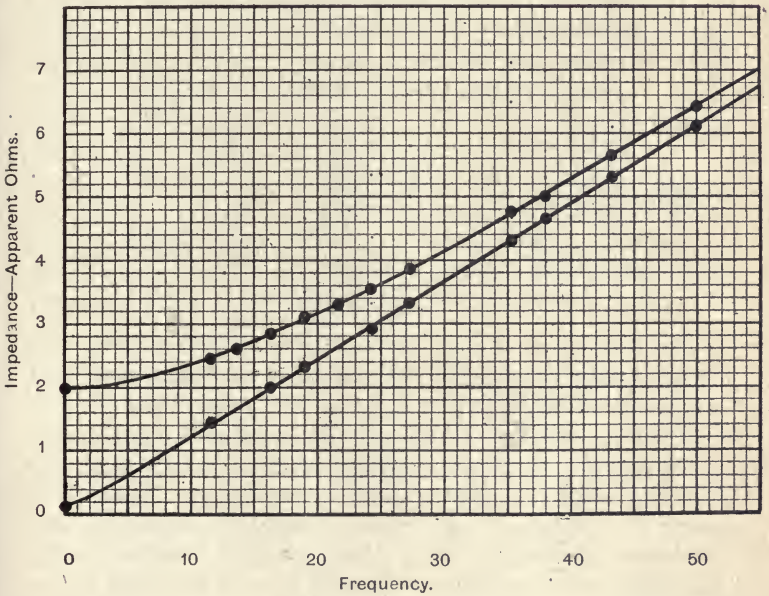


FIG. 19.—CURVES SHOWING DEPENDENCE OF IMPEDANCE ON FREQUENCY.
Lower Curve, Coil alone. Upper Curve, Coil and Resistance.

The upper curve shows readings of the impedance of the coil, and a resistance of 1·85 ohms in series taken at the same time. The resistance in this case is much larger compared with the reactance, and the curve is affected much more by the resistance near its lower end. At very high frequencies indeed, the reactance would become so great that the resistance would be negligible, and the two curves would then coincide, since the reactance factor of the impedance is the same for both curves.

The important point to be learnt from the results of this experiment is that the impedance of any circuit consists of

two elements, resistance and reactance. The resistance is independent of frequency; the reactance depends directly on the frequency. The frequency of the current supplied determines the relative importance of the two elements.

It would be easy to calculate the value of the self-induction of the resistance from the readings shown on the curve.

If the ohmic resistance is known, this can be done directly from a single observation.

If the resistance is not known, two points on the curve will enable the calculation to be made.

Thus the upper curve shows

at frequency 10 impedance = 2·35

,, 40 ,, = 5·3

In 1st case $R^2 + (2\pi n L)^2 = (2{\cdot}35)^2 = 5{\cdot}5$

,, 2nd ,, $R^2 + (2\pi n_1 L)^2 = (5{\cdot}3)^2 = 28{\cdot}0$

$\therefore$ By subtraction $(2\pi L)^2 (n_1^2 - n^2) = 22{\cdot}5$

$$L^2 = \frac{22{\cdot}5}{(n_1^2 - n^2)\, 4\pi^2} = \frac{22{\cdot}5}{1500 \times 4 \times 9{\cdot}87} = {\cdot}000379$$

$\therefore L =$ ·0195 henries,

or 19·5 millihenries.

The determination of the effect of self-induction upon impedance is a problem which arises in many ways.

Choking coils for arc lamps are frequently made with a movable core so that the self-induction can be varied, and the impedance thereby increased or decreased. In an alternator the armature coils have a variable amount of self-induction depending upon their position relative to the magnet poles. In induction motors cases of a similar nature are of frequent occurrence.

The self-induction also depends on the degree of saturation of any iron forming part of the magnetic circuit, since this affects its permeability. The current in a coil thus frequently alters its self-induction.

The following experiment is consequently of very great practical importance, although the full calculations based upon it will not be given until later.

EXPERIMENT V.—DETERMINATION OF DEPENDENCE OF IMPEDANCE UPON SELF-INDUCTION.

(1) Due to alteration of magnetic circuit.

DIAGRAM OF CONNECTIONS.

As for Experiment IV. page 39.

It is assumed that the coil r in this case has a movable core, or is arranged so that its magnetic circuit can be varied by alteration of the air gap in it.

Instructions.—Make connections exactly as described for Experiment IV. The current must in this case also be kept approximately constant if the effect of alteration of the magnetic circuit is to be considered alone. The current must, as before, be regulated by adjusting the resistance R. The frequency must also remain constant.

For a series of positions of the movable part of the magnetic circuit take readings of the current and of the voltage across the terminals of the inductive winding. From these readings calculate in each case the impedance as in the previous experiment, and tabulate the results.

The alteration in the magnetic circuit should be carried out in such a way that an equal movement of the moving portion is made between each pair of readings; the observations may then be plotted as a curve with displacement measured horizontally and impedance plotted vertically.

Determination of Impedance with Varying Air Gap.

Description of coil..... Resistance ..... ·15 ohms.

Current employed .....3 amps. at .....30 periods.

Length of Air Gap.	Current. $= C$	Volts Across Coil. $= V$	Impedance of Coil. $= \frac{V}{C}$
Inches.			
9	3·0	1·2	·4
6	3·0	1·9	·63
3	3·0	5·7	1·9
0	3·0	17·0	5·7

A set of readings taken from a choking coil with a movable laminated core are reproduced in the form of a curve shown in Fig. 20.

The coil employed was the same as that referred to in the last two experiments. The air gap was varied by drawing out the core, leaving the measured gap between the end of the core and a projection on the base, which completed the magnetic circuit when the core was pushed home.

The value of the coefficient of self-induction for each position of the movable core may be calculated from the

readings obtained in this experiment if the resistance of the coil and frequency of the current are noted. This is sometimes important in the case of rotating machinery, as the self-induction affects the wave form of the voltage generated. A more obvious use of the results is to show the range of application of a choking coil required to work on arc lamp circuits of various voltages.

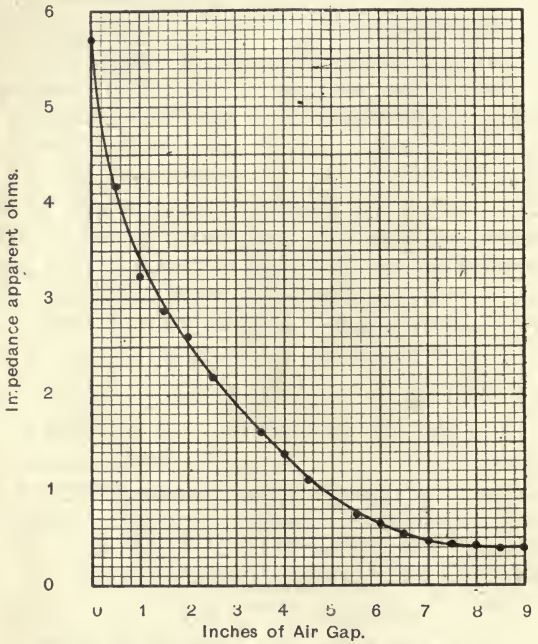


FIG. 20.—DEPENDENCE OF IMPEDANCE ON AIR GAP.

EXPERIMENT VI.—DETERMINATION OF DEPENDENCE OF IMPEDANCE UPON SELF-INDUCTION.

(2) Due to alteration in strength of current.

General.—The strength of the magnetic field produced in an iron core by a given current depends upon the permeability of the iron.

Since the self-induction of a coil having a magnetic circuit partly composed of iron is proportional to the strength of field produced by each ampere of current flowing in the coil, any variation in the permeability of the iron will change the coefficient of self-induction. The permeability of iron changes

as its magnetic saturation is altered. Consequently the permeability of the iron will depend upon the strength of the current producing the field, and the self-induction of the coil will have a different value for each value of the current.

Obviously there will actually be a constant and rapid change in the permeability of the iron corresponding to the changing value of the alternating current. It is only the mean value for a given current with which we are at present concerned.

Diagram of Connections.

As for Experiment IV. page 39.

Instructions.—Connect in series with a supply of alternating current a variable resistance, the inductive coil to be experimented upon, and an ammeter. Connect a voltmeter to the terminals of the inductive coil.

By means of the resistance R vary the current in the circuit from a low value upwards. The frequency must be maintained constant. For each value of the current read ammeter and voltmeter, and tabulate the reading as follows:

Determination of Impedance at Various Currents.

Coil No..... Frequency of Current 37

Current. $=C$	Volts. $=V$	Impedance. $=\frac{V}{C}$
·349	·92	2·75
·706	2·42	3·38
1·572	6·5	4·15
1·183	4·62	3·9

The readings should also be recorded in the form of a curve, as in Fig. 21, the current being plotted horizontally, and the impedance vertically.

The results shown in the upper curve Fig. 21 were obtained on the choking coil already experimented upon with currents having a periodicity of 37.

For low currents* the impedance is seen to be small. This is owing to the well-known fact that the permeability of iron is low for very low inductions, but rises rapidly as the iron becomes more magnetised, until it reaches a point where it begins to be saturated. The permeability then falls again, but more gradually than it increased at first. These variations are very clearly to be traced in the curve. With a non-magnetic core the impedance would be constant at all currents.

The curve represents, in fact, the permeability curve for the core of the choking coil, and has the same properties as the permeability curves of iron, although the shape of the curve shown is affected by the fact that the coil had a heavy cast-iron base, which may have slightly affected the slope of the curve.

The reason for the curve approximating so closely to the shape of a permeability curve is due to the fact that in the impedance $\sqrt{R^2 + (2\pi n L)^2}$ the value of R was ·15 ohm, and was therefore small compared with the reactance $2\pi n L$, the lowest value of which was 2·44, rapidly rising to a much higher value. Hence the impedance was nearly the same as $2\pi n L$. Now the value of L is directly proportional to the mean permeability of the magnetic path, and consequently the impedance varies in proportion to the permeability.

The curve which has just been discussed must not be confused with the magnetisation curve, which shows the dependence of the strength of the magnetic field (not permeability) upon the magnetising current.

---

* With these low currents a correction had to be made for the voltmeter current which was not negligible compared with the total current. The voltages were read upon an electro-magnetic voltmeter, as the ordinary electrostatic type would not read low enough. In making this correction it was found to be permissible to consider the voltmeter current to be in phase with the voltage at its terminals. Since this was not the case for the current in the coil, the current through the coil had to be determined by subtracting the voltmeter current from the component of the current in the coil which was in phase with the voltage, and determining the resultant of this energy voltage and the idle component of the voltage. The resultant current in the coil determined in this way is the value plotted in the curve. For the larger currents this correction was unnecessary. The lowest reading was obtained by calculation from the coefficient of self-induction (10·5 millihenries) measured on the secohmmeter.

A simple method of avoiding the correction for the voltmeter current would have been to employ a key in series with the voltmeter, which would be closed when the voltage is read and opened when the current is read. This would only be permissible when the voltmeter resistance is high compared with the resistances in the circuit.

The lower curve in Fig. 21 is practically the magnetisation curve of the coil, and resembles the magnetisation curve discussed later in connection with transformers, &c.

The lower curve is obtained by plotting the voltage at the terminals of the coil instead of the impedance.

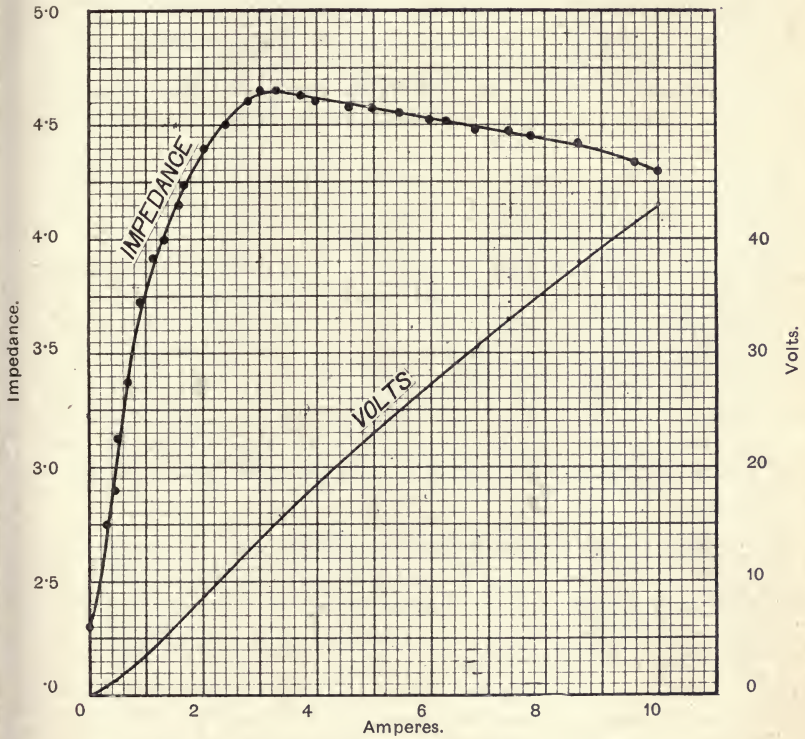


FIG. 21.—VARIATION OF IMPEDANCE WITH CURRENT.

The voltage represented by this curve is almost entirely due to the back electromotive force of self-induction referred to on page 16, the drop in the resistance of the windings being only very slight. For simplicity, we may assume the curve to show the back electromotive force corresponding to each value of the current in the coil. From the results it would then be easy to calculate the number of magnetic lines formed in the core. The formula giving the back voltage in terms of the magnetic flux, which will be obtained later, is:—

$4{\cdot}44\ Z\ n\ t\ \times\ 10^{-8} = V$ where

V = back electromotive force
Z = number of lines
t = number of turns,
n = frequency.

In the coil experimented upon the number of windings was 160. Hence, the number of lines corresponding to each volt $= \frac{10^8}{37 \times 160 \times 4{\cdot}44} = 3{,}800$ nearly. The vertical scale might, consequently, be plotted in terms of lines of force, and similarly the horizontal scale might have been given in ampere turns. A curve of considerable theoretical interest would thus be obtained, although for practical purposes the scales actually employed are generally more useful as well as more easily obtained.

As regards the form of Curve II. it is practically straight after a small initial bend, but bends slightly to the right at the upper end. If the current were increased to much higher values the curve would bend decidedly to the right, showing the well-known "knee" of a magnetisation curve. All cores which are excited by alternating currents are employed with magnetic densities far below the knee, in order to avoid the heavy hysteresis losses which would occur at higher saturation.

Graphic Calculation of Impedance.—The triangle of impedance shown in Fig. 15 forms the basis of a convenient method of obtaining the value of the impedance of a circuit from values of the self-induction and the reactance, requiring the use of *accurately ruled* squared paper only.

Thus, set out from the same point, the resistance of the circuit horizontally and the reactance ($= 2\ \pi\ n\ L$) vertically. The value of the impedance is then given by joining the extremities of the lines thus drawn. This is really the converse of the operation for determining the reactance and self-induction from observations of resistance and impedance given on page 35.

Example.—If the coil experimented upon, with the results shown in Fig. 17, page 38, had a reactance of 10·4 ohms, instead of 5·2 ohms, find graphically the points to give a curve similar to that in Fig. 17. This is equivalent to calculating the curve which would have been observed at double the frequency.

Inductive and Non-inductive Resistance.—Strictly speaking, no resistance is entirely non-inductive, and no conductor can have inductance without some resistance. It is desirable to form a general idea as to what form of resistance may be considered for practical working purposes non-inductive. Convenient resistances in which the self-induction is entirely negligible at all ordinary frequencies may be made by grouping together incandescent lamps connected in parallel or series-parallel to suit the voltage required. Liquid resistances are also specially suitable for alternating-current work since there is little eating away of the electrodes. A solution of

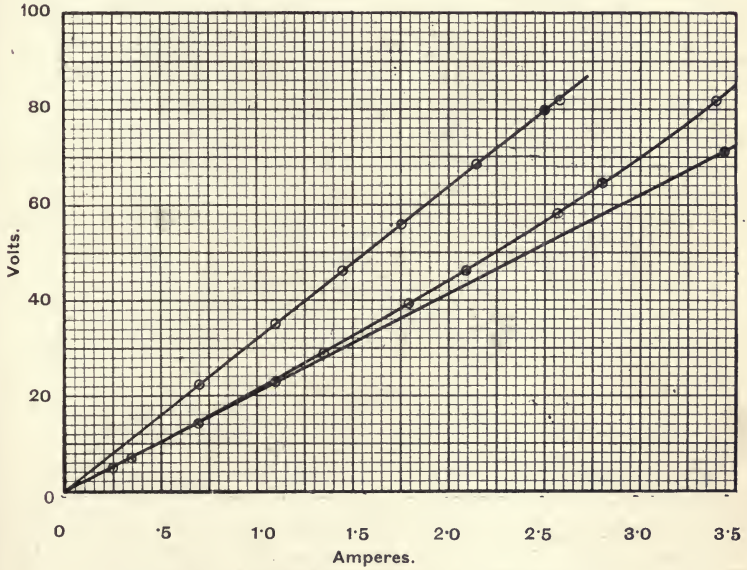


FIG. 22.—IMPEDANCE AND RESISTANCE OF WIRE COILS.
● Direct current. O Alternating current.

common washing soda works satisfactorily, the strength of the solution being chosen according to the resistance required. The electrodes may be movable iron plates or tubes of any convenient form.

For general purposes the ordinary resistance frame made of spirals of resistance wire may be considered to be non-inductive, if any of the high-resistance alloys are used. Iron spirals, on the other hand, have considerable self-induction on account of the magnetic nature of the wire, which gives rise to what is

generally known as "skin effects," and if employed for alternating currents must be looked upon as "partially inductive" resistances. For specially high frequency currents any form of spiral may lead to errors if looked upon as non-inductive; the same caution should be applied in cases where the wave form of the current is specially "peaky."

Some tests made upon ordinary spiral resistance frames are recorded in the curves shown in Fig. 22.

The upper curve shows readings taken with alternating current at 42 cycles, and with continuous current upon a resistance consisting of spirals 1in. diam. of No. 22 S.W.G. resistance wire, the coils being spaced at an average of 5·5 turns per inch. The resistance consisted of 3 sets of 4 coils 27 inches long, the three sets being in parallel, and the 4 coils of each set being in series. It will be seen that at this frequency no appreciable difference between the apparent resistance with alternating and direct current can be traced. This result can easily be verified by calculation.

By measurement with a secohmmeter and standard self-induction the inductance of the coils was found to be ·26 millihenries, while the ohmic resistance was 69 ohms.

The reactance of the coils was consequently $= 2\pi n L$

$= 2\pi 42 \times \cdot 00026$

$= \cdot 069$

Hence ratio $\frac{\text{resistance}}{\text{reactance}} = \frac{1000}{1}$ and the reactance can safely be neglected.

The lower curve in Fig. 22 applies to a wire resistance formed of No. 14 S.W.G. galvanised iron wire, coils 1·1in. diam., and wound 5 coils to the inch, 50 coils 26 inches long being in series.

This curve has been drawn for a double purpose. Firstly, it serves to show to what extent a spiral wound with thick wire of fairly low resistance may be considered non-inductive. Secondly, it shows the influence of the magnetic properties of iron when employed as a resistance.

As may be seen from the curve, the apparent resistance of the iron spirals for alternating and continuous currents is approximately the same with very small currents. The exact relation may be calculated from the following data.

Self-induction measured by secohmmeter = 2·3 millihenries.
Ohmic resistance = 20·66 ohms.
Reactance = $2\pi n L$ = ·604 when n = 42.
$\therefore$ Impedance = $\sqrt{20{\cdot}66^2 + {\cdot}604^2} = \sqrt{426{\cdot}84 + {\cdot}365}$
= 20·664 apparent ohms.

Hence the error introduced in neglecting self-induction would only affect the impedance to the extent of about one-quarter per 1,000, when used for such small currents.

With a non-magnetic material the self-induction would be the same for all current densities, and the resistance might safely be considered non-inductive for any current of this frequency.

The important influence of the magnetic material of which the resistance in this case is composed is plainly shown by the wide and increasing divergence between direct and alternating current readings, as the current density increases, showing that iron resistances cannot be considered as non-inductive with larger currents, although a similar resistance of non-magnetic material may generally be considered to be so.

The figures given above form a useful rough basis for estimating the self-induction of any resistance spiral, as the self-induction varies directly as the square of the number turns in the spiral, directly as the sectional area of the coil, and inversely as the length.

Thus self-induction $\propto \frac{t^2 A}{l}$

Impedance of Armature of an Alternator.—A practical example of a circuit in which the impedance is affected by all the factors determined separately in the foregoing experiments is the core of the armature of an alternator or synchronous motor.

The self-induction of the armature will depend on the exact position of the armature relative to the poles, since the magnetic circuit of the armature is completed through the poles or the pole faces. The extent to which the position affects the self-induction will depend on the nature of the winding and form of the magnetic system of the machine.

A distributed winding will show a small variation, since some of the conductors will always be opposite to part of the magnets, while the maximum value of its self-induction will not be high.

A winding concentrated into single slots, on the other hand, if the air gap between armature and poles is small, will have a high self-induction when opposite to a pole, and a very much lower self-induction when situated between the poles.

The self-induction will also be affected by the amount of the armature current flowing, since the core will be more highly saturated with heavy currents, and its permeability will be lower. The main field will also affect the self-induction, for the same reason.

EXPERIMENT VII.—DETERMINATION OF THE IMPEDANCE OF THE ARMATURE OF AN ALTERNATOR WHILE STATIONARY.

DIAGRAM OF CONNECTIONS.

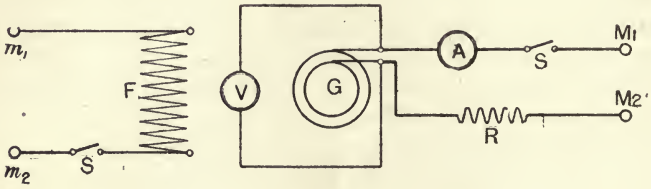


FIG. 23.

M_1 M_2. Source of alternating current.
m_1 m_2. Source of direct current.
G. Alternator armature.
F. Alternator field windings.
A. Ammeter.
V. Voltmeter.
R. Variable resistance.
S, S. Switches.

Instructions.—Connect the alternator armature to a source of alternating current which should give the same periodicity and similar wave form to that of the alternator whose armature is under test. Insert in the same circuit an ammeter, variable resistance, and switch. Connect a voltmeter to the armature terminals.

Close the switch, and give the resistance R its maximum value. Take readings on the ammeter and voltmeter for a series of positions of the armature, which should be turned through the same angle between each pair of readings, from a position in which the armature coils are midway between the

poles, to the position bringing the coils symmetrically under the poles.

Repeat these readings with several values of the current, which should finally be increased to the full-load current of the machine.

Again take the same readings with the alternator fields excited to their normal extent. A field regulator and ammeter may be required in the exciting circuit for the purpose of adjusting the excitation. They are not shown on the diagram.

Finally, measure the ohmic resistance of the armature with direct current.

The readings should be entered in the manner indicated below, a separate table being taken for the experiments with the field unexcited and excited.

Determination of Armature Impedance.

Alternator No..... Type.....
Output.... amps..... volts.... frequency
Normal excitation ....amps.

Armature Position.	Current.	Voltage.	Impedance.

For each set of readings corresponding to one particular current, the *mean* impedance for the various positions of the armature should be calculated and inserted in the table. Strictly, the "square root of mean squares" value should be taken as the effective impedance corresponding to any value of the current, but the error introduced by taking the *mean* value is very small, and the calculation is much simplified.

Some difficulty may possibly be experienced in keeping the armature steady in the desired position when the magnets are excited, as it will tend to rotate into the position giving the maximum value of the impedance. With a little care it can usually be held steady with a wooden wedge under the pulley or a similar device.

The self-induction of the armature can be easily calculated from the results obtained above, since

$$\text{Impedance} = \sqrt{R^2 + (2\pi n L)^2}$$

When R = resistance of armature.
L = self-induction armature.
n = periodicity of current.

The self-induction should be calculated for two values obtained with the magnets excited, and the load current respectively, at its maximum and minimum values.

Fig. 24 gives two curves obtained in the manner just described, the experiment being carried out on the armature of a small Pyke & Harris inductor alternator. The lower curve

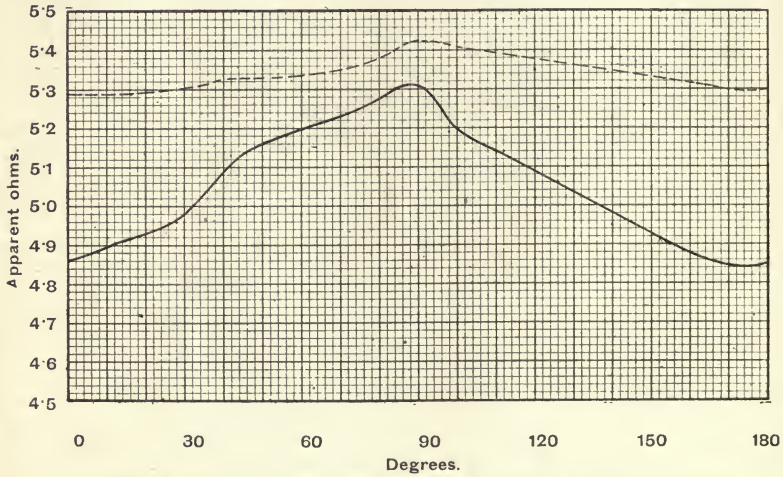


FIG. 24.—CURVES OF IMPEDANCE OF ARMATURE OF INDUCTOR ALTERNATOR.

shows the results with the field fully excited with 3·8 amps., and the armature carrying its normal current of 5 amperes. The upper dotted curve shows the results obtained with the field unexcited, and the same armature current.

In this type of machine the armature is wound in two sets of windings between which the soft-iron inductors rotate ; the variation of self-induction is therefore specially large in this case. The armature resistance was high, being 3·65 ohms. The effect of the field excitation is to partially saturate the iron cores of the armature and of the inductors. This reduces the permeability of the magnetic path, and consequently also

the self-induction of the armature. Hence the impedance of the armature when the fields are unexcited is considerably higher. The difference between the two curves would be still greater if the armature resistance had not been so high. The impedance was so largely composed of resistance that variation in self-induction makes comparatively little difference to the total value of the impedance.

In order to make sure that the difference between the two curves was not due to induced currents in the field winding, the voltage at the terminals of the winding was measured when open. No induced voltage was found. In most types of alternators this effect may be noticed. It is referred to later.

Another effect which has an important influence on the self-induction and impedance of the armature must be mentioned. This is the currents which are induced in the pole-faces, poles, and field-windings by the alternating flux set up by the current in the armature. This is referred to at greater length in the chapter on alternators.

Chapter III.

Power and Power-factor.

Power in an Alternating Circuit.—The nature of power is that of a force exerted at a certain speed: for instance, the resistance to turning of a shaft overcome at the rate of some number of revolutions per minute.

In a direct-current electric motor the force exerted by the armature is proportional to the current in the armature, since the turning effort is produced by the magnetic action of the armature current upon the field. The speed of the motor is independent of current, but is proportional to the voltage applied to the motor. Hence the power, which is proportional to the product of turning effort and speed, is proportional to the product of current and volts supplied to the motor armature. The actual power supplied to the motor is measured in watts, which are numerically equal to the product of current and voltage.

It would, however, be possible to apply to the armature current and voltage alternately, instead of simultaneously.

Thus, if the armature were held stationary a large current could be sent through it at a very low voltage. If the motor, on the other hand, is allowed to run without load, a high voltage may be applied, causing the motor to run very rapidly, while taking only a very small current. In either of these cases the power supplied by the motor would be very small.

A somewhat analogous case to the one just suggested may arise in an alternating circuit, and with corresponding results. The voltage and current may be considerable, but unless they have a high value *simultaneously* the power which they represent may be small.

Power of an Alternating Current.—The power of an alternating current at any instant in any part of a circuit is numerically equal to the product of the current in the circuit multiplied by the voltage measured at the terminals of the portion of the circuit under consideration. Since both current and voltage change their direction twice in every cycle, this product may be either positive or negative, and will usually be positive during part of each period, and negative during the remainder.

If the current and electromotive force act in opposing directions in the circuit, their product must be considered as negative, *i.e.*, the power represented by the current is negative. This is illustrated in Fig. 25, which shows a curve of current C and a curve of voltage V which differ in phase by 30°. The power of the circuit varies continuously, since the product of current and volts changes with these quantities. A third curve of *watts* W obtained by calculating the product of current and volts is drawn, and represents the fluctuation of the power in the circuit.

Since ordinates below the base line represent quantities directed in a negative direction along the circuit, the power curve passes below the line, *i.e.*, represents *negative watts* when given by the product of two quantities on opposite sides of the base line (since the product of a positive quantity by a negative quantity is always negative).

The meaning of this negative power is that when the power of the circuit is negative, the circuit is not receiving power from the generator to be spent in heating conductors driving motors, &c., but is at such times giving back power to the generator (or to some part of the circuit) in virtue of the self-induction (analogous to inertia) possessed by the circuit. When the watts are negative, the power is therefore actually to be considered as given back by the circuit to the generator to assist in driving it. Consequently the energy delivered to the resistances or apparatus in any portion of the circuit is the mean *arithmetic* sum of the power of the circuit multiplied by the time of its duration.

The **Average Power** developed in the circuit is the average value of the product (amperes × volts) in the circuit. This value is shown in Fig. 25 by a horizontal dotted line A B, obtained by adding together the ordinates of the curve of watts corresponding to each vertical line of the squared paper and dividing by the number of these ordinates. Ordinates below the base line must be subtracted from the sum of the ordinates above the line. The value of the average power is seen in this case to be 430 watts.

The average value of the power in a circuit can be measured directly by a wattmeter.

The wattmeter has two coils, one carrying the current of the circuit, and one connected in the same manner as a voltmeter so as to carry a small current proportional to the voltage of the circuit to which it is connected.

The deflection of the instrument is produced by the mutual action of these two coils, so that it is proportional to the product (amperes × volts). Since the inertia of the moving parts of the instrument is far too great to allow them to follow the rapid variations occurring at every period, the deflection is steady, and is proportional to the mean value of the power in the circuit.

Power Factor.—It was pointed out that in Fig. 25 the curves of current and voltage are not in phase, since they do not pass through their zero values simultaneously. In consequence of this a portion of the power represented by the product (current × volts) is always negative during each period.

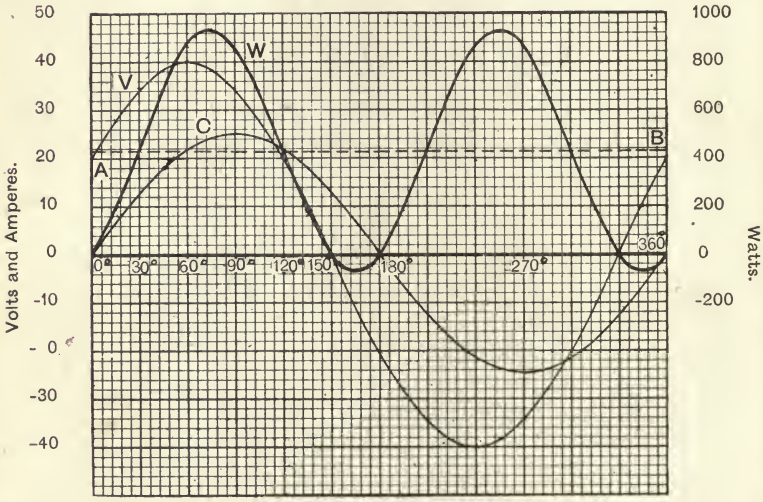


FIG. 25.—CURVES OF CURRENT, VOLTAGE, AND WATTS.
Angle of lag=30°.

If the same current and voltage existed in a circuit without self-induction there would be no difference in phase between them, and the two curves would always pass through zero at the same time, and at every instant would both be on the same side of the horizontal axis. Hence in this case their product would always be positive, and the average useful power of the circuit would be correspondingly greater than is the case for the condition illustrated in Fig. 25, where the power is partly positive and partly negative.

In order to illustrate this, the same curves are redrawn in Fig. 26 with the volts and amperes coincident in phase, together with the curve showing the power which would be in the circuit in this case. The dotted line *C D* gives the average height of this curve, and shows what would be the average power given to the circuit if this had been non-inductive. The value of this average power is 500 watts.

From this it is evident that with given values of current and voltage the power developed will not be simply the

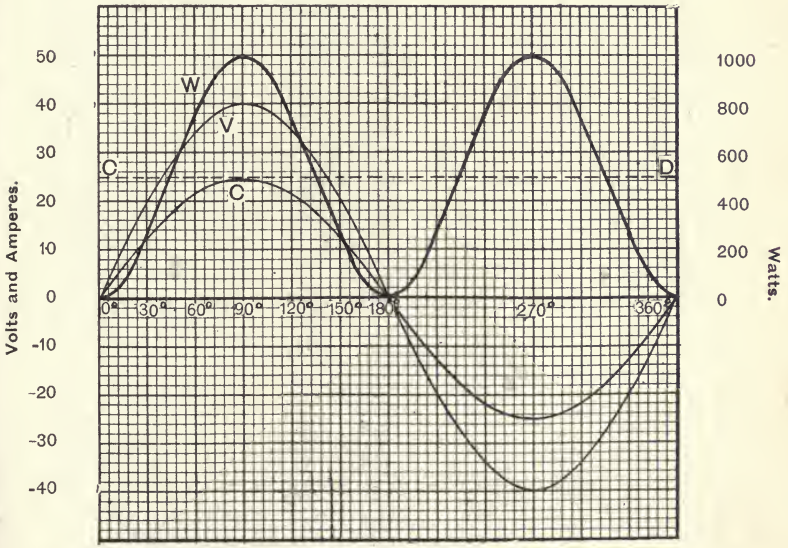


FIG. 26.—CURVES OF CURRENT, VOLTAGE, AND WATTS. Current and voltage in phase.

product of these quantities, but will depend upon the difference in phase between them. When the current and voltage are in phase there is no negative power, and the power developed in the circuit has its greatest value.

This value is the product of the current and voltage as read upon an ammeter and voltmeter.* The product of current and voltage is often called the *apparent watts* of the circuit, or sometimes the *volt-amperes*.

* The relation between the reading of an ammeter or voltmeter and the maximum value of the variable current and voltage is given in the next chapter.

The readings of an ammeter or voltmeter are independent of the phase-difference between current and voltage. Thus, the product of ammeter and voltmeter readings will only give the true value of the power in the circuit when current and voltage are in phase with one another. Under these conconditions the *apparent* power, *i.e.*, the product of amperes and volts, is also the real power.

Under any other circumstances the product of ammeter and voltmeter readings will give a value of the "apparent watts," which is greater than the true power.

The greater the difference of phase between current and voltage, the greater is the negative power, and the less the resultant output. If the phase difference is exactly 90°, or a quarter of a period, the positive and negative power are equal, and the average power developed is zero. This should be verified by the student, who should repeat the curves shown in Figs. 25 and 26 with 90° difference of phase, and calculate the average power under these conditions.

Thus it follows that only in a non-inductive circuit is the power equal to the product (volts × amperes).

The **Power-factor** is the ratio of the true watts to the apparent watts or volt-amperes.

$$\text{Power-factor} = \frac{\text{actual power in watts}}{\text{volts} \times \text{amperes.}}$$

Power of circuit = volts × amperes × power-factor.

As explained in the next chapter, an ammeter in a circuit carrying a current with a maximum value of 25 amps., would read 17·68 amps., and similarly the voltmeter would give a reading of 28·3 volts with a maximum voltage of 40.

The product of these readings, $17{\cdot}68 \times 28{\cdot}3 = 500$, is, therefore, the apparent power in both cases shown in Figs. 25 and 26.

Thus the power-factor of the circuit for which Figs. 25 and 26 were drawn is the ratio of the height of the line $A\,B$ to the height of $C\,D$, *i.e.*, $\frac{430}{500} = {\cdot}86$, in the case of Fig. 25, and $\frac{500}{500} = 1{\cdot}0$ in the case of Fig. 26.

The comparison of the power in a circuit as measured on a wattmeter with the "apparent watts" or volt-amperes forms one of the most direct methods of determining the power-factor of a circuit.

The following experiment illustrates the measurement of the power-factor in both inductive and non-inductive portions of a circuit.

EXPERIMENT VIII.—DETERMINATION OF THE POWER-FACTOR OF A CIRCUIT.

DIAGRAM OF CONNECTIONS.

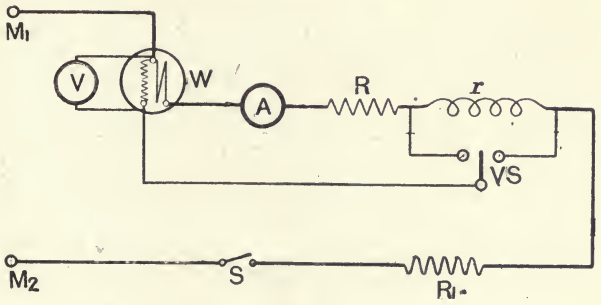


FIG. 27.

M_1 M_2 source of alternating current.
r inductive portion of circuit in which power-factor is to be determined.
R non-inductive portion.
R_1 resistance for varying current.
W wattmeter for measuring power of circuit.
V voltmeter for measuring voltage of circuit.
A ammeter for measuring current of circuit.
VS 2-way voltmeter switch.

Instructions.—Connect in series an inductive resistance, a non-inductive resistance, and variable resistance.

Put an ammeter in the circuit, and connect a voltmeter across the portion in which the power-factor is to be determined.

Connect a wattmeter to read the same voltage and current as the voltmeter and ammeter. In order to be able to measure the power-factor in the non-inductive portion of the circuit alone, connect the common lead from the voltmeter and wattmeter shunt to a two-way switch, in such a way as to be able to put the instruments across the non-inductive part of the circuit or across both resistances at will.

Take simultaneous readings of voltmeter, ammeter, and wattmeter for each position of the voltmeter switch. Repeat the readings with several different values of the current obtained by altering the resistance R_1 outside the portion of the circuit on which the measurement is being made. If it is desired to determine the power-factor in a definite portion of the circuit, the resistances must of course not be altered in the portion of the circuit in which the watts are being measured, since this would at once alter the power-factor. In cases where the power-factor of a motor, coil, or other piece of apparatus is required, the current should always be varied to give the range of readings likely to occur subsequently, since the power-factor will probably vary with the current on account of iron saturation.

For the purpose of forming a suitable experiment, it is convenient to keep r constant and to determine the power-factor of the circuit for various values of R.

Enter the readings under headings as follows :—

Determination of Power-factor in an Inductive Circuit.

Current $= C$	Inductive Circuit.				Non-inductive Circuit.			
	Volts $= V_1$	Watts $= W_1$	Volt-amps. $C \times V_1$	Power-factor $= \frac{W_1}{C \times V_1}$	Volts $= V_2$	Watts $= W_2$	Volt-amps. $= C \times V_2$	Power-factor $= \frac{W_2}{C \times V_2}$
2·1	108·5	226	228	·991	105	220	220·7	1·0
5·4	109·0	570	589	·967	97	524	524	1·0
14·67	100·7	1185	1478	·802	64	939	939	1·0

The power-factor in the non-inductive portion of the circuit should, of course, be constant and equal to unity. Its measurement is introduced into the experiment chiefly in order to illustrate the difference between the power-factors in the two cases. In practical measurements the readings taken as just described on the non-inductive circuit form a most valuable check on the correctness of the readings of the instruments, and the extra connections and time involved are well repaid by the extra certainty of the results whenever a non-inductive resistance forms a part of the circuit. It is therefore strongly recommended that the watts and volt-amperes in a non-inductive portion of the circuit should be

taken *with the same instruments* employed for ascertaining the power-factor of an inductive circuit whenever possible.

The results of the test should be plotted in a curve, with current measured horizontally and power-factor vertically. This is illustrated by the curve in Fig. 28, which was taken on a circuit composed of a choking coil and non-inductive resistance in series. In this case the non-inductive portion of the circuit was varied for each reading.

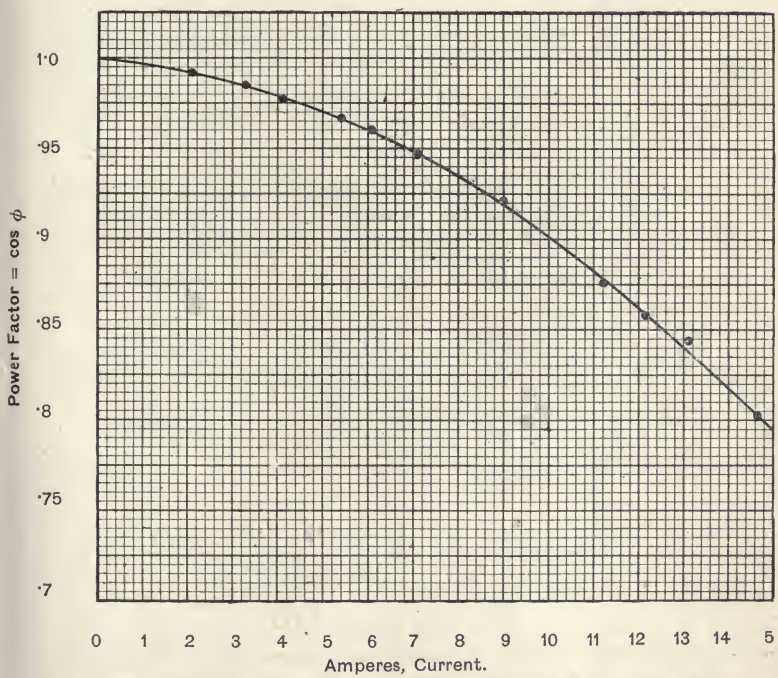


FIG. 28.—CURVE SHOWING VARIATION OF POWER FACTOR WITH CURRENT.

A resistance of about 100 ohms was put in series with the coil, giving a power factor for the whole circuit of nearly unity. This resistance was then gradually cut out, enabling an increased current to pass, and at the same time reducing the power-factor of the circuit.

The curve shows the power-factor for the whole circuit, including both inductive and non-inductive portions. Three of the readings actually taken are shown in the table above as an example.

Energy and Idle Voltage.

It has been shown already that the electromotive force of the circuit consists of two components which differ by a quarter of a period in phase (see page 18). One of these components (equal to $C\ R$ when there are no iron losses in the circuit) is in phase with the current, the other (equal to $2\pi n\ L\ C$) is a quarter of a period out of phase with it.

It has also been stated that the average product of the instantaneous values of the current and volts is zero when the variations of current and voltage have a quarter period phase difference, whereas the value of this product is numerically equal to the product (current × voltage) when the current and voltage are in phase.

Hence the two components of the voltage correspond respectively to the portion of the voltage which does not affect the power of the circuit, and the portion which when multiplied by the value of the current represents the total power of the circuit.

These two components may be suitably called the "Idle" and "Useful," or "Energy" components of the electromotive force.

Hence power given out by current in circuit = useful component of electromotive force × current = energy voltage × C.

Referring to Fig. 29 representing the three electromotive forces of the circuit $\cos F\ E\ O = \dfrac{F\ E}{O\ E}$

or $F\ E = O\ E \cos F\ E\ O$;

but the angle $F\ E\ O$ is the angle of phase difference between the current and voltage of the circuit = φ

Hence $F\ E = O\ E \cos \varphi$

or energy voltage = total voltage × $\cos \varphi$

or useful component = total voltage × $\cos \varphi$.

Consequently if V = total voltage of circuit, the power given out = C × energy voltage = $C \times V \cos \varphi$.

But the power of the circuit = $C \times V \times$ power-factor, and consequently we have power-factor = $\cos \varphi$.

The **power-factor** of a circuit is the cosine of the angle of phase difference between current and voltage.

Each set of readings obtained in the experiment just described would enable us to construct a voltage diagram for the circuit.

The energy voltage E_e is obtained by dividing the watts by the current, since

watts = energy voltage × current.

The following construction gives the idle voltage overcoming the self-induction of the circuit.

Draw a horizontal line (see Fig. 29) $O\,E$ to represent the total voltage E. On $O\,E$ describe a semicircle, and from E

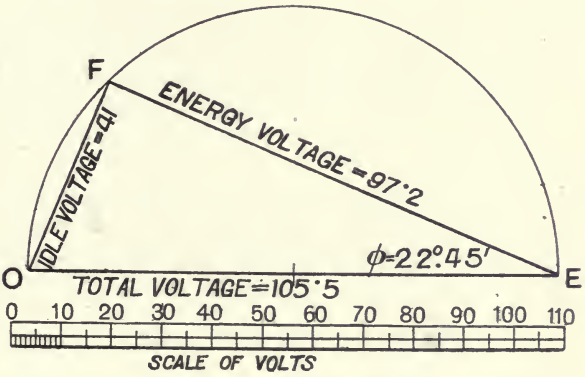


FIG. 29.—DIAGRAM OF ELECTROMOTIVE FORCE.

with radius equal to the energy voltage describe a circle cutting the semicircle in F. Then $O\,F$ represents the magnitude and phase of the self-induction electromotive force, as the sides $F\,E$, $O\,E$ represent in phase and magnitude the energy voltage and total voltage respectively.

The angle of lag is the angle $F\,E\,O$ between $F\,E$ and $O\,E$, since $O\,E$ is the voltage of the circuit and $F\,E$ is in phase with the current.

Fig. 29 represents such a diagram for the point on the curve in Fig. 28, for which total voltage of circuit was 105·5 the current 9 amperes, and the true watts 875. The energy voltage is consequently $\frac{875}{9} = 97{\cdot}2$. The curve in Fig. 28 shows the value of the power factor or $\cos\varphi$ to be ·921, whence $\varphi = 22°\ 45'$. This value for φ is seen to correspond with that of the angle in Fig. 29.

The construction adopted is based on the fact that the angle contained by a semi-circle is always a right angle. The useful and idle voltage vectors are always mutually perpendicular, and have as a resultant the voltage represented by the diameter of the semicircle.

Choking Coil.—A resistance having a high self-induction may have a high **apparent resistance,** although its true, or ohmic, resistance is small. It may consequently be used in series with an arc lamp in order to reduce the current to the value required, and answers then the purpose of the "ballast" resistance which is necessary to make the lamp burn steadily.

Since the voltage absorbed by the self-induction does not represent loss of power (*i.e.*, is not energy voltage), a coil possessing low ohmic resistance and high self-induction may be used more economically than an ordinary resistance in many cases to reduce the current in an alternating circuit.

The following experiment illustrates the use of a choking coil and also the saving which may be effected by it.

EXPERIMENT IX.—DETERMINATION OF POWER LOST IN A CHOKING COIL AND POWER SAVED BY ITS USE.

DIAGRAM OF CONNECTIONS.

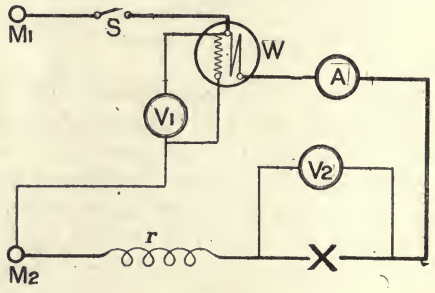


FIG. 30.

M_1 M_2 = Source of alternating current.
X = Arc.
r = Choking coil.
V_1 = Voltmeter reading voltage of circuit.
V_2 = Voltmeter reading voltage of arc.
A = Ammeter reading current in circuit.
W = Wattmeter reading power of circuit.
S = Switch.

Instructions.—Connect an alternating arc lamp and choking coil in series. Join them to a source of alternating current in series with an ammeter and the series coil of a wattmeter. Connect the shunt coil of the wattmeter and a voltmeter across the terminals of the circuit. Connect also a voltmeter across the arc.

Adjust the choking coil* so that the lamp burns with its correct current and voltage. Take several readings at short intervals of the current and watts, and of the voltages of the whole circuit and the arc.

The readings should be tabulated as in the following table. The watts taken by the arc (which forms a non-inductive resistance) are numerically equal to the product of the current and the voltage across the arc.†

The watts lost in the choking coil and leads are the difference between the total watts of the circuit and the watts taken by the arc.

The power saved by the use of the choking coil instead of an equivalent resistance is the difference between the total volt-amperes of the circuit, *i.e.*, the "apparent watts," and the volts registered by the wattmeter, since with a non-inductive resistance in place of the coil the total power taken would be the product of current and voltage of the circuit.

Determination of Power Absorbed in Circuit Containing Arc Lamp and Choking Coil.

Voltage of Circuit = V_1.	Current = C.	Voltage Across Arc V_2.	Watts in Circuit = W.	Watts of Arc = CV_2.	Watts Lost in Circuit = $W - CV_2$.	Watts Saved $CV_1 - W$.
100	10	40	568	400	168	432

The power factor of the circuit is the value of the fraction $\frac{W}{CV_1}$ and the angle of lag is the angle whose cosine is $\frac{W}{CV_1}$ or $\cos^{-1}\frac{W}{CV_1}$.

* Or if this cannot be done, adjust a resistance in the main circuit, so as to alter the voltage of the circuit shown in the diagram.

† The shunt coil and the series coil forming part of the mechanism of the lamp will be inductive, and may modify this statement very slightly.

The watts spent in heating the coil are chiefly spent in heating the windings ($= C^2 R$ watts). Power will also be spent in hysteresis of the core and in producing eddy currents in the metal of the core, screws, &c. The copper and iron losses can be separated by making a careful measurement of the resistance of the winding of the coil at the working temperature and calculating the power lost in this resistance.

The relation between the voltages in a circuit composed of an arc lamp and choking coil is so important that another experiment is given by way of illustration.

Experiment X.—Determination of Voltage of Choking Coil Required by an Arc Lamp.

Diagram of Connections.

Same as Fig. 13 (page 29), with substitution of the arc for the non-inductive resistance R.

Instructions.—Connect the arc lamp in series with a variable choking coil and an ammeter to an alternating current circuit of the given voltage. Connect voltmeters to read respectively the voltage across the arc, coil, and main circuit.

Adjust the coil until the current and voltage of the lamp are those required, then read the voltage at the terminals of the coil.

It will be noticed that the total voltage of the circuit is less than the sum of the voltages of the coil and arc. This is due to the fact that the voltage in the coil is not in phase with the voltage of the arc, because it is the resultant of an energy voltage and voltage of self-induction. The energy voltage is in phase with the voltage of the arc, since the arc itself has no self-induction.

The three voltmeter readings should be drawn as the three sides of a triangle, of which the total voltage forms the hypotenuse (see Fig. 31), in which the values chosen represent those of the circuit in which the readings in the table (page 67) were taken.

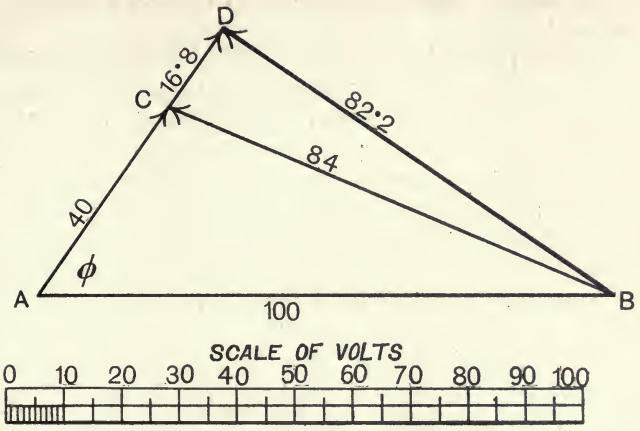


FIG. 31.—DIAGRAM OF E.M.F. FOR ARC LAMP AND CHOKING COIL.

Let $A\,B$ represent the total voltage.
$A\,C$ represent voltage of arc.
$B\,C$ represent voltage of coil.

Produce $A\,C$ and from B draw $B\,D$ perpendicular to $A\,C$, cutting it in D. Then the $\triangle$ $B\,D\,C$ will be the $\triangle$ of voltages for the coil, $D\,C$ being the energy voltage in phase with the current, and $D\,B$ being the self-induction voltage. The power lost in the coil will be the product of the voltage represented by $D\,C$ $\times$ current in the circuit. The voltage represented by $D\,B$ is the product of the reactance of the coil $\times$ current $= 2\,\pi\,n\,L \times C$. Consequently the coil would have to be designed so as to have the self-induction L obtained in this way. This may be put in other words by saying that the back voltage of the coil must be the voltage represented by $D\,B$.

Calculation.—As an example to illustrate the preceding statements, the following example of a choking coil and arc lamp are given. The figures are those for which Fig. 31 is drawn.

An arc lamp requires 40 volts and 10 amps., a choking coil being put in series with it to reduce the current to the required value. The voltage of supply is 100 volts. It is found by calculation, the details of which are explained later, that the power spent in heating the windings and overcoming hysteresis and eddy current losses is 168 watts.

The energy voltage represented by this lost power = $\frac{\text{watts}}{\text{current}} = \frac{168}{10} = 16{\cdot}8$. The watts taken by the arc, which acts like a non-inductive resistance, = (current × voltage) = 10 × 40 = 400 watts, hence, total power given to circuit = 400 + 168 = 568 watts.

The total energy voltage = 16·8 + 40 = 56·8.

The total voltage applied to the circuit = 100, hence, the idle voltage, or voltage spent in overcoming the self-induction of the coil, will be the third side of a right-angled triangle, having its hypotenuse 100 and one side 56·8. This is shown in Fig. 31.

The remaining voltage may be got by measurement or calculation. Its value = $E_0 = \sqrt{100^2 - (56{\cdot}8)^2} = 82{\cdot}2$ volts. The total voltage across the coil is the resultant of this voltage and the energy voltage of the coil 16·8.

Total volts across coil = $E_1 = \sqrt{(82{\cdot}2)^2 + (16{\cdot}8)^2} =$ 84 volts.

This could also have been obtained by direct measurement from the figure as shown. The angle of lag for the whole circuit is the angle between the energy voltage $A\,D$ and the total voltage $A\,B$. Referring to the figure, $\cos \phi = \frac{56{\cdot}8}{100} = {\cdot}568$ which is the power factor of the circuit. The angle of lag for the coil alone is the angle $D\,C\,B$. For the coil, $\cos \varphi = \frac{16{\cdot}8}{84} = {\cdot}2$.

The coil has consequently to be designed so that the back electromotive force produced in it by self-induction is 82·2 volts. From the formula given previously,

$$V = 4{\cdot}44\ Z\,N\,n\,10^{-8}.$$

Where V = back voltage due to self-induction.
Z = number of magnetic lines in core.
N = number of windings.
n = frequency of current.

Hence the product $Z\,N$ can be calculated. Usually the value of Z will be determined by the size of core it is desired to use, and the magnetic density found desirable to avoid excessive iron losses.

Suppose in the present instance periodicity = 50. Then $Z\,N = \frac{82{\cdot}2 \times 10^8}{4{\cdot}44 \times 50} = 37{,}000{,}000.$

If it is decided to use a type of coil having a core with an iron section of 3 sq. in., and the maximum induction to be employed is 30,000 lines per square inch, the value of $Z = 3 \times 30{,}000 = 90{,}000$.

Hence the number of turns of wire must be $\frac{37{,}000{,}000}{90{,}000} = 410$.

Resultant of Two Currents Not in Phase.—Exactly as with an alternating electromotive force, an alternating current may be looked upon as the sum of two component currents. In this case the instantaneous value of the total current is at any moment the sum of the instantaneous values of the two component currents.

Also, if represented on a vector diagram, the line representing the total current is the diagonal of the parallelogram having the component currents as sides drawn to scale and correct difference of phase.

A case in which currents differing in phase and magnitude are combined together occurs when a circuit divides into two branches of unequal resistance and inductiveness, these branches again uniting to form the main circuit. Thus in Fig. 32 two circuits of this kind lie between the points X and Y. Between X and Y is an alternating voltage. One branch circuit is non-inductive, and consequently the current in this branch is in phase with the voltage maintained between X and Y. The other branch is, however, partly inductive, and the current lags behind the voltage in phase.

Thus in the two branch circuits there are two currents differing in phase and in magnitude, but the resultant current flowing from X to Y must be the same as flows in the single conductor forming the remainder of the circuit. On account of the difference in phase between the two currents, the sum of the currents measured in each circuit separately will not be equal to the total current. They must be combined in exactly the same manner as two electromotive forces which are not in phase.

The composition of currents in this way is best illustrated by the following experiment, in which the currents first flow in two branch circuits, where they are measured separately, and are then made to flow together through a single conductor where the resultant current is measured.

Experiment XI.—Determination of Relation between Currents in Branch and Main Circuits.

Diagram of Connections.

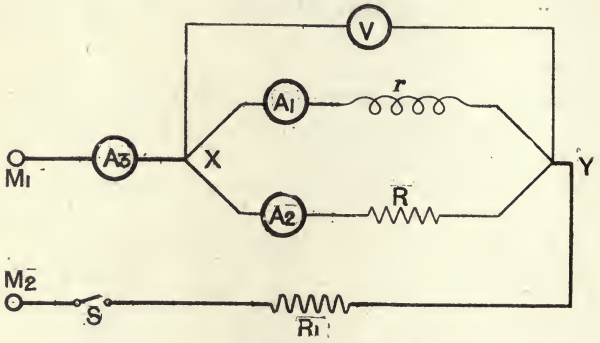


Fig. 32.

M_1 M_2 Source of alternating current.
R Non-inductive resistance.
r Inductive resistance.
R_1 Variable resistance.
A_1 Ammeter reading current in r.
A_2 Ammeter reading current in R.
A_3 Ammeter reading total current.
V Voltmeter reading voltage of branch circuits.
S Switch for breaking circuit.

Instructions.—Connect in parallel to one terminal of the source of supply of alternating current an inductive and a non-inductive resistance with an ammeter in series with each.

Join them both to the other terminal of the supply in series with a variable resistance, ammeter, and switch.

Connect a voltmeter to the points of junction of the two branch circuits.

Take simultaneous readings on the three ammeters and the voltmeter for several values of the current, which may be varied either by alteration of the voltage of the alternator supplying the circuit, or by varying the non-inductive resistance in the common branch of the circuit.

For each set of readings construct a triangle to represent the magnitude and phase relations of the

currents. From this triangle obtain by measurement the value of the angle of lag between current and voltage of the complete circuit ($= \phi$), which will be the angle between the lines representing C_2 (in the non-inductive resistance) and C_3 the total current.

This is because C_2 is in phase with the voltage of the circuit, and would therefore be parallel to it in the diagram of current and voltage. Similarly the angle of

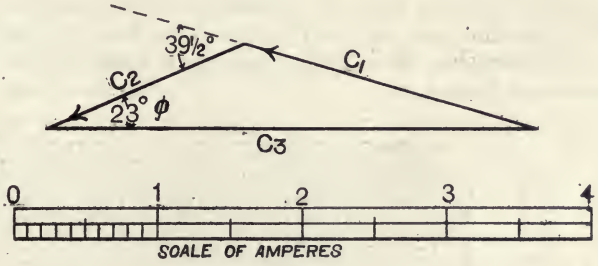


FIG. 33.—DIAGRAM OF CURRENTS.

lag in the inductive branch circuit r is the angle between C_2 and C_1. Calculate also the impedance of each branch circuit, and the joint impedance of the two circuits by dividing the voltmeter reading respectively by each branch current and by the main current. These values may be drawn as a triangle of impedance for the part of the circuit between X and Y.

As an example of the readings taken and of the method of entering up, one out of a series of actual values, together with the corresponding diagram, is given below and on Fig. 33.

Voltage = V.	Current in $R = C_2$.	Current in $r = C_1$.	Current in $R + r = C_3$.	Angle of Lag.		Impedance.		
				In Joint Circuit = ϕ.	In r = ϕ.	Of $R = \frac{V}{C_2}$.	Of $r = \frac{V}{C_1}$.	Of Joint Circuits = $\frac{V}{C_3}$.
20·6	1·48	2·12	3·41	23°	39½°	13·9	9·72	6·04

Idle Current.—In the discussion previously given of the conditions in a circuit in which a difference of phase exists between current and voltage, it was shown that the total voltage of the circuit might be considered as

being composed of two components, viz.: the energy component ($= E \cos \phi$) in phase with the current, and the idle component ($= E \sin \phi$) $\frac{1}{4}$ period out of phase with the current.

For some purposes it is preferable to treat the total *current* as if formed of two components respectively in phase and $\frac{1}{4}$ period out of phase with the *voltage*. It is immaterial from a mathematical standpoint whether the current or voltage is looked upon as consisting of two components, since there is actually one current and one voltage in the circuit, and it is only for purposes of investigation that we consider either of them to be of a composite character.

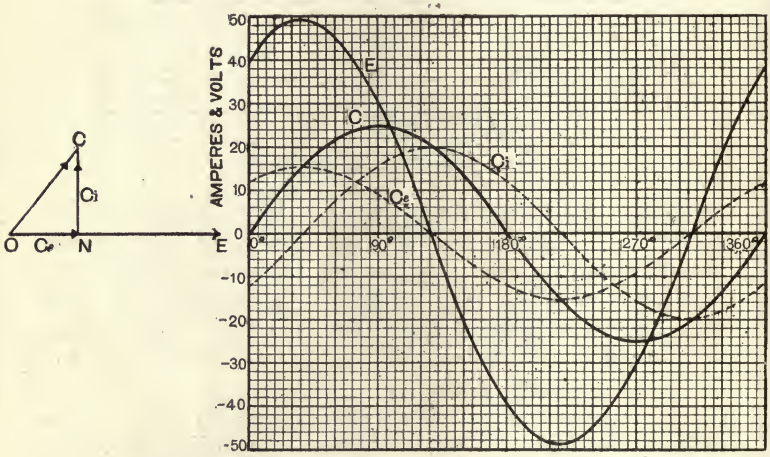


FIG. 34.—CURVES OF VOLTAGE AND COMPONENT AND RESULTANT CURRENTS.

Following out this idea, Fig. 34 has been drawn to represent the same current, voltage, and angle of lag as shown in Fig. 7, page 17. In this case, however, the current is resolved into two components whose values are $C \cos \phi$ in phase with the voltage, and $C \sin \phi$ $\frac{1}{4}$ period behind the voltage in phase. These components are usually termed the *energy current* and the *wattless* or *idle current* respectively, and are shown by the dotted lines marked C_e and C_i. It is evident from the curves that at any instant the sum of the instantaneous values of energy and wattless currents is equal to the total current.

The currents are shown in Fig. 34 both in the form of curves and in the form of vectors, in which $O N$, $N C$ are the two components of $O C$, which represents the total current.

As in the previous discussion, it is only the current and voltage which are coincident in phase which represent power given to the circuit. Hence in this case

power of circuit = energy current × total voltage.

$= C \cos \phi \times V = C V \cos \phi$ as before.

Actually the two points of view illustrated by Figs. 7 and 34 lead to the same result, the difference lying in this, that in one case we speak of an *idle component of the voltage* overcoming the effects of self-induction, whereas in the other case we speak of an *idle current*, which may also be termed the magnetising current, and which is the portion of the total current spent in producing the magnetic field which is the manifestation of the self-induction.

The following experiment is given as an example of this method of regarding the quantities in the circuit, and should be carefully compared with Experiment VIII. The measurements are a repetition of those made in Experiment VIII., but the conclusions drawn from them are obtained by a different course of reasoning.

EXPERIMENT XII.—DETERMINATION OF IDLE AND ENERGY CURRENT IN A CIRCUIT.

DIAGRAM OF CONNECTIONS.

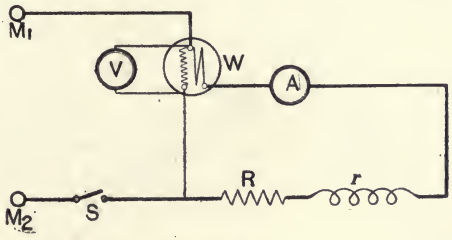


FIG. 35.

$M_1 M_2$ Source of alternating current.
W Wattmeter.
R Non-inductive resistance.
r Inductive resistance.
A Ammeter measuring current of circuit.
V Voltmeter ,, voltage ,, ,,
S Switch for breaking circuit.

Instructions.—Connect in series an inductive and a non-inductive resistance, ammeter, the series coil of a wattmeter and switch. Connect a voltmeter and the shunt coil of the wattmeter across the two resistances, so as to read the total voltage of the circuit.

Take readings on the ammeter, voltmeter, and wattmeter for several values of the resistances.

For each set of readings determine graphically the idle and energy current and angle of lag as described below, and tabulate the results as shown in the following table:—

Determination of Idle and Energy Currents.

Voltage $= V$	Watts $= W$	Total Current $= C$	Energy Current $\frac{W}{V}$	Idle Current.	Angle of Lag.
100·7	1185	14·67	11·7	8·89	$36\frac{1}{2}°$

To separate the idle and energy currents, proceed as follows:—

Draw a horizontal line OC (see Fig. 36) to represent the total current of the circuit to a scale of amperes. Describe a semi-circle on O C as diameter. Determine the value of the energy current by dividing the measured watts by the voltage, and describe a circle with centre C and radius equal to the energy current on the scale of amperes,

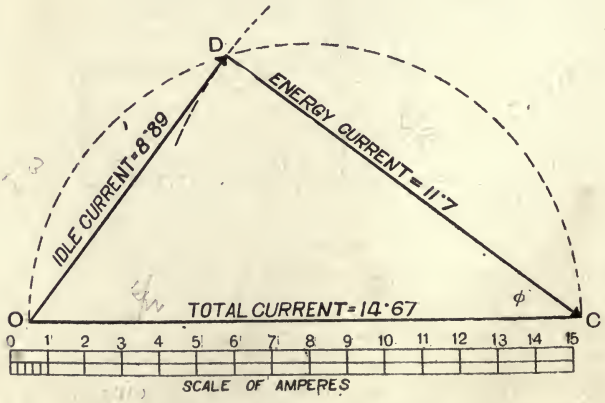


FIG. 36.—DIAGRAM OF CURRENTS.

cutting the semi-circle in D, then the line O D represents the idle current in phase and magnitude.

The figures inserted in the table above give a sample reading, and Fig. 36 is the diagram corresponding to these figures.

The value of the angle of lag may be obtained by measurement from the triangle, since the voltage of the circuit is in phase with the energy current, and consequently the angle D C O is the angle of lag. ϕ may also be got from the value of the power factor; thus

$$\frac{\text{watts}}{\text{amperes} \times \text{volts}} = \text{power factor} = \cos \phi$$

As a check on the accuracy of the figures inserted in the table, it should be remembered that

$$(\text{energy current})^2 + (\text{idle current})^2 = (\text{total current})^2;$$

indeed it is by the use of this equation that the idle current would generally be calculated from the total and energy currents.

3-Voltmeter Measurement of Power.—A direct useful application may be made of the measurements made by 3 voltmeters in Experiment II., page 29. It was shown how the 3-voltmeter measurements could be represented graphically as the sides of a triangle of voltages. Since the voltage in the non-inductive portion of the circuit is in phase with the current, the angle of lag in the whole circuit is the angle between the lines representing the voltage in the non-inductive part and the total voltage respectively.

Similarly, the angle of lag for the inductive portion is the angle between the lines representing non-inductive voltage and voltage of inductive portion.

Thus, referring to Fig. 14, page 31, the voltage represented by A C is in phase with the current in the circuit, and consequently the angle C A B is the angle of phase-difference between the current and total voltage, *i.e.*, the angle of lag for the whole circuit. Similarly, the angle D C A is the angle of lag for the inductive portion of the circuit.

Since the power in any circuit or portion of a circuit is given by $C \times E \times \cos \phi$, where C and E are the current and voltage of the part considered, we have in the diagram of the three electromotive forces all the information necessary to determine the power in the whole circuit and each portion of it, when we know also the value of the current.

Consequently, in the diagram Fig. 14 we have

Energy in whole circuit $= C \times AB \times \cos CAB$.

Energy in inductive part of circuit ... ... $= C \times CB \times \cos DCA$.

Energy in non-inductive part ... ... $= C \times AC$.

Where in each case the lines are taken as the voltages which they represent.

If, therefore, a non-inductive resistance forms part of a circuit, or if a non-inductive resistance can be inserted in the circuit for the purposes of the measurement, three readings of a voltmeter and a reading of the current enable us to measure the power of the circuit. This is frequently of great advantage where an accurate wattmeter is not available.

The diagram of connections necessary for such a measurement is Fig. 13. In the following experiment a wattmeter is introduced into the circuit so that a comparison may be made between the readings obtained with the wattmeter and by the 3-voltmeter method.

The ammeter is unnecessary if the value of the non-inductive resistance is known, as will be seen from the calculation given below.

Experiment XIII.—Measurement of Power by the 3-Voltmeter Method.

Diagram of Connections.

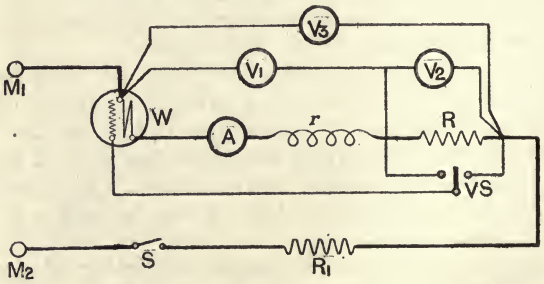


Fig. 37.

M_1 M_2 Source of alternating current.
r Inductive resistance.
R Non-inductive resistance.
R_1 Resistance for varying current in circuit.

V_1 Voltmeter measuring voltage in inductive portion of circuit.
V_2 Voltmeter measuring voltage in non-inductive portion of circuit.
V_3 Voltmeter measuring voltage of total circuit.
A Ammeter reading current in circuit.
W Wattmeter reading either total power or power in inductive portion.
VS Voltmeter switch for changing connections of wattmeter shunt coil.

Instructions.—Connect in series with an ammeter and the series coil of a wattmeter an inductive and a non-inductive resistance. Connect these to a source of alternating current through a variable resistance in order to enable the current in the circuit to be varied.

Connect either three voltmeters, or a single voltmeter with 3-way switch, to read the voltages of the whole circuit, of the inductive and the non-inductive portions. Connect the shunt coil of the wattmeter to a 2-way switch so as to read the power in either the total circuit or the inductive portion only, as desired.

Take readings on all the instruments for several values of the current, noting both total watts and watts in inductive resistance. Tabulate the results as shown below, where a few readings actually determined in the manner described are given as examples :—

Measurement of Power by Three Voltmeters.

Current.	Volts.			Watts observed.		Watts calculated.	
	Total.	Inductive.	Non-inductive.	Total.	Inductive.	Total.	Inductive.
2·1	108·5	11	105	226	6	226·2	7·5
4·09	110	22·2	103	440	20	440	19·6
10·25	105	49	79	965	155	964·5	154·5
13·12	103	58	69	1140	236	1141	236·1

If the value of cos ϕ used in calculating the power from the readings of the ammeter and voltmeter is obtained by actual measurement, a high degree of accuracy is not possible. The power can be obtained entirely by calculation from the ammeter and voltmeter readings, as is shown by the following investigation :—

By the ordinary trigonometrical relations in a triangle we have (see Fig. 14, page 31) :

$$V_1^2 = V_2^2 + V_3^2 - 2\ V_2\ V\ \cos\ \phi,$$

but $V_2 = CR$ where R is the resistance of the non-inductive portion of the circuit.

$$\therefore\ V_1^2 = V_2^2 + V_3^2 - 2\ R\ C\ V_3\ \cos\ \phi.$$

Since the total power of the circuit $= CV_3\ \cos\ \phi$, we have

$$\text{Power in circuit} = \frac{V_2^2 + V_3^2 - V_1^2}{2\ R}$$

The power in the inductive portion r is obtained by subtracting the watts spent in the non-inductive portion from this expression. The watts in the non-inductive resistance are $C^2R = \frac{V_2^2}{R}$

Hence watts in inductive part of circuit

$$= \frac{V_3^2 - V_2^2 - V_1^2}{2\ R}$$

In making a measurement of the power of the circuit it is not necessary to measure the current if the value of the non-inductive resistance is known. It will often be the easiest method of determining this resistance to put an ammeter in the circuit, as shown in the diagrams given.

For greatest sensitiveness of measurement the readings of the voltmeters connected to the inductive and non-inductive parts of the circuit should be as nearly equal as possible.

3-Ammeter Measurement of Power.—This method is analogous to the preceding, but employs three ammeters instead of three voltmeters for making the measurement. From the readings, a triangle of currents, instead of a triangle of voltages, gives the construction for determining the angle of lag.

In this case a non-inductive resistance must form a parallel circuit to the inductive part of the circuit, and must be added if not already part of the connections. The connections for making the measurement are those given in Diagram 32, page 72.

As in the previous case, the following experiment is arranged for a comparison between the measurement made by this method, and the measurement with a wattmeter :—

Experiment XIV.—Measurement of Power by the 3-Ammeter Method.

Diagram of Connections.

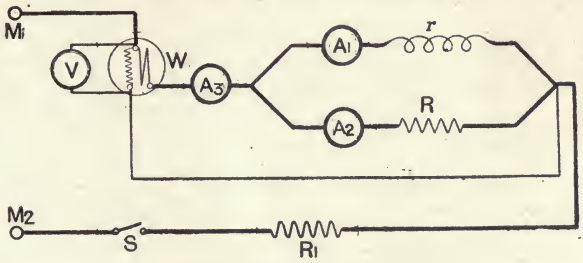


Fig. 38.

M_1 M_2 Source of alternating current.
r Inductive resistance.
R Non-inductive resistance.
R_1 Resistance for varying current.
A_1 Ammeter measuring current in inductive circuit.
A_2 Ammeter measuring current in non-inductive circuit.
A_3 Ammeter measuring total current.
V Voltmeter measuring voltage of circuit.
W Wattmeter measuring total power of circuit.

Note.—The wattmeter may be connected so as to read the power in the inductive circuit only by placing the current coil in the branch circuit instead of the main circuit.

Instructions.—Connect in parallel an inductive and a non-inductive resistance, each in series with an ammeter. Connect the joint resistance thus formed to a source of alternating current in series with an ammeter and a variable resistance for changing the current. Connect a voltmeter to measure the voltage applied to the branch circuits, and insert a wattmeter to read the power in them.

For several values of the current read all the instruments, and tabulate the results as below. The figures in the table are taken from actual readings and given as an example:—

Measurement of Power by Three Ammeters.

Voltage.	Current.			Watts Observed.	Watts Calculated.	
	Total.	Inductive.	Non-Inductive.	Total.	Total.	Inductive.
20·6	3·41	2·12	1·48	63·5	64·2	33·7
27·7	4·70	2·92	2·12	117·4	117·7	59·3
32·0	6·13	3·80	2·56	185·0	185·9	103·9
45·8	7·98	4·80	3·76	333·3	334·3	162·0

The watts may be calculated after drawing a triangle of currents for each set of readings from the formula: Power $= C\,V\,\cos\phi$. The value of ϕ is the angle between the line representing the current in the non-inductive branch and in the circuit for which the power is to be calculated.

As in the last experiment, the results should be checked by a calculation. In this case the calculation depends upon the formula obtained as follows:—

In the triangle of currents (see Fig. 33, p. 73) we have by trigonometry

$$C_1^2 = C_2^2 + C_3^2 - 2\,C_2\,C_3\cos\phi.$$

Also $C_2 = \frac{E}{R}$ where E is the voltage at the terminals of the branch circuits and R is the value of the non-inductive resistance.

$$\therefore C_1^2 = C_2^2 + C_3^2 - \frac{2}{R}\,C_3E\cos\phi.$$

But $C_3E\cos\phi$ is the total power given to the branch circuits. Hence

$$\text{Total power in branch circuits} = \frac{R}{2}\left[C_2^2 + C_3^2 - C_1^2\right]$$

The power in the non-inductive branch $= C_2^2R$.

$$\therefore \text{Power in inductive branch} = \frac{R}{2}\left[C_3^2 - C_2^2 - C_1^2\right]$$

For maximum sensitiveness the currents in the two branches should be approximately equal.

If the value of the resistance in the non-inductive branch is known, it is not necessary to employ the voltmeter. Usually the addition of the voltmeter forms the simplest method of determining the resistance.

In the case of both 3-voltmeter and 3-ammeter measurements, the power is usually calculated directly by the use of the formulæ given, and not by construction of a triangle, or by the calculation of cos ϕ.

It will be seen that in order to measure the power in an inductive circuit by the 3-voltmeter method it is necessary to insert a non-inductive resistance in the circuit, and thus to absorb a considerable proportion of the power during the measurement.

In making a measurement with three ammeters it is necessary to have a branch circuit which takes a large part of the total power.

For these reasons it is not usual to employ these methods except in such cases as the measurement of the load of a machine under test where the manner in which the power of the circuit is absorbed is not of importance. For continuous reading of the power in a circuit employed in doing useful work the method is not admissible.

In carrying out the experiments just given, if fairly small currents are used, the losses in the voltmeters, ammeters, and wattmeter shunt coils may be found to seriously affect the accuracy of the measurement. A knowledge of the resistance of each instrument is necessary to determine the best arrangement for minimising these errors, and the experiments, if carefully carried out, form a valuable exercise in judgment as to their use.

Chapter IV.

Virtual Value of an Alternating Current.

Curve-tracing by Contact Maker.—The easiest method of determining experimentally the wave form of an alternating current or electromotive force is to fix a rotating contact maker to the shaft of the generator. This contact maker acts as a switch, which is only closed for an instant once during each revolution of the shaft. If the terminals of the alternator are connected to an electrostatic voltmeter through this contact switch, the voltmeter will indicate the voltage of the machine at the instant when the circuit is closed. By varying the position of the contact maker the voltage of the alternator can be observed for a series of positions of its rotating part, and the variation of voltage during a complete cycle may be traced.

One of the simplest forms of such a contact maker is a disc of ebonite, with a metal strip let into the edge at one point. Two insulated brushes, each formed of a strip of spring steel, are mounted side by side so as to press on the edge of the disc. Once in each revolution they are connected together by the rotating strip. By mounting the brushes on a movable arm which can be clamped to a divided sector, the point of contact can be varied by any desired angle. A convenient form of this arrangement is shown in Fig. 39.

Another device which is very easy to apply is to provide a rotating pin on the shaft or coupling of the alternator. The pin comes into contact with a light spring once during each

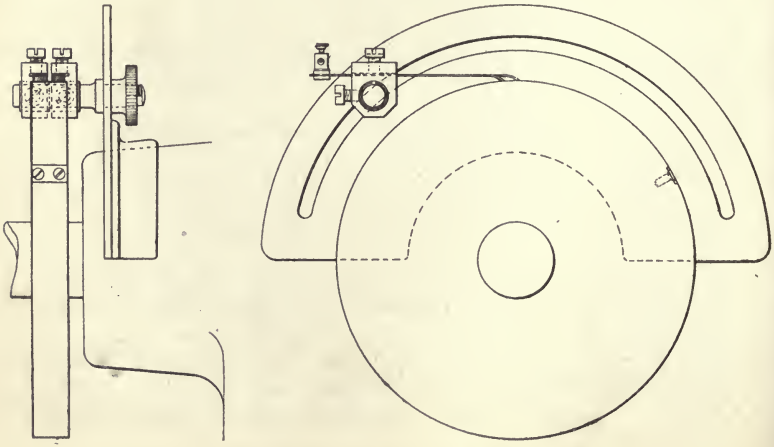


Fig. 39.—Rotating Contact Maker.

revolution, and so completes the circuit. The spring in this case also must be insulated, and provided with means for moving it round a graduated dial.

A fine jet of conducting fluid has been employed instead of a spring, in order to insure more uniform contact. This is less easy to apply. In either of the latter arrangements contact is usually made to the rotating pin through the frame and shaft of the machine. In some cases this would be objectionable, and the pin must then be insulated, and connected to a slip ring.

The readings are generally taken upon an electrostatic voltmeter, since the readings of such an instrument are independent of the duration of the time of application of the voltage. It is usually necessary to put a condenser in parallel with the voltmeter, in order to insure steady readings, since the leakage which takes place is often sufficient to discharge the voltmeter between successive contacts. By adding a condenser, a practically uniform voltage is maintained at the terminals of the voltmeter, in spite of a small leakage. In Fig. 39 it will be noticed that the spring brushes are put one in advance of the other, so that the contact is made only momentarily, and not during the whole time necessary for the contact to pass under the brushes.

Another method of taking the readings is to replace the electrostatic voltmeter by a condenser, which is then discharged by a switch through a ballistic galvanometer, when a reading is to be made. By this method a momentary throw, instead of a steady reading, is obtained.

EXPERIMENT XV.—DETERMINATION OF WAVE FORM OF AN ALTERNATOR.

DIAGRAM OF CONNECTIONS.

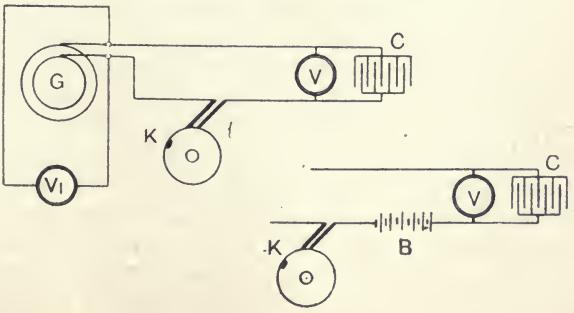


FIG. 40.

G Alternator armature.
K Rotating contact maker.
V Electrostatic voltmeter for reading instantaneous value of voltage.
C Condenser.
V_1 Voltmeter for reading virtual value of voltage of alternator.

Instructions.—Connect the alternator terminals to an electrostatic voltmeter in series with the rotating contact maker, so that the voltmeter is only momentarily connected to the alternator by the contact maker once during each revolution. It will usually be found necessary to connect a condenser in parallel with the electrostatic voltmeter in order to get reliable readings. Connect a second voltmeter, which need not be electrostatic, to the terminals of the alternator. Insert a regulating resistance in the alternator field circuit.

Throughout the experiment keep the alternator voltage as read on voltmeter V_1 constant, by regulation of the field when necessary.

For a series of positions of the contact maker take readings of the voltmeter V.

Plot the results as a curve, measuring displacement of contact horizontally and voltage vertically.

It is probable that difficulty will be experienced in reading the lower voltages on account of the uneven scale of the electrostatic voltmeter. Thus a voltmeter reading up to 80 volts cannot be used for voltages much below 20, and a voltmeter reading to 160 will probably have a scale not extending below 80. For taking the lower readings, a small battery may be inserted in the position indicated by B in the lower part of Fig. 40. The voltage of this battery is then in series with the voltage to be measured, and should be chosen of a suitable value to bring even the lowest reading on to the scale. The true value of the voltage to be measured is obtained by subtracting the battery voltage from the voltage recorded on the voltmeter. This auxiliary battery has not to supply any appreciable current, and may consequently be formed of very small cells. These should give a fairly constant voltage, and must be well insulated. A battery of small secondary cells answers the purpose well, or a set of small dry cells, which are inexpensive.

An additional advantage gained by the addition of the battery is that it enables a distinction to be made between positive and negative values of the voltage, since the battery will be in series with the voltage when acting in one direction and in opposition to it when it changes sign and the volt curve crosses the zero line. Without such a device it is often difficult to judge the exact point of reversal, since the voltmeter readings are always in the same direction, irrespective of reversal of the applied voltage. A key should be arranged for throwing the battery directly on to V, so that its voltage may be read at short intervals, and subtracted from the total readings.

The same method of tracing curves may be applied to read the instantaneous values of the current in an alternating circuit, and so to obtain the current wave form. In this case the terminals of a non-inductive resistance in the circuit are connected periodically to the voltmeter by means of the contact maker. In a non-inductive resistance the voltage is always proportional to the current, and consequently the voltmeter readings when divided by the constant resistance give the values of the current.

By means of a throw-over switch the simultaneous values of current and voltage in a circuit may be obtained by connecting the contact maker and electrostatic voltmeter alternately to the terminals of the alternator, so as to read the voltage of the circuit, and to the terminals of a non-inductive resistance, so as to read the voltages proportional to the current.

This is the principle of the following experiment.

EXPERIMENT XVI.—DETERMINATION OF SIMULTANEOUS CURVES OF CURRENT AND VOLTAGE IN A CIRCUIT.

DIAGRAM OF CONNECTIONS.

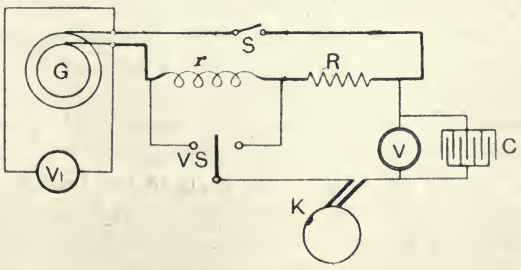


FIG. 41.

G Alternator armature.
R Non-inductive resistance.
r Resistance partly inductive.
K Rotating contact maker.
V Electrostatic voltmeter for reading instantaneous voltage.
V_1 Voltmeter for reading virtual voltage of alternator.
VS Throw-over switch.
C Condenser in parallel with electrostatic voltmeter.

Instructions.—Connect in series to the terminals of the generator a non-inductive resistance and the inductive resistances which are to form the circuit. Connect to the centre terminal of a throw-over switch an electrostatic voltmeter in series with the rotating contact maker.

Connect the other pole of the voltmeter to the terminal of the non-inductive resistance which is supplied direct from the alternator.

To one side of the 2-way switch connect one terminal of the generator; to the other side connect the terminal of the non-inductive resistance nearest to the same generator terminal. Connect a voltmeter to the terminals of the generator. If found necessary, connect a condenser in parallel with the electrostatic voltmeter. A battery may be used in series with the electrostatic voltmeter as described in the previous experiment, if necessary to read the low voltages.

For each position of the contact maker read the instantaneous voltage of the alternator, and the instantaneous voltage of the non-inductive resistance by putting the switch first to one side and then to the other. Throughout the experiment maintain the voltage of the alternator constant by regulating the field if necessary. Results should be entered in tabular form, and curves of current and voltage should be plotted on the same sheet of squared paper. The resulting curves will be similar to those shown in Fig. 7 if the alternator gives a true sine curve. Usually this will not be exactly the case, and the curve may more nearly resemble some of the less simple curves given later. Whatever be the shape of the curve, the angle of lag between current and voltage should be measured as the angle (plotted horizontally on the curve) between the points at which the curves of current and voltage cross the zero line.

A little care will have to be used in deciding which points near the zero line are positive and which are negative, since the voltmeter will always give readings in the same direction. If the contact maker is moved rapidly at this point, the sign of the voltage may change before the voltmeter needle has time to go to zero and rise again as the zero point is passed. By moving the contact maker slowly, and carefully watching the voltmeter, the approximate point of reversal can be obtained with certainty; the exact point can only be seen after plotting the curve. As already mentioned, the use of a battery is a great assistance at this part of the curve.*

Comparison of Values of Alternating and Direct Current.—The simplest standard of comparison between an alternating and direct current is that of their heating effects when passed through a resistance, since this is independent of direction and dependent only on the strength of the current.

The rate at which heat is developed in a conductor whose resistance is R ohms is C^2R joules per second where C = current in amperes. Thus a direct current of C amperes would develop C^2R joules in one second. An alternating current of *equivalent value* will also develop C^2R joules in one second. Hence the alternating current of C amperes, which is equivalent to a direct current of C amperes, must be such that the average value of C^2 (alternating) = average value of C^2 (direct), *i.e.*, $\sqrt{\text{average value of } C^2 \text{ alternating}} = \sqrt{\text{average value of } C^2 \text{ direct}}$.

Now $\sqrt{\text{the average value of } C^2 \text{ direct}}$ is C direct, but $\sqrt{\text{the average value of } C^2 \text{ alternating}}$ is not the same as the average value of C alternating.

The fact that the $\sqrt{\text{average value of } n^2}$ is not equal to the average value of n can be shown at once by taking a numerical example.

For instance, the average value of the numbers 1, 2, 3, 4 is

$$\frac{1 + 2 + 3 + 4}{4} = \frac{10}{4} = 2{\cdot}5.$$

* A slight modification of the arrangement shown in Fig. 41 enables the low values of current and voltage to be measured with increased accuracy. By connecting the voltmeter to read alternately the voltage across r, R and $R+r$ (instead of R and $r+R$ only), the separate readings will check each other. Thus, since the quantities read are *instantaneous* values, the sum of the voltages across r and R must be equal to the voltage of $R+r$, and thus the two readings across the separate resistances should always be together equal to the third reading. Since the circuit is inductive, it will seldom happen that more than one of the quantities is small at the same time. Thus the value of a low reading can generally be calculated as the difference of two readings of more convenient magnitude.

The average value of their squares is

$$\frac{1 + 4 + 9 + 16}{4} = \frac{30}{4}$$

The square root of this $= \frac{\sqrt{30}}{2} = \frac{5 \cdot 47}{2} = 2 \cdot 73.$

Consequently we see that the current which has the same heating effect as a direct current is not the current whose *average* value is the same as the direct current, but the current whose average value of the squares of its instantaneous values is the same as the square of the direct current.

The student should convince himself by actual trial of the difference between the average value and the root of the mean of values squared. For this purpose he should draw a curve such as that shown in Fig. 4, and measure the length of a number of equi-distant ordinates. By adding these lengths together and dividing by the number of ordinates taken, he will obtain the *average* value of the current represented by the curve. By squaring each ordinate, adding the ordinates together, dividing by the number of ordinates, and then extracting the square root, he will obtain the value of the alternating current which is the equivalent as regards heating of a direct current of that number of amperes.

It can be proved mathematically that the average ordinate of a sine curve is $\frac{2}{\pi} = \cdot 635$ of the maximum ordinate, whereas $\sqrt{\text{the average value of the squares of the ordinate}} = \frac{1}{\sqrt{2}}$ or $\cdot 707$ of the height of the maximum ordinate.

It is important always to remember that the *average* value of an alternating current is never employed, since it would not be equivalent to a direct current of the same number of amperes.

The value of the alternating current which is equivalent to a direct current as regards heating effect is called the *virtual, effective,* or *R.M.S.* (root of mean squares) value, and it is this which is recorded on an ammeter and is generally understood when an alternating current of a given number of amperes is spoken of.

The virtual value of an alternating current of regular wave form is therefore $\frac{1}{\sqrt{2}}$ or $\cdot 707$ of its maximum value.

Exactly the same statement applies to an alternating electromotive force, the virtual voltage being $\frac{1}{\sqrt{2}}$ of the maximum value.

Scales of Instruments. — Both ammeters and voltmeters give readings proportional to the mean values of the square of current or voltage, and would consequently give, in most cases, a nearly even scale for (amperes)^2 or (volts)^2, the amperes or volts being the virtual value of the current or voltage. The reason for this is as follows: The reading of all alternating-current measuring instruments is produced either by the expansion of a heated wire or by the mutual action of a fixed and a movable part. In hot-wire instruments the average heating effect on the wire must obviously be due to the average value of $C^2 R$ as explained above, and the expansion of the wire is approximately proportional to the heat given to it. In all other instruments the deflection depends upon the force with which the fixed and movable parts act upon one another. But both fixed and moving portions exert forces proportional to the current or voltage being measured. The resultant force depends, consequently, on the product of the two forces exerted by the fixed and moving part, *i.e.*, on the product of two quantities, each separately proportional to the quantity to be measured. It thus results that the average force producing the deflection is the average of a force due to (current × current) or (voltage × voltage), *i.e.*, to the average of the square of the quantity to be measured.

As stated above, the scale of an alternating current instrument would usually be evenly divided if graduated in (amperes)^2.

An excellent example of this is the Siemens dynamometer, in which the scale is uniformly divided, and the readings give the values of the average value of (current)^2. The effective current is obtained by extracting the square root of the reading.

For convenience in reading, the scale of most instruments is usually marked in volts instead of (volts)^2. The scale of a Cardew voltmeter would have divisions nearly equal in size if its readings were proportional to (volts)^2 instead of volts.

Thus it is seen that practically all direct-reading alternating-current ammeters and voltmeters have an uneven scale, since the distance between successive sub-divisions

would increase in the ratio of the square of the value of the quantity being measured, if the deflection is proportional to the deflecting force. In many instruments arrangements are adopted by which a more even graduation is obtained by making the deflection corresponding to a given force smaller at higher points of the scale, or by arranging that the action between fixed and moving parts becomes less as the deflection increases.

In order to make a distinction between the *maximum* value of alternating currents, voltages, &c., and their *effective* or virtual values, it is convenient to adopt the general convention of indicating the maximum value by a capital letter and the virtual value by the corresponding small letter.

Thus C, V, W represent maximum values;
c, v, w are virtual, or *R.M.S.* values.

Consequently for harmonic wave form we have

$$C = \sqrt{2}\, c \qquad V = \sqrt{2}\, v., \text{ \&c.}$$

Example.—As an illustration which should help to make clear the subject of the preceding paragraph, we may consider the relation between the reading of a Cardew voltmeter and the voltage applied to its terminals.

The Cardew voltmeter is of the hot-wire type, a long, thin wire of high resistance being heated by the current passing through it, and caused thereby to expand. The expansion of the wire produces a movement in a fine cord attached to a spring at one end of the wire, and causes the deflection of a needle attached to a small pulley round which the cord passes. The deflection of the needle is thus proportional to the expansion of the wire, which is, in turn, practically proportional to the rate at which heat is communicated to the wire by the current passing through it.

The rate at which heat is developed in the wire has already been stated to be C^2R joules per second. Since the value of C is constantly changing in the case of an alternating current, it is the average value of $C^2\ R$, which represents the average rate at which the wire is heated, and which, therefore, determines the deflection of the needle.

Suppose that the resistance of the wire of a Cardew voltmeter is 300 ohms, and suppose that the voltage represented graphically in Fig. 4, is applied to the terminals of the instrument. For the sake of convenience, however, we will assume the scale to be doubled, so that the maximum

voltage is 50. The maximum current, taken at any time by the wire, will be $\frac{50}{300} = \cdot 166$ amps., since 50 volts is the maximum value of the voltage. The maximum rate at which heat is given to the wire, is, therefore, $(\cdot 166)^2 \times 300 = 8 \cdot 33$ joules per second.

The average rate at which heat is communicated to the wire may be calculated roughly from the curve in the following manner. Dividing the complete cycle into six equal parts and estimating the average value of the electromotive force during each of these intervals, we may tabulate our results as follows :—

Interval.	Average E.M.F.	C	$C^2 R$
0° – 30°	12·9	·043	·555
30° – 60°	35·4	·118	4·18
60° – 90°	48·3	·161	7·78
90° – 120°	48·3	·161	7·78
120° – 150°	35·4	·118	4·18
150° – 180°	12·9	·043	·555

Mean value of $C^2 R$ for $\frac{1}{2}$ cycle = 4·172 joules per sec.

Since the curve is symmetrical and repeats itself, the average rate at which heat is given to the voltmeter wire when connected to the supply will be 4·172 joules per second and this will determine the deflection.

Comparing this value with the maximum rate of 8·33 joules per second, it is seen that the average rate of generation of heat in the wire is almost exactly one-half the maximum rate. (If the average value had been determined more accurately it would have been found to be exactly one-half.) Thus the deflection of an alternating-current voltmeter or ammeter is exactly one-half the deflection which would be observed if the value of the voltage or current were maintained constant at its maximum value.

In order to ascertain the direct-current voltage which would produce the same reading we must write

$$V_D^2 R = \frac{1}{2}(V_{A\,MAX.})^2 R$$

$$\therefore V_D = \frac{1}{\sqrt{2}} V_{A\,MAX}$$

where V_D = the value of the equivalent direct voltage.

$V_{A\,MAX.}$ = maximum value of alternating voltage.

The equivalent direct voltage is thus in the present case $\frac{50}{\sqrt{2}} = 35 \cdot 35$, and the alternating voltage would be spoken of as a virtual voltage of 35·35 volts.

Suppose, now, that the voltage applied to the voltmeter wire is doubled, the current which it will send through the wire will be doubled, and the value of C^2R throughout each cycle will be four times as great as before. Thus heat will be developed four times as rapidly and the expansion of the wire will be four times as great. Consequently, the deflection corresponding to $\frac{100}{\sqrt{2}}$ or 70·7 volts will be four times as great as for the original $\frac{50}{\sqrt{2}}$ or 35·35 virtual volts. The nature of the scale of a Cardew voltmeter is shown in Fig. 42, where the inner

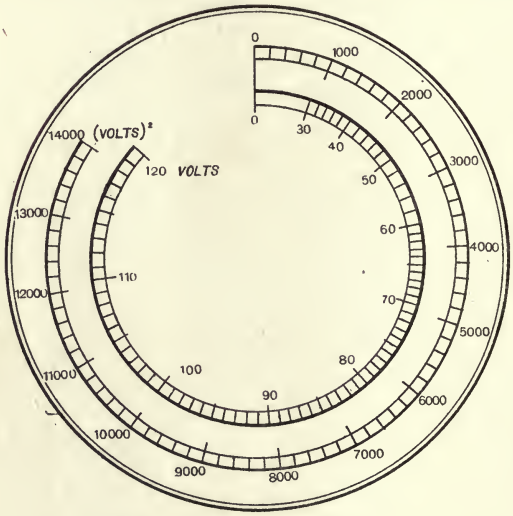


FIG. 42.—SCALES OF VOLTS AND (VOLTS)².

scale represents the scale of volts, and the outer evenly divided scale shows the graduation, which would be obtained by plotting the squares of the voltages, which are proportional to the expansion of the wire producing the deflections. It is thus seen on the figure that the deflection corresponding to 70 volts is four times the deflection for 35 volts, and that a practically uniform scale would be obtained if the graduations were measured in (volts)² instead of volts.

Average Power in a Circuit.—In the example which has just been given, it was shown that the *average* value of C^2R in an alternating circuit is half the *maximum* value. In a non-inductive circuit $C^2R = C\,E$, since the voltage E is equal to the product $C\ R$ of current resistance. Thus the *average* value of the power developed in a non-inductive circuit is half the *maximum* value of the power. In the case of measurement of power, it is the *average* value which is required for all purposes of comparison, since the total work done by the current in the circuit is the sum of all products of the form (power at any moment × time during which it is developed), this is equivalent to (*average power* × *time considered*). The work done in a circuit in five minutes, for instance, is (average value of watts × 5 × 60 secs.)

The maximum value of the watts in a non-inductive circuit = maximum amperes × maximum volts, hence

$$\text{average watts} = \frac{1}{2}\ (\text{maximum amperes} \times \text{maximum volts})$$

$$= \frac{1}{\sqrt{2}}\ \text{maximum amperes} \times \frac{1}{\sqrt{2}}\ \text{maximum volts.}$$

$$= \text{virtual amperes} \times \text{virtual volts.}$$

Hence we have the important relation for a non-inductive circuit.

Average power in circuit = product of virtual amperes and virtual volts.

The relation just obtained may be extended to an inductive circuit if we substitute the *energy* voltage for the total voltage. This is permissible, since the energy voltage is in phase with the current, and is the only part of the voltage which influences the power of the circuit.

The energy voltage = $e \cos. \phi$, and consequently in an inductive circuit

average watts = virtual amperes × virtual volts × cos. ϕ.

The above relations may be summarised in symbols as follows, capital letters representing *maximum* values, and small letters virtual values.

For non-inductive circuits

$$w = \frac{1}{2}\,W = \frac{1}{2}\,C\,E$$

$$\therefore \quad w = c.e.$$

For inductive circuits

$$w = \frac{1}{2} W = \frac{1}{2} C E \cos. \phi.$$

$$\therefore \quad w = c e \cos. \phi.$$

The same results may be obtained from slightly different considerations, as follows :—

In Fig. 26, page 59, is shown the curve obtained by multiplying the simultaneous instantaneous values of current and voltage together. This curve, being the product of the values plotted in the sine curves, is itself a sine curve, although displaced above the zero line. Its average value, with due regard to signs, can be shown mathematically to be half its maximum value. For the instantaneous value of the voltage $= E \sin \theta$, and the instantaneous value of this current $= C \sin \theta$, consequently the instantaneous value of the watts $= E C \sin^2 \theta$.

The average value of $\sin^2 \theta$ is $\frac{1}{2}$.

Hence average watts $= \frac{1}{2} C E$.

This has been shown experimentally from the curve on page 59. Its maximum value $= C.E = \sqrt{2} c \times \sqrt{2} e = 2 c e$. Consequently the average value of the power curve $= \frac{C E}{2} = \frac{2 c e}{2} = c e$. That is, the average power in an alternating circuit is the product of the virtual values of the current and voltage.

The same statement holds good when the current and voltage are not in phase, if we substitute the energy voltage for the total voltage. The energy voltage is $e \cos \phi$ (see page 64), and the power in an alternating circuit having an angle of lag ϕ is $c e \cos \phi$. This is, consequently, the value given by a wattmeter, connected to the circuit. The maximum value $= C E \cos \phi$, and the average value $= \frac{1}{2} C E \cos \phi$.

It is important to remember that it is the *average* and not the R.M.S. value of the watts which gives the true power in a circuit. A wattmeter deflection is always proportional to the watts at any instant (and not to watts ^{2}), and consequently gives the average power.

Chapter V.

Effect of Capacity.

Capacity in an Alternating Circuit.—All insulated conductors have the property of being able to hold electricity in its static form. When an insulated conductor is connected to a source having a different potential, electricity will flow to or from it until its potential is the same as that of the source. The amount of electricity held by a conductor when its potential is unity is called its *capacity*. Hence a conductor has a capacity equal to unity when one unit of quantity of electricity raises it to a potential of unity.

The practical unit of capacity is the *Farad*.

The unity of quantity of electricity is the *Coulomb*, and is the quantity which is transmitted by a current of 1 ampere in 1 second.

Thus the *charge* of a condenser = number of coulombs which have flowed into it = number of amperes × duration of flow in seconds.

$$\text{Also capacity} = \frac{\text{amount of charge}}{\text{potential due to charge}} = \frac{\text{amperes} \times \text{seconds}}{\text{volts}}.$$

The Farad being too large to correspond to the dimensions of capacities occurring in practice, the *microfarad*, *i.e.*, one-millionth of a Farad, is usually employed.

The capacity of a conductor depends on its size and shape and upon its surroundings.

A *condenser* may be any conductor having an appreciable capacity, although it usually denotes a special piece of apparatus consisting of thin sheets of metal separated by insulating material, alternate plates being connected together respectively to two terminals, so as to form two conductors of very large surface separated by a very thin layer of insulation. A condenser of this kind when connected to opposite poles of a generator will take considerable charges on the two sets of conducting plates, *i.e.*, the generator will supply a certain *quantity* of electricity, so as to charge the condenser. If the generator supplies alternating current, the charges given to the condenser plates will be alternatively positive and negative, and there will thus be an alternating flow of electricity

(*i.e.*, a current) between the machine and both poles of the condenser.

The following general explanation may help to make the succeeding more precise reasoning understood by those not already familiar with the nature of capacities.

Effect of Capacity in the Circuit: General Explanation.—If a constant electromotive force be applied to a circuit possessing neither self-induction nor capacity, a current is instantly produced the value of which is given by Ohm's law:

$$\text{Current} = \frac{\text{Electromotive force}}{\text{Resistance}}.$$

If, however, the circuit possesses electrostatic capacity as well as resistance, a charging current will have to flow into the circuit before the voltage acting upon the resistance rises to the full value of the applied electromotive force. Consequently, at the first moment of application of the voltage a current will flow into the circuit, and the strength of this current will depend mainly upon the capacity of the circuit, and only partly upon its resistance. It is only after this initial current has charged the circuit that the normal value of the current will be established, and the current strength will be that given by Ohm's law. Thus in a circuit containing a condenser or having electrostatic capacity there will be an initial rush of current which may be considered as flowing in advance of the normal current of the circuit.

This may be illustrated by the diagram Fig. 43. Suppose

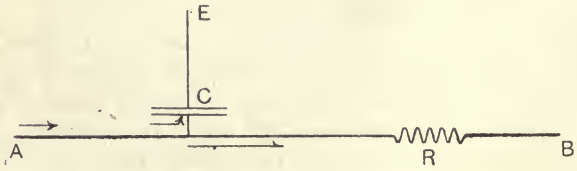


FIG. 43.—CIRCUIT POSSESSING CAPACITY AND RESISTANCE.

that C represents a condenser with one pole connected to the circuit, and the other pole earthed. A is a point in the circuit which is suddenly connected to a source of high potential. This will cause a flow of current from A to B. Before the high potential applied at A can produce a current through the resistance R, the potential at the end of R nearest to A

must be raised by the flow of electricity from A. The flow will, however, not reach R until the condenser has been charged, as the electricity flows into the condenser first, this being nearer to A. Consequently it is only after the charging current has raised the potential of the condenser plates to the potential of A that the full potential of A can act upon R so as to send current through it.

In the case of an alternating-current circuit possessing capacity there will be a charging current in the circuit at each reversal of the electromotive force, and the total current flowing will be the sum of the capacity-charging current and the normal current obeying Ohm's law. The charging current will be a quarter of a period in advance of the electromotive force, and hence the resultant current will always have its variations in advance of the voltage by the amount of an angle between zero and 90°, depending upon the relative magnitude of the resistance and capacity of the circuit.

When a condenser is first connected to the terminals of a source of electromotive force, equal and opposite charging currents will flow into the condenser from the two sides. That is, if we consider a positive charge to flow in at the positive terminal, a negative charge will simultaneously flow in at the negative terminal. But an oppositely-directed flow of electricity of opposite sign is equivalent to a direct flow of the same sign. Consequently current may be considered as actually flowing in the same direction in the circuit on both sides of the condenser, and is often said to flow "in" at one terminal and "out" at the other, or to flow *through* the condenser. As soon as the condenser is charged, the flow must necessarily cease until the potential of the source to which it is connected changes.

Relation Between Voltage and Current.

The following symbols will be employed :—

Q Quantity of electricity in coulombs.

K Capacity of condenser in farads.

E Potential (in volts) of terminal connected to source of higher electromotive force if other terminal is considered to be at zero potential, or E = difference of potential between terminals, if both are connected to sources of electromotive force.

C Current in amperes flowing into condenser.

When connected to a constant source of electromotive force, no current flows through the condenser. In this case

$$\text{Charge} = \text{coulombs} = K\,E.$$

During the process of charging,

Current flowing to condenser = rate of change of charge.

When connected to an alternating source of electromotive force the difference of potential between the terminals will change at the same rate as the alternating source. Consequently $Q = K\,E$ will change also. Since current = rate at which quantity flows, the rate of change of $K\,E$ (measured in coulombs per second) will be numerically equal to the current flowing to the condenser.

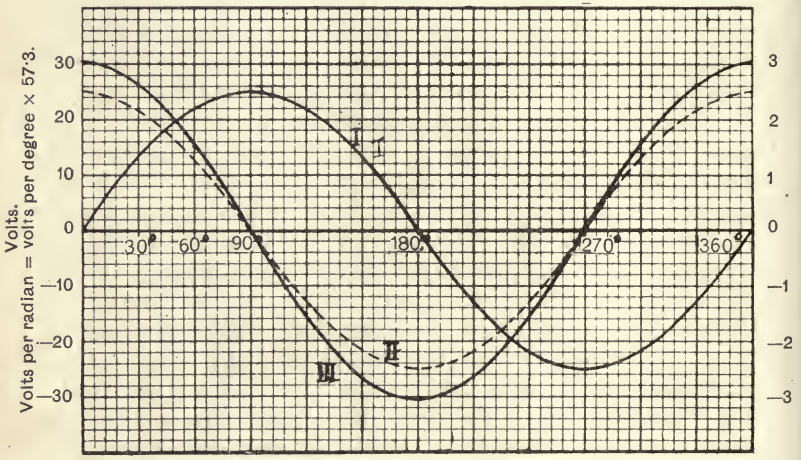


FIG. 44.—CURVES OF ELECTROMOTIVE FORCE, CHANGE OF ELECTROMOTIVE FORCE AND CHARGING CURRENT. CAPACITY = 200 MICROFARADS.

Hence charging current of condenser = capacity × rate of change of voltage at terminals.

It is now possible to represent graphically the relation between the voltage applied to a condenser and the charging current.

This is done in Fig. 44, where curve I is a curve of voltage whose maximum value is 25 volts. Curve II. is a curve of rate of change of voltage (measured in volts per radian*),

* The radian is the unit angle in circular measure, and is equal to about 57·3°. There are 2π radians in a complete circle, or 360°.

and may be obtained by a similar construction to that given on page 13 for finding the "rate of change" of a current. Since in this case the rate of change is measured per radian, instead of per cycle, the maximum height of the curve obtained is equal to the maximum height of the curve of voltage. The maximum rate of change of volts per cycle would be 2π times as great. Curve III. is the charging current plotted to a scale of amperes, each value being obtained from curve II. by multiplying by the factor ·125, which is equal to the product of the capacity of the condenser ($=200$ microfarads) and $2\pi n$ the number of radians per second at a frequency of 100. The maximum current is thus found to be 3·14 amperes.

If E = maximum value of voltage
then instantaneous value of voltage $= E \sin \theta$
and instantaneous value of rate of change of voltage
$= E \sin (\theta + 90°) = E \cos \theta.$
Hence value of current at any instant $= K E \cos \theta.$

It will be seen, both from the figure and from the expressions just given, that the variation of the charging current will be similar to those of the voltage in character, but will always occur *one-quarter period earlier in phase.* Hence the charging current will be similar to the *idle current* discussed on page 74, except that its phase is $\frac{1}{4}$ period *before* that of the voltage of the circuit instead of being $\frac{1}{4}$ period later. By the same reasoning as that given previously it is evident that the charging current is a wattless or idle current.

In a circuit possessing both capacity and resistance there will be a resultant current and voltage differing in phase.

In a circuit possessing both capacity and resistance two currents will arise when an electromotive force acts on the circuit, viz., (1) the energy current in phase with the voltage, (2) a charging current leading the voltage in phase by $\frac{1}{4}$ period. The total current flowing at any instant will be the algebraic sum of the instantaneous values of these two. The resultant current may therefore be represented by a rotating line obtained in magnitude and relative phase as the diagonal of a rectangle having the maximum value of the two component currents as the length of its sides. The reason in this case follows exactly as in the case of the resultant of two electromotive forces given on page 32.

This resultant current will differ in phase from the voltage, but will lead by an angle ϕ depending on the relative magnitude of the resistance and capacity.

The charging current will be the component which is one-quarter period in advance of the voltage, while the energy current will be the component in phase with the voltage. As in the case of an inductive circuit, the power given to the circuit is the product of energy current and voltage or the product of total current × total voltage × cos ϕ, when ϕ is the angle of lead of current and cos ϕ is the power factor of the circuit.

The relation between these quantities is shown in diagram form in Fig. 45, where OE is the total voltage and OC the

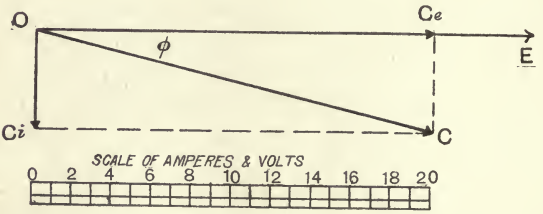


FIG. 45.—DIAGRAM OF VOLTAGE AND CURRENTS IN CIRCUIT, WITH CAPACITY.

resultant current in the circuit. The charging current is OC_i, and the energy current OC_e.

The angle of lead is $C_e\ O\ C$, *i.e.*, the angle between total voltage and total current.

The values of the quantities represented are

Total voltage $= E = 25$ volts.

Energy current $= OC_e = 20$ amps.

Idle current $= OC_i = 2\pi n\ KE = 5$ amps.

Total current $= OC = \sqrt{20^2 + 5^2} = 20{\cdot}6$ amps.

Impedance of Circuit having Capacity.—In the case of a circuit with capacity as with self-induction, the impedance or apparent resistance is the value of the fraction

$$\frac{\text{voltage of circuit}}{\text{current in circuit}}$$

and in this case also the impedance, or apparent resistance, is always greater than the true, or ohmic, resistance.

Its value may be obtained by drawing a triangle of impedance similar to that obtained on page 35 for an inductive circuit. The components of the impedance at right

angles to each other will be, respectively, R, the resistance of the circuit, and $\frac{1}{2\pi nK}$ due to the capacity, since

$$\frac{\text{total voltage}}{\text{charging current}} = \frac{E}{2\pi nKE} = \frac{1}{2\pi nK}$$

The resultant of these components representing the total impedance of the circuit $= \sqrt{R^2 + \left(\frac{1}{2\pi nK}\right)^2}$

The results now obtained are illustrated in the following experiments :—

EXPERIMENT XVII.—DETERMINATION OF THE EFFECT OF A CONDENSER UPON THE PHASE RELATIONS IN A CIRCUIT.

DIAGRAM OF CONNECTIONS.

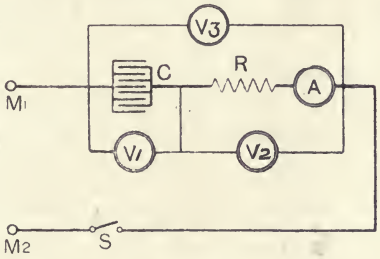


FIG. 46.

M_1 M_2 Source of alternating current.
R Non-inductive resistance.
C Condenser.
A Ammeter.
V_1 Voltmeter for reading voltage across condenser.
V_2 Voltmeter for reading voltage non-inductive resistance.
V_3 Voltmeter for reading total voltage of circuit.

Instructions.—Connect in series to the source of supply a condenser or other form of capacity, a non-inductive resistance, ammeter, and switch.

Connect voltmeters, or a single voltmeter with 3-way switch, to read respectively the voltage across the capacity, resistance, and total circuit.

For several values of the current obtained by variation of the resistance R take readings on the voltmeters and ammeter.

For each set of readings construct a triangle of electromotive forces and deduce the angle of lead of the circuit by measurement of the angle from the diagram. Calculate in each case the power factor by dividing the energy voltage by the total voltage. From the value of the power factor ($= \cos \phi$) find the value of ϕ from a table of cosines.

Readings should be tabulated as follows :—

Determination of Phase Relations in Circuit with Capacity.

Voltage of Circuit $= V_3$	Voltage of Non-inductive Resist. $= V_2$	Voltage across Condenser $= V_1$	Current $= C$	Angle of Lag from Diagram.	Power Factor $\frac{V_2}{V_3}$	Angle of Lag by Calculation $\cos \frac{V_2}{V_3}$	Impedance $= \frac{V_3}{C}$
114	26	111	·76	76·5	·228	76·75	150
112·5	53·4	99	68	61·5	·475	61·75	165·5
110	85·7	69	·47	38·5	·779	38·75	234

Periodicity of circuit = 54.

From results of this experiment the value of the capacity can be calculated, as well as the value of the resistance of the circuit.

Thus the idle voltage $V_1 = \frac{C}{2 \pi n K}$, from which the value of K can be calculated*.

Further, the energy voltage $= V_2 = C R$, from which the value of R can be determined.

The figures given in the table above are three sets of readings taken from a series by way of illustration. The diagram of voltage corresponding to these results is shown in Fig. 47. In the case of the third set of readings, the value of the capacity is given by the formula thus :—

$$K = \frac{C}{2 \pi n V_1}$$

$$= \frac{\cdot 47}{2 \pi \times 54 \times 69} = \cdot 00002 \text{ farad.}$$

$$= 20 \text{ microfarads.}$$

Also the resistance of the circuit is given by

$$R = \frac{V_2}{C} = \frac{85 \cdot 7}{\cdot 47} = 182 \text{ ohms.}$$

This resistance was actually a single 16 c.p. 100-volt incandescent lamp.

* This calculation can only be employed when the wave form of the current is practically a sine curve.

Obviously the readings taken would enable us to determine the power of the circuit, since we have determined the total current and voltage, and also the power factor of the circuit. The total power = amperes × volts × power factor.

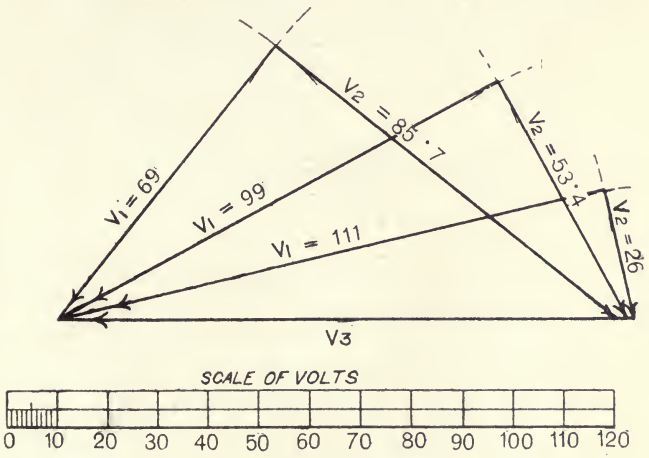


FIG. 47.—DIAGRAM OF VOLTS FOR CIRCUIT, WITH CAPACITY.

The calculation might also be carried out as in the Experiment XIII. by the 3-voltmeter method.

This calculation was carried out for all the readings of the series from which the readings in the table above were taken. At the same time the power was measured on a wattmeter, and except in the case of the first reading, where the power factor is very low, the results of the calculation agreed with the readings of the wattmeter within a fraction of 1 per cent.

The idle and energy currents in the circuit might be calculated from the readings taken in the last experiment, since idle current = $C \sin \phi$, and energy current = $C \cos \phi$.

Instances of circuits containing sufficiently large capacities to seriously affect the nature of the current chiefly arise where long feeders have to transmit current to a considerable distance. Instances of a current which leads the voltage in phase from other causes are, however, not at all infrequent, and will be referred to later in connection with the running of synchronous motors.

The following experiments illustrate the effect of the various quantities in an alternating circuit possessing capacity upon the impedance.

EXPERIMENT XVIII.—DETERMINATION OF DEPENDENCE OF CURRENT IN A CIRCUIT UPON CAPACITY.

DIAGRAM OF CONNECTIONS.

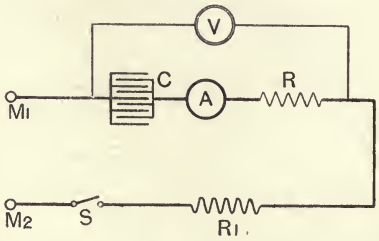


FIG. 48

M_1 M_2 Source of alternating current.
R' Non-inductive resistance of known value.
R Variable non-inductive resistance for maintaining the voltage of V constant.
C Condenser capable of giving several values of capacity.
A Ammeter for measuring current in circuit.
V Voltmeter for measuring voltage of portion of circuit consisting of C and R.
S Switch for breaking circuit.

Instructions.—Connect in series a known non-inductive resistance R, condenser capable of giving several values of capacity, an ammeter, and variable non-inductive resistance. Connect a voltmeter across the condenser and resistance R, so as to read the drop in both together.

Supply the circuit with alternating current, and maintain constant the periodicity and the voltage as read on the voltmeter.

First, short-circuit the resistance R, so as to read the current in the circuit having capacity and a negligibly small resistance. Take a series of readings, each corresponding to a different value of the capacity.

Then, for one or two values of the resistance R take similar sets of readings, keeping the resistance constant during each set.

Unlike the case of an inductive resistance with an iron core, the capacity of the condenser, and consequently the impedance of the circuit, will be the same for all values of the current.

Determination of Dependance of Current upon Capacity.

Resistance in Series with Condenser.	Capacity Microf.	Current.	Voltage.	Impedance.
0	2	·059	100	1695·0
0	10	·295	100	339·0
0	20	·590	100	169·5

Periodicity = 32 cycles per second.

The voltage should be kept constant by means of an additional variable resistance R_1 in the circuit or by regulation of the generator voltage. For slight variations the current readings may be corrected by increasing them in the same ratio as any decrease in the voltage observed, and vice versâ.

In this experiment, since the total current is small, care must be taken not to introduce errors due to the voltmeter current. If the voltmeter is not of the electrostatic type, it should be connected so that its current does not pass through the ammeter.

Enter the readings in tabular form as shown, and plot a curve for each set of readings, plotting capacity horizontally and current vertically.

Plot also a curve comparing impedance and capacity with the same horizontal scale as before.

The curves given in Fig. 49 show the results obtained with a subdivided condenser of 20 microfarads capacity, three of the readings being shown on the table above. The curves drawn as a continuous line show the readings with the condenser alone in circuit. The dotted curves show the results with a resistance of 4 ohms in series with the condenser. Throughout the experiments the voltage was maintained at 100, the frequency being 32.

With low values of the capacity the current is very small, and the resistance is practically without effect; it becomes relatively more important as the capacity and

current in the circuit increase. Evidently when the capacity of the condenser is zero no current will flow,

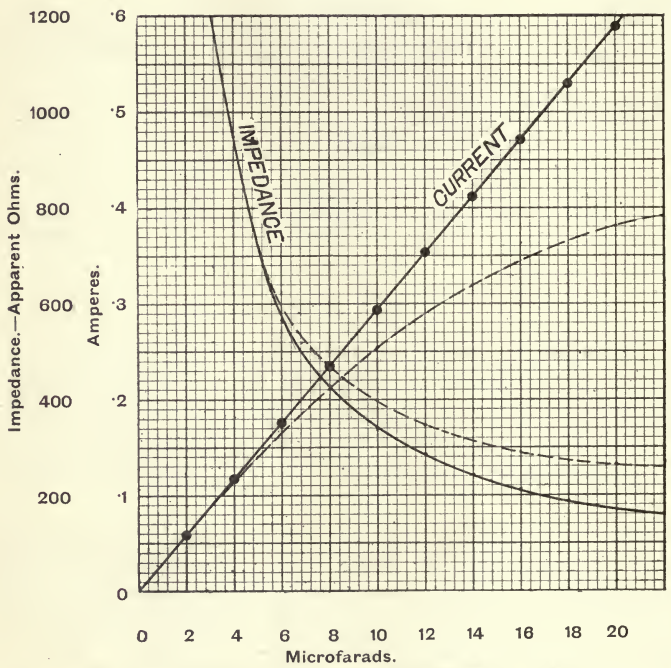


FIG. 49.—VARIATION OF CHARGING CURRENT WITH CAPACITY.

since the condenser will then be simply a break in the continuity of the circuit, making all current impossible.

The flow into and out of a condenser depends not only on the capacity of the condenser, which determines the amount of the charging current at each reversal, but also on the frequency of charge and discharge, *i.e.*, on the periodicity of the current. As already expressed in the formulæ given on page 102, the current varies directly with the frequency. This follows immediately from the fact that a current is a rate of flow, or displacement of electricity, and consequently varies directly as the number of charges per second.

The next experiment illustrates these statements.

EXPERIMENT XIX.—DETERMINATION OF DEPENDENCE OF CHARGING CURRENT UPON FREQUENCY.

DIAGRAM OF CONNECTIONS.

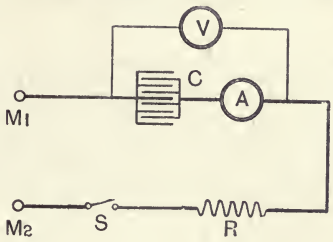


FIG. 50.

M_1 M_2 Source of alternating current.
C Condenser.
V Voltmeter.
A Ammeter.
R Adjustable resistance for keeping voltage across condenser constant, if this cannot be done by other means.
S Switch for breaking circuit.

Instructions.—Connect in series a condenser and ammeter and switch. Connect these to a supply of alternating current of variable frequency. Unless this supply can be maintained at a constant voltage, a regulating resistance must be put in the circuit, so that the voltage across the condenser may be maintained at a constant value. Connect a voltmeter across the condenser. If this voltmeter is not of the electrostatic type it must be connected across the ammeter also, so as to prevent the ammeter reading the voltmeter current, or must be disconnected when the current readings are being taken.

For a range of different frequencies of current, adjust the condenser voltage to a constant value, and observe the frequency and current. The frequency will usually have to be determined by a speed counter applied to the alternator supplying the circuit, the frequency being given by—

$$\text{Frequency} = \frac{n \times p}{60}$$

n = revs. per min.
p = No. of pairs of poles.

The readings should be entered in tabular form, and a curve plotted with frequency, or revolutions per minute, horizontal and charging current vertical.

Fig. 51 shows the results of an experiment carried out in this way on a 20 microfarad condenser, the current

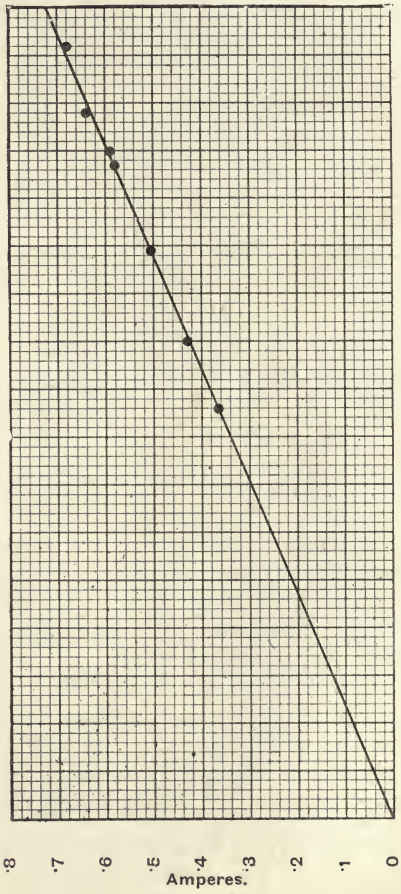


FIG. 51.—VARIATION OF CHARGING CURRENT WITH FREQUENCY.

being read on a hot-wire ammeter. The alternator was a 4-pole machine, and the voltage was adjusted by variation of its excitation and not by a resistance as indicated in the diagram of connections above. The voltage was maintained at 100. The curve given by the readings is

seen to be a straight line, showing that the current increases in direct proportion to the frequency.

The curve goes through zero, since with a frequency of *0* the current would become a continuous current, which can only charge the condenser once, and then cease to flow.

The results may be used to check the accuracy of the formula given previously, viz. :—

$$C = 2\pi\, n\, K\, E.$$

For the last reading at a speed of 1620, we should have as the value of the current

$$C = 2\pi \frac{2 \times 1620}{60} \times \cdot 0002 \times 100 = \cdot 678 \text{ amp.}$$

The actual value observed was ·68 amp.*

The preceding experiments have shown how the capacity and frequency of charge and discharge of a condenser affect the amount of current in the circuit.

The quantity of each charge depends not only on the capacity of the condenser, but also on the voltage to which the condenser is charged each time, *i.e.*, on the effective terminal voltage. This is the point to be illustrated in the next experiment.

EXPERIMENT XX.—DETERMINATION OF EFFECT OF TERMINAL VOLTAGE UPON CHARGING CURRENT OF A CONDENSER.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XIX., Fig. 50, page 109.

Instructions.—Connect a condenser in series with an ammeter to the source of alternating current. Arrange matters so that the terminal voltage of the condenser can be varied, either by means of a variable resistance in the circuit, as shown in the diagram, or by means of a field-regulating resistance in the generator field circuit. Connect a voltmeter to the condenser terminals if the voltmeter is electrostatic, or so as to measure the voltage

* Again it must be mentioned that such results are only obtainable with an alternator giving sine wave form. With any other wave form we should still obtain a straight line as a result of the experiment, but its inclination would be different, and the current would not be exactly that given by the formula. The formula would only be true of each of the harmonics of which the wave is formed.

across both ammeter and condenser, if the voltmeter takes current which might affect the ammeter readings.

Throughout the experiment keep the frequency constant, and vary the voltage at the condenser terminals. For each value of the voltage read the value of the current.

Enter readings in tabular form. A curve should be plotted with voltage horizontal and charging current measured vertically.

Fig. 52 shows the curve plotted from a set of readings obtained with a 20 microfarad condenser at 54 periods per second. Again the results give a straight line, showing that the charging current varies in direct

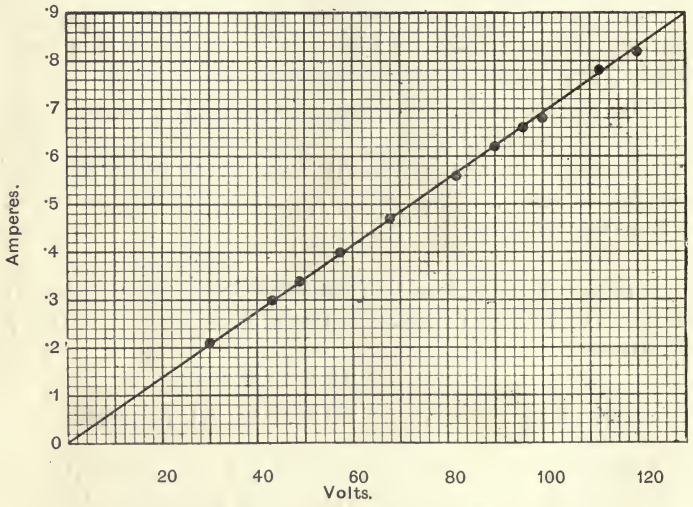


FIG. 52.—VARIATION OF CHARGING CURRENT, WITH VOLTAGE.

proportion to the voltage applied to the condenser. As these readings were taken with an alternator giving a sine wave form, a similar calculation might be made to confirm the results as given in the case of the previous experiment.

Measurement of Power in a Circuit possessing Capacity.—From what has already been said, it will be apparent that the power in a circuit possessing capacity cannot be determined by a simple measurement of the current and

voltage. The power can, however, be obtained by the use of a wattmeter, or by either of the methods given in Experiments XIII. and XIV. for inductive circuits. The diagrams of connections there given apply to the present case also, with the substitution of the capacity for the inductance. A special form of experiment is consequently not necessary.

The following record of an actual experiment will serve as a guide as to the observations to be taken. The readings practically form a companion experiment to the 3-voltmeter method given as Experiment XIII.

As in that case, the total power in the circuit is given by the formula

$$W = \frac{V_3^2 + V_2^2 - V_1^2}{2R}$$

while the power in the portion exclusive of the non-inductive resistance is

$$W^1 = \frac{V_3^2 - V_2^2 - V_1^2}{2R}$$

When V_3 = total voltage of circuit.
V_2 = voltage of non-inductive portion.
V_1 = voltage across portion including capacity.
R = value of non-inductive resistance.

Example of Determination of Power in Circuit containing capacity.—The actual connections made are shown in the diagram Fig. 53, which is identical in principle with that shown in Fig. 37, page 78, but shows the modification, already frequently alluded to, of employing a single voltmeter with two-way switches. It is to be noted that

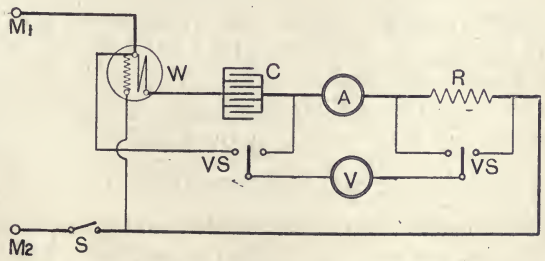


FIG. 53.—DIAGRAM OF CONNECTIONS FOR DETERMINATION OF POWER IN CIRCUIT WITH CAPACITY.

in this case the voltmeter was of the dynamometer pattern, and the switches are arranged so that its current does not pass through the ammeter, as its amount would not be negligible compared with the small current in the circuit.

The non-inductive resistance R was a bank of lamps; the condenser had a capacity of 20 microfarads. The periodicity was 54 per second.

Determination of Power in Circuit with Capacity.

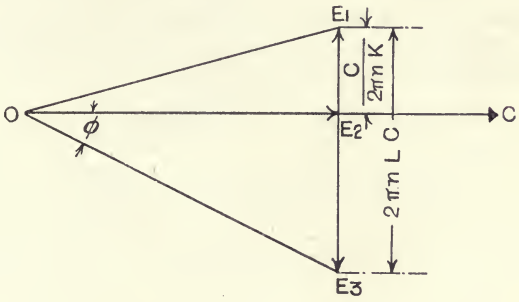
Watt-meter Reading.	Total Volts V_3	Non-inductive Resistance Volts V_2	Condenser Volts V_1	Current C	Calculated Power of Circuit $\frac{V_3^2+V_2^2-V_1^2}{2R}$
0	115·0	0	115	·79	0
20	114·0	26·0	111	·76	19·4
30	113·0	41·7	105	·72	30·0
37	112·5	53·4	99	·68	37·0
40	111·5	65·8	90	·62	40·0
40	110·0	85·7	69	·47	40·0

Capacity and Self-induction in Series.—It has been shown that the effect of self-induction in a circuit is to cause the current to lag in phase behind the voltage, and that a capacity causes the current to lead in phase. When both capacity and self-induction are present, the effect is to produce a difference of phase between current and voltage, which is the difference between the angle of lag or lead, which would be due to either acting separately.

This result is best shown in diagram form by drawing a line $O\ C$ to represent the current horizontally (see Fig. 54). The energy voltage spent in overcoming the non-inductive resistance of the circuit ($=C \times R$) and the iron losses is then marked off as the length $O\ E_2$ to the scale of volts. At E_2 the line $E_2\ E_1$ is drawn vertically upwards to represent the volts spent in overcoming the capacity of the circuit $\left(=\frac{C}{2\pi n K}\right)$. From E_1 the length $E_1\ E_3$ is measured downwards to represent the voltage overcoming self-induction ($=2\pi n L C$). The triangle $O\ E_2\ E_3$ then represents the following quantities, viz.: The total voltage $O\ E_3$, the resultant idle voltage $E_2\ E_3$, and the energy

voltage $O E_2$. The angle ϕ, which may be either an angle of lead or of lag, is then the angle $E_2 O E_3$.

The importance of this combination is chiefly due to the use of condensers in certain special cases, in order to



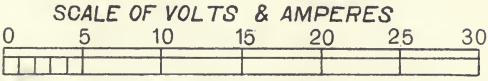


FIG. 54.—DIAGRAM OF CURRENT AND VOLTAGE IN CIRCUIT WITH CAPACITY AND SELF-INDUCTION.

reduce the angle of lag in an inductive circuit and thus to improve the power factor.

Impedance of Circuit containing both Inductance and Capacity.—Since the energy voltage is $C R$, and the idle voltage of the circuit is $\left(2\pi n L C - \frac{C}{2\pi n K}\right)$, the expression for the total voltage may be written

$$E^2 = C^2 \left[R^2 + \left(2\pi n L - \frac{1}{2\pi n K} \right)^2 \right]$$

$$\text{Hence impedance} = \frac{E}{C} = \sqrt{R^2 + \left(2\pi n L - \frac{1}{2\pi n K} \right)^2}$$

If ϕ = the angle of lag of the circuit, from the Fig.

$$\tan \phi = \frac{C\left(2\pi n L - \frac{1}{2\pi n K}\right)}{C R}$$

$$= \frac{2\pi n L}{R} - \frac{1}{2\pi n K R}$$

$$\text{and the angle of lag} = \tan^{-1}\left(\frac{2\pi n L}{R} - \frac{1}{2\pi n K R}\right)$$

These are the general expressions for the impedance and angle of lag in an alternating circuit. If the circuit is non-inductive L becomes zero.

The following experiment serves to illustrate what has just been said.

EXPERIMENT XXI.—DETERMINATION OF POWER AND PHASE RELATIONS IN A CIRCUIT CONTAINING RESISTANCE, SELF-INDUCTION, AND CAPACITY.

DIAGRAM OF CONNECTIONS.

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

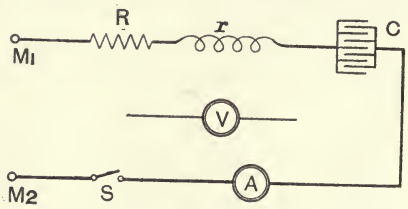


FIG. 55.

$M_1 M_2$ Source of alternating current.
R Non-inductive resistance.
r Inductive resistance.
C Condenser.
A Ammeter for measuring current in circuit.
V* Voltmeter for measuring respectively voltage of inductive and non-inductive resistance, condenser, and total circuit.
S Switch for breaking circuit.

Instructions.—Connect in series to the source of supply a non-inductive resistance, inductive resistance, condenser, and ammeter. Connect a voltmeter to read in turn the total voltage of the circuit, and the voltage across each resistance and the condenser, and across the inductive and non-inductive resistances together. It is an advantage to insert a wattmeter in the circuit to serve as a check on the accuracy of the other readings.

For several values of the non-inductive resistance and current, take readings of the current and voltage of inductive and non-inductive resistances, and also the resultant voltage across both resistances. Read also the voltage at

* Only a single voltmeter is actually necessary, and should be connected successively to the different terminals indicated in the diagram.

the terminals of the condenser and the total voltage of the circuit. For each set of readings construct a diagram showing the relation between the voltages of the circuit, proceeding as in the example given below. Hence calculate the total power of the circuit and the power in the inductive and non-inductive portions.

Power and Voltages in Circuit with Capacity, Resistance, and Self-induction.

Volts Across.							Watts in		
Total Circuit V_6	Capacity V_3	Inductive Resistance V_1	Non-inductive Resistance V_2	Capacity + Inductance V_5	Inductive and Non-inductive Resist. V_4	Amps.	Total Circuit	Non-inductive Resistance	Inductive Resistance
97·0	125	92·0	45 (45)	61 (60·9)	119	·78	67 (67·2)	34·8 (35·7)	32·2 (32·5)
90·0	110	84·0	40·8 (40·7)	55·5 (55·24)	110	·69	57 (56·77)	28·3 (28·2)	28·7 (28·57)
80·0	91	74·8	35 (34·5)	49·5 (49·6)	98	·57	43 (42·23)	19·56 (19·67)	23·5 (23·26)
72·2	76	69·5	31 (30·2)	43 (43·2)	90·5	·475	33 (32·92)	14·3 (14·35)	18·7 (18·53)
60·0	62	52·6	27 (26·9)	35 (35·4)	71·5	·38	21·74 (21·62)	10·43 (10·22)	11·31 (11·40)
70·0	47·5	42·0	42 (42·4)	30 (29·6)	76	·297	20·43 (20·315)	12·6 (12·6)	7·83 (7·72)
50·2	32	22·0	39 (38·9)	(16)	52·5	·20	9·78 (9·72)	7·83 (7·78)	1·95 (1·94)

The method of drawing the diagram and determining the phase relations between the voltages of the various portions of the circuit are best discussed in connection with an actual example.

The readings tabulated in the annexed table show the readings obtained in an experiment in which a 20 microfarad condenser formed the capacity, incandescent lamps were used as the non-inductive resistance, and two sections of a small transformer were employed as inductive resistance. In this case a wattmeter was also used to check the accuracy of the results. The figures given in brackets are calculated values, the other figures were those actually observed.

It will be convenient to adopt a convention frequently employed, of drawing energy voltage on the diagram

horizontally, so that idle voltages due to self-induction (which precede the current in phase) are drawn vertically downwards, and idle voltages due to capacity (and which lag behind the current) are drawn vertically upwards.

The voltage V_2 in the non-inductive resistance is drawn horizontally as OA (see Fig. 56), since

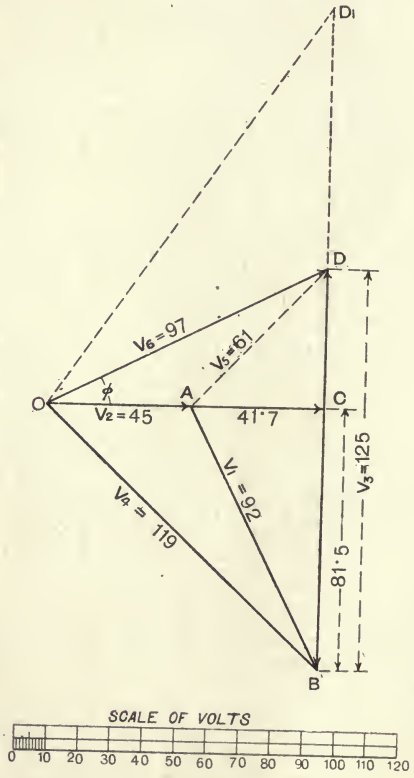


FIG. 56.—DIAGRAM OF VOLTAGES IN CIRCUIT WITH CAPACITY AND SELF-INDUCTION.

this is an energy voltage, and in phase with the current. The voltage V_1 taken across the inductive resistance r will consist partly of energy and partly of idle volts, and is drawn downwards from A, making an obtuse angle with OA. In order to determine the correct position for V_1 on the diagram, describe a circle with

radius equal to V_1 from A as centre, and a second circle about O of radius V_4, the resultant voltage across both inductive and non-inductive resistances. The intersection of these circles will give the point B, and AB represents in phase and magnitude the voltage V_1.

The triangle OAB is exactly similar to the triangle obtained in Experiment XIII. in a circuit with inductance and resistance only. The total energy voltage of the circuit is obtained by drawing a vertical line BC to meet OA produced in C. Then OC is the energy component of the total voltage of the circuit. The line CB represents the idle voltage due to self-induction

The condenser voltage will be in phase with BC, but opposite in direction, or, more accurately, it will be 180° out of phase with BC.

Hence the condenser voltage V_3 is represented by the vertical line BD measured from B upwards. Finally, the total voltage of the circuit is represented in phase and magnitude by the line OD, which in the case for which Fig. 56 is drawn is seen to be later than the current in phase, so that there is a leading current in the circuit. The line BC represents the portion of the condenser voltage which is neutralised by the idle voltage of self-induction of r. Thus, if the circuit had been non-inductive, the resultant voltage of the circuit would have been the dotted line OD^1, where CD^1 is equal to the condenser voltage V_3 or BD. It is thus seen that the voltage required to send a given current may be reduced by the presence of both inductance and capacity. If the inductance had been greater, so that $CB = BD$, the self-induction and capacity would exactly have neutralised each other, and the voltage of the circuit would only be of the magnitude necessary to send the given current through a non-inductive resistance.

The diagram shown in Fig. 56 is drawn for the first set of readings given on the table above. It would form an excellent exercise for the student to draw diagrams for the remaining readings, if he has not the opportunity of carrying out a set of readings for himself.

Determination of Power from the Diagram.—Having drawn this diagram, it is an easy matter to calculate the power in each part of the circuit from it.

In each case the power = current × energy component of voltage. Thus the power in the inductive portion r is the product of the current, and the energy component of AB, *i.e.*, AC.

$$\text{Power} = W_r = C \times AC.$$
$$= \cdot 78 \times 41 \cdot 7 = 32 \cdot 5 \text{ watts.}$$

Similarly in the total circuit, the energy component is represented by the line OC, and the power

$$= W_t = C \times OC.$$
$$= \cdot 78 \times 86 \cdot 7 = 67 \cdot 63 \text{ watts.}$$

In the non-inductive resistance R the power

$$= W_R = C \times OA.$$
$$= \cdot 78 \times 45 = 35 \cdot 1 \text{ watts.}$$

In the condenser the power is zero, since the voltage is entirely idle voltage $\frac{1}{4}$ period behind the current.*

Evidently the power in the circuit could be calculated directly from the voltmeter readings, in the same way as in Experiment XIII.

The triangle OAD would in this case be the triangle whose sides would represent the voltages to be employed in the calculation. Another and more direct way would be to calculate the power from the triangle OAB obtained from the inductive and non-inductive portions of the circuit, and omit the consideration of the voltage across the condenser. Obviously this might not be possible if the capacity were not confined to one portion of the circuit.

Resonance.—The conditions just alluded to may give rise to what is called "Electric Resonance." It has been shown in the preceding discussion that when special relations exist between the capacity K, the self-induction L, and the frequency in a circuit, the voltage at the terminals of the self-induction and capacity connected in series, may be less than the voltage at the terminals of one of them. Thus, if the voltage of an alternator be connected to a circuit containing a condenser and choking coil, it is possible that the voltage at the terminals of the condenser or choking coil might be greater than the voltage supplied by the alternator. An example will serve to illustrate this.

* The small losses due to resistance and dielectric hysteresis in the condenser are neglected as being too small to affect the results.

The total voltage of the circuit

$$= \sqrt{(\text{energy voltage})^2 + (\text{idle voltage})^2}.$$

Suppose the energy voltage to be due to 10 amperes flowing through ·5 ohm, its value will be

$$10 \times \cdot 5 = 5 \text{ volts.}$$

Let the inductance idle voltage $= 2\pi n L c =$ 200 volts, and the capacity idle voltage $= \frac{-C}{2\pi n K}$ $= -160$ volts, the negative sign indicating that the phase of the capacity idle voltage is opposite to that of the inductance idle voltage.

The total idle voltage of the circuit is then $= 200 - 160 = 40$ volts.

The total voltage of the circuit $= \sqrt{5^2 + 40^2} = \sqrt{25 + 160} = \sqrt{185} = 13{\cdot}6$ volts, whereas the voltage in the inductance is 200, and across the condenser is 160.

Cases of this kind frequently occur in practice in connection with long concentric mains which have considerable capacity. The self-induction of the generator armature, of the cable itself, or of machines connected to the cable, may supply the inductance in the circuit.

In the ideal case where no losses occur in the circuit due to resistance, &c., if $2\pi n L c = \frac{c}{2\pi n K}$ the voltage of the condenser and choking coil will be exactly equal and opposite. Consequently, a very small voltage applied to the circuit might theoretically give rise to an indefinitely high voltage in each portion. The action is due to the periodicity of the current being exactly so timed in this case as to correspond with the natural period of charge and discharge of the condenser under the conditions existing in the circuit. A repetition of a small periodic electromotive force of this periodicity is thus sufficient to produce very large currents and consequently high voltages at the terminals of the condenser.

The expressions given above enable a determination to be made of the conditions under which resonance can occur, and the extent to which it will exist in each portion of the circuit.

Capacity and Inductance in Parallel.—In the previous paragraph it was found that the effect of a capacity and self-induction connected in series was that the idle electro-

motive force produced by each was partly or entirely neutralised by the other. Thus, for the particular case, when $2\pi n L c = \frac{c}{2\pi n K}$ the resultant idle electromotive force is zero, and the combination offers no opposition to the changes of the alternating current.

When the capacity and self-induction are in parallel, the electromotive force at their common terminals depends only upon the voltage of the supply, and is clearly independent of the nature of the two branch circuits. It is, however, possible for the inductance and capacity to have a combined effect on the current in the external circuit, for there may be a lagging current produced in the branch containing self-induction which may partially or entirely neutralise the leading current flowing in the branch containing capacity as regards the external circuit.

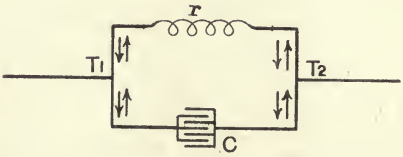


FIG. 57.—CONDENSER AND INDUCTANCE IN PARALLEL.

Suppose, for instance, that an inductive resistance r with an inductance L henries, and a condenser C with capacity K farads, are connected in parallel, as shown in Fig. 57.

If $2\ \pi\ n\ L\ c = \frac{c}{2\ \pi\ n\ K}$, the lagging current flowing from T_1 to T_2 (Fig. 57) through the branch r will be equal to $\frac{V}{2\ \pi\ n\ L}$ if the resistance of the circuit is negligible, and this will be equal to the leading current $2\ \pi\ n\ K\ V$ flowing through the branch C. The total current flowing from T_1 to T_2 is thus zero so far as the external circuit is concerned, since the currents in the two branches are always simultaneously equal and opposite in phase, although a circulating current will flow in alternate directions round the circuit $T_1 C\ T_2\ r$. It will thus appear that whatever voltage may be applied between the terminals

T_1 and T_2 no current will apparently pass from one to the other. This condition could, of course, only exist when both branches are without resistance and sources of loss, such as hysteresis or eddy currents.

Example.—As an example of what has just been said may be quoted an experiment of Mr. W. N. Mordey.*

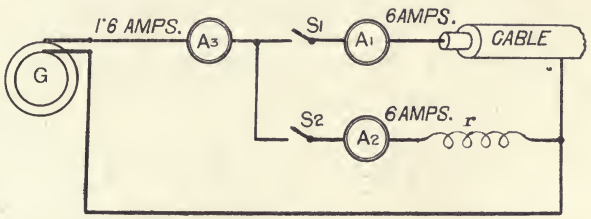


FIG. 58.—INDUCTIVE RESISTANCE NEUTRALISING IDLE CURRENT DUE TO CAPACITY OF CABLE.

Mr. Mordey connected an alternator giving 2,050 volts at 100 cycles to a $5\frac{1}{2}$-mile length of 37/15 concentric rubber-covered cable, one pole of the alternator being connected to the inner core and the other pole to the outer core, which was earthed, as was also the steel sheathing.

The capacity of the cable between conductors was ·86 microfarad per mile, thus the virtual current taken from the alternator was calculated to be

$$C = 2\pi n E K = \frac{2\pi \times 100 \times 2{,}050 \times \cdot 86 \times 5\cdot 5}{1{,}000{,}000}$$

$$= 6\cdot 092 \text{ amps.}$$

The current actually observed was 6 amps. A choking coil was then put in a circuit connected to the alternator in parallel with the cable. The connections were then as shown in Fig. 58. The coil was so designed that with switch S_1 open and S_2 closed, so that the alternator supplied current to the coil only, the current taken by the coil was 6 amps.

On closing both switches, so that the alternator supplied the two circuits in parallel, the current given out by the alternator was found to fall to 1·6 amps. Thus ammeters A_1 and A_2 both indicated 6 amps., while A_3 only registered 1·6 amps.

* Proc. Inst. Elec. Eng., vol. xxx., p. 378.

The explanation was that the current in the inductive circuit lagged behind the voltage of the alternator, while the current in the capacity circuit was in advance of the voltage, so that the two branch currents were nearly opposite in phase, and neutralised each other to a large extent.

Chapter VI.

The Transformer.

The Transformer.—One of the greatest advantages of alternating over direct currents is the ease with which an alternating current may be converted from a low current at high pressure to a high current at low pressure of approximately equal power, and vice versâ, without the employment of rotating machinery.

The transformer is the means by which this transformation is accomplished. The principles underlying its construction and behaviour are illustrated in the experiments which follow.

General Principles. — Fig. 59 represents a rectangular frame consisting of a number of soft-iron stampings of the shape shown, placed side by side, so that the section of the frame is approximately square. This forms a magnetic

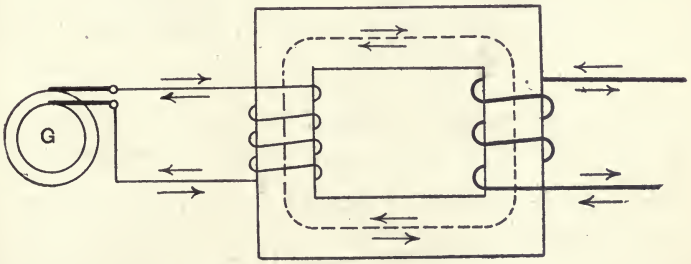


Fig. 59.—Principle of the Transformer.

circuit of very high permeability without any air gap. Two separate windings of insulated wire are indicated as being wound upon this magnetic circuit, one winding being connected to an alternator. A current sent through either winding will produce a magnetic flux in the *core*, which is the name usually applied to the magnetic circuit of a transformer. The general direction of this flux is indicated in the diagram by a dotted line. An alternating current flowing in the coil connected to the alternator will produce an alternating field in the magnetic circuit as indicated in the diagram, where the upper arrows show the direction of current and

flux during one-half period, and the lower arrows indicate their direction after reversal of the current.

Let Z = maximum number of lines of force produced in the core by the current,

n = periodicity of current,

N_1 = number of turns composing the winding I. connected to the alternator,

N_2 = number of turns composing the other winding II.

E_1, E_2 = maximum value of the voltage at the ends of the windings I. and II.

The number of lines will change with the current producing them, and will vary harmonically, if the alternator voltage supplies current having a sine wave form.

The electromotive force generated in one turn of winding = rate of change of magnetic lines $\div 10^8$.

The maximum rate of change of the lines may be shown to be $2\pi n Z$ by similar reasoning to that given on page 13 in connection with the rate of change of current.

Hence, maximum electromotive force induced in a single turn $= E = \frac{2\pi n Z}{10^8}$,

and the maximum voltage induced in winding (II.) having N_2 turns $= E_2 = \frac{2\pi n Z N_2}{10^8}$.

The *virtual* voltage corresponding to this maximum value $= e_2 = \frac{1}{\sqrt{2}} \cdot \frac{2\pi n Z N_2}{10^8} = \sqrt{2}\pi n Z N_2 10^{-8}$, or $e_2 = 4{\cdot}44\, n Z N_2 10^{-8}$, which is a fundamental formula for the voltage induced in the winding of a transformer.

But the magnetic flux passes through the windings of the coil I. connected to the alternator, as well as through winding II. Consequently, there will be induced a back voltage in coil I., which we have previously called the back electromotive force of self-induction, opposing the voltage of the alternator. The value of this back voltage is obviously

$$e_1 = 4{\cdot}44\, n Z N_1 10^{-8}$$

since it may be calculated by the same reasoning as that just given, and it is induced in a winding having N_1 turns.

On account of the complete magnetic circuit of soft iron on which the coil I. is wound, its self-induction is exceedingly high, and the impedance of the winding is very great, and

due almost entirely to self-induction. Consequently, the applied electromotive force is nearly all spent in overcoming the self-induction, and only an exceedingly small part of it is required to produce an energy voltage overcoming the resistance of the winding.

Thus in the case of a transformer with unloaded secondary winding, the back voltage very nearly equals the applied voltage. In such a case if v is the alternator voltage $e_1 = v$ very nearly, consequently the current in the winding is exceedingly small.

In a transformer that winding which has a voltage applied to it from an external source and which carries the magnetising current is called the **Primary winding.**

The **Secondary winding** is the winding which also embraces the magnetic circuit, and through which consequently the magnetic lines also pass, and which has a voltage induced in it due to the magnetic changes set up by the change of current in the primary.

Thus with no load on the secondary the ratio of voltages will be very nearly

$$\frac{\text{Primary voltage}}{\text{Secondary voltage}} = \frac{4{\cdot}44\, n\, Z\, N_1}{4{\cdot}44\, n\, Z\, N_2} = \frac{N_1}{N_2}$$

$$= \frac{\text{No. of primary windings.}}{\text{No. of secondary windings.}}$$

Effect of Current in Secondary Circuit. — The preceding statements were based on the assumption that the magnetising current in the primary winding was the only current acting upon the magnetic circuit of the transformer. If the secondary circuit is closed through a resistance, the voltage induced in the winding by the alternating field will produce a current in it.

The direction of this current will be opposite to that of the current in the primary, because the voltage induced in the secondary is due to the same causes, and is in the same direction relative to the magnetic circuit as the *back* voltage of the primary.

The current in the primary is sent by the alternator in opposition to the back voltage of the winding, but in the secondary it is the induced voltage which produces the current, which, therefore, circulates in the opposite direction round the magn tic circuit to the current in the primary winding.

In other words, this current will be ½ period out of phase with the current in the primary (this is shown to be the case in the diagram given later), and hence the field due to the secondary current varies similarly to that due to the primary winding, but is always opposite to it in direction. The effect of the secondary current is consequently to weaken the main field produced by the primary current in the magnetic circuit of the transformer. The weakening of the alternating field by the secondary current will at once diminish the back electromotive force in the primary winding, since this is due to the alternating flux. In consequence of this the applied voltage, being now opposed by a less back electromotive force, will send an increased current through the primary winding; this will in turn again increase the strength of the field. Matters will finally adjust themselves to the condition in which the alternating flux is equal to its former value* (*i.e.*, practically of the strength sufficient to produce a back electromotive force equal to the applied electromotive force), the primary current having increased exactly in the proportion necessary to counteract the opposing magnetising effects of the current in the secondary winding.

The magnetising effect of the current in either winding is proportional to the number of turns in the winding multiplied by the current flowing in the winding. Hence a certain current in the secondary will not produce an *equal* increase of current in the primary, but an increase which bears to the secondary current the inverse ratio of the number of turns in the two windings. Thus if the primary have 10 times the number of turns of the secondary, the increase in current in the primary will be one-tenth of the current of the secondary. This may be put more briefly by saying that the total magnetic flux is constant for a constant applied voltage, and consequently the resultant ampere-turns acting on the circuit must be a constant, and hence, if we neglect the no-load current, $N_1 C_1 = N_2 C_2$ In general, the following relation is very nearly true †

$$\frac{\text{Primary current}}{\text{Secondary current}} = \frac{\text{Secondary turns}}{\text{Primary turns}} = \frac{N_2}{N_1}$$

The experimental verification of the statements made regarding the voltage and current ratios will be found in all

*The question of drop in the conductors is considered later.

† The no-load current which makes it not exactly true is discussed later.

the experiments which follow, and a special experiment for this purpose is therefore not given.

Voltage Drop in Transformer.—The current flowing in both primary and secondary windings requires a certain amount of voltage to be spent in overcoming the resistance of these windings. Let

c_1 = current in primary winding.
c_2 = current in secondary winding.
r_1 = resistance in primary winding.
r_2 = resistance in secondary winding.
T_1 = number of primary turns.
T_2 = number of secondary turns.

The loss of voltage in primary windings due to their resistance = $c_1 r_1$ volts.

The effect of this drop is to make the effective voltage spent in producing the alternating flux in the core smaller by this amount. Consequently, if e_1 = primary applied voltage, the effective voltage which produces the flux affecting the secondary winding = $e_1 \backsim c_1 r_1$.

Here the symbol $\backsim$ indicates the *vector difference* between the two quantities and not the simple algebraic difference. It must be remembered that the applied voltage will not be in phase with the energy voltage $c_1 r_1$. Thus the voltage induced in the secondary winding is

$$(e_1 \backsim c_1 r_1) \times \frac{T_2}{T_1}.$$

The voltage at the terminals of the secondary will only have this value when the secondary current is zero. When a current flows in the secondary, there will be a loss of voltage due to the resistance of the windings, and consequently the secondary voltage

$$= e_2 = e_1 \frac{T_2}{T_1} \backsim \left(c_1 r_1 \frac{T_2}{T_1} + c_2 r_2 \right)$$

But $$\frac{c_1}{c_2} = \frac{T_2}{T_1} \text{ or } c_1 = c_2 \frac{T_2}{T_1}.$$

$$\therefore e_2 = e_1 \frac{T_2}{T_1} \backsim \left(c_2 \frac{T_2}{T_1} r_1 \frac{T_2}{T_1} + c_2 r_2 \right)$$

$$= e_1 \frac{T_2}{T_1} \backsim c_2 \left[r_1 \left(\frac{T_2}{T_1} \right)^2 + r_2 \right]$$

or putting $\frac{T_1}{T_2} = k$, the ratio of the windings,

$$e_2 = \frac{e_1}{k} \backsim c_2 \left[\frac{r_1}{k^2} + r_2 \right]$$

If there were no loss of voltage e_2 would be $\frac{e_1}{k}$, consequently the drop of voltage in the transformation is given by $c_2 \left[\frac{r_1}{k^2} + r_2 \right]$, which is directly proportional to the secondary current.

The result of the resistance of the windings is thus to produce a variation in the ratio of transformation, the secondary voltage being less with increased loads. It is usual to take account of this in designing the number of turns of the windings, and to make this ratio such that the secondary voltage will be slightly too high at light loads, and slightly too low at the maximum load. In this way the exact voltage required is approximated to at average outputs.

The current given out by the secondary winding is usually called the "load current," or sometimes simply the "load."

EXPERIMENT XXII.—DETERMINATION OF REGULATION OF A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

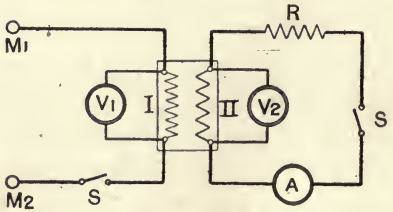


FIG. 60.

M_1, M_2 Source of alternating current.
I., II. Primary and secondary windings of transformer.
R Variable non-inductive resistance for varying transformer load.
V_1, V_2 Voltmeters for reading primary and secondary voltage.
A Ammeter for reading output of secondary.
S, S Switches for breaking primary and secondary circuits.

Instructions.—Connect the primary of the transformer to a source of alternating current.

Connect the secondary to a variable non-inductive resistance in series with an ammeter and a switch.

Connect voltmeters to the terminals of the primary and secondary windings.

Beginning with open secondary circuit, read primary and secondary voltage. Then close the switch, and repeat the readings for increasing values of secondary current. The primary voltage and frequency should be maintained constant throughout the experiment. It is most important to remember that it is no use to determine the drop in the transformer windings until they have attained their normal working temperature. It will therefore be necessary to run a large transformer for several hours before taking readings in order to make them correspond with working conditions. Small transformers will arrive at steady temperatures much sooner. It is usually convenient to make the regulation test during, or after, the trial run in the case of a new transformer.

Tabulate the results under headings, as follows :—

REGULATION TEST OF A TRANSFORMER.

Transformer No..... Type.....

Output.....kw.. .....cycles per second.

Ratio of transformation .....to.....volts.

Current Output $= C_2$.	Primary Voltage $= V_1$.	Secondary Voltage $= V_2$.	Ratio of Transformation $= \frac{V_1}{V_2}$.	Drop in Transformer† $= \frac{V_1}{k} - V_2$ or $V_{02} - V_2$

† $k = \frac{\text{Number of primary windings}}{\text{Number of secondary windings}} = \frac{\text{No-load primary voltage.}}{\text{No-load secondary voltage.}}$

V_{02} = no-load secondary voltage.

At no-load the ratio of primary to secondary volts will be practically equal to the ratio of the number of the windings.

If no loss occurred in the transformer this would, therefore, be the ratio of transformation at all loads. The difference between this theoretical value of the secondary voltage and the actual value is termed the " drop " of the transformer.

Two curves should be plotted from the results of this experiment; the first one is ratio of transformation, which should be plotted vertically upon a base representing secondary current.

Plot also a curve, comparing secondary voltage and output current, plotting current horizontally and voltage vertically. Draw through the no-load voltage a horizontal line. The ordinate between this horizontal line and the curve of voltage gives the value of the drop corresponding to any load.

The curve shown in Fig. 61 gives the results of a test made in this way on a small 1 kw. transformer with a nominal voltage ratio of 100 : 50. The primary voltage was kept constant at 100. In this case a single curve represents both secondary voltage and ratio of secondary to primary voltage on the two scales given at the side of the Fig. 61.

A careful measurement of the resistance of the windings of this transformer made while they were still warm gave the following results:

$$r_1 = \cdot 685 \text{ ohm.}$$
$$r_2 = \cdot 183 \text{ ohm.}$$

The ratio of the number of turns was $\frac{248}{128} = 1 \cdot 935$.

Inserting these quantities in the formula just given the following values for the copper drop in the secondary voltage are obtained.

$$\text{Drop} = C_2 \left[\frac{r_1}{k^2} + r_2 \right]$$

At full load of 20 amps. $C_2 = 20$.

$$\therefore \text{Drop} = 20 \left[\frac{\cdot 685}{1 \cdot 935^2} + \cdot 183 \right] = 20 \ (\cdot 183 + \cdot 183)$$
$$= 7 \cdot 3 \text{ volts.}$$

Referring to Fig. 61 the actual loss of voltage between no load and full load is seen to be $51 \cdot 4 - 41 \cdot 2 = 10 \cdot 2$ volts.

Probably magnetic leakage will account for the greater part of the difference between the 10·2 observed and the calculated drop of 7·3 volts. The magnetic leakage increases, like the drop in the conductors, with the load on the transformer. The value, $10 \cdot 2 - 7 \cdot 3 = 2 \cdot 9$ volts, will not, however, represent exactly the value of the magnetic

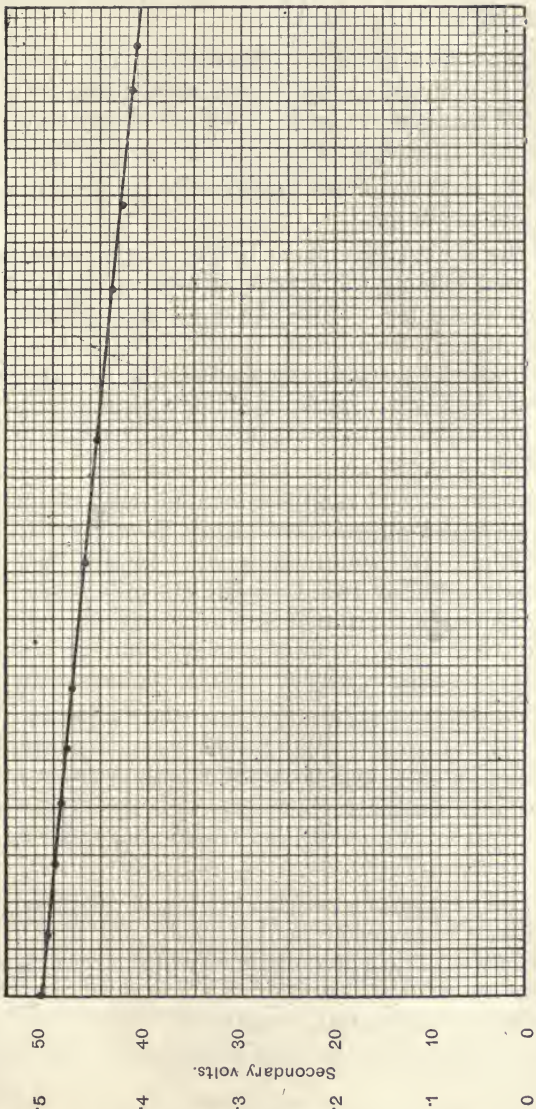


FIG. 61.—CURVE OF SECONDARY VOLTAGE AND RATIO OF TRANSFORMATION OF TRANSFORMER.

leakage drop, since the copper drop and the applied voltage are not exactly in opposition of phase, although they are nearly so at full load.

The method just given for determining the drop in a transformer is difficult to carry out with great accuracy, especially if the voltages to be dealt with are fairly high.

For instance, an error of 1 per cent. in the readings of the voltmeter may produce an error of 10 to 20 per cent. or more in the value of the drop.

Thus if the secondary voltage is 200, it will probably be difficult to read reliably the difference between 199 and 200 volts on the voltmeter. Since the total drop even at high loads would possibly be only six to eight volts, it is evident that the percentage accuracy in readings of the drop will not be high. Also, if the primary voltage is high, say 1,000 volts, it is impossible to regulate this so as to maintain it at exactly the same value.

The following method has the advantage that the drop of voltage is measured directly, instead of being only a small fraction of the total volts registered. In this case two similar transformers, or two transformers giving the same ratio of transformation, are required, and are both connected together to the supply mains—primary and primary or secondary and secondary, whichever is the most convenient.

When two similar transformers have their primary windings connected to the same supply, their secondaries will then give the same voltage, and if joined together in opposition would exactly neutralise each other. Consequently a voltmeter connected in series with the two windings under these conditions would read zero.

If, now, one transformer be loaded, it will give a slightly lower secondary voltage on account of the drop in its windings. The voltmeter would then give a reading which is the difference between the secondary voltage of the transformer without drop and that of the loaded transformer. That is to say, the voltmeter will read the drop in the loaded transformer directly.

It is obvious that the second transformer is prevented from contributing current to the load circuit by the voltmeter in series with it. (See Fig. 62.)

Experiment XXIII.—Determination of Drop in Transformer. (Direct Reading Method.)

Diagram of Connections.

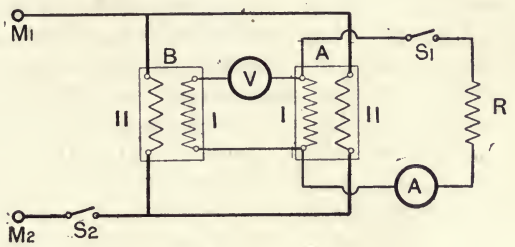


Fig. 62.

M_1, M_2 Source of alternating current.
I., *II.* Primary and secondary windings of transformers *A* and *B*.
R Variable non-inductive resistance for altering load of transformer *A*.
A Ammeter for reading output of *A*.
V Voltmeter for reading drop of transformer *A*.
S_1 Switch for breaking load circuit.
S_2 Switch for connecting transformers to supply.

Instructions.—Connect* both transformers to the source of supply through a switch, primary or secondary windings being chosen according to the voltage available.

Connect the free windings of the two transformers together in opposition, inserting a low-reading voltmeter in the circuit. Connect one of these windings also to a load circuit comprising a variable resistance, ammeter, and switch.

Before putting the low-reading voltmeter in the circuit, it is necessary to make sure that the windings connected to it are in opposition and not in series. If they are in series, the voltmeter will get double the voltage of a single transformer. First, therefore, put in place of the voltmeter a set of lamps in series or a high-reading voltmeter, or a thin piece of fuse wire, according to voltage to be expected. Close switch S_2, and note if there is considerable voltage. If so, interchange the connections of one transformer.

* The advantage of making the connection of both transformers to the supply by a single switch is that the voltmeter does not receive a high voltage, as might be the case if one transformer were connected to the supply before the other.

After connecting to the supply, note the reading of the voltmeter with the switch S_1 open. If the transformers are exactly similar, this will be zero.

Close the switch, and gradually increase the load on the transformer A, until the maximum load contemplated is reached. For each value of the load read the voltmeter. The voltage applied to the transformers should be maintained constant, although a small variation will not affect the results, since the voltage of both transformers will vary together.

The readings should be tabulated and plotted in the form of a curve, load current being measured horizontally and voltage drop vertically.

The curve shown in Fig. 63 shows results obtained in this way on a 1 kw. transformer having a ratio of transformation

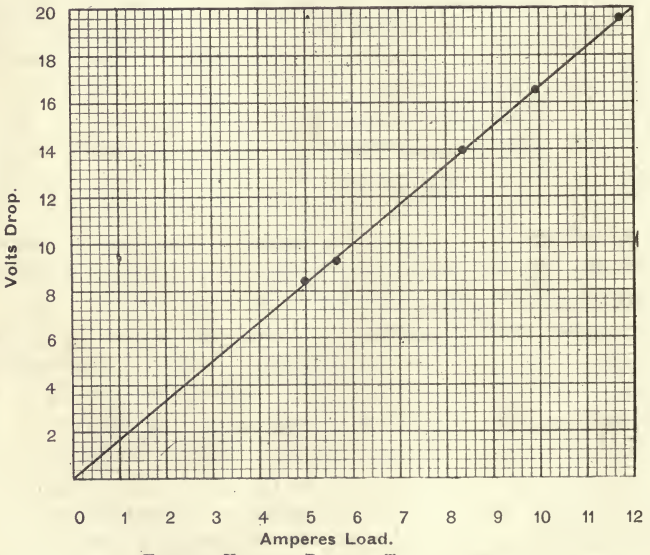


FIG. 63.—VOLTAGE DROP IN TRANSFORMER.

of 100 to 50 volts. In accordance with the formula given on page 130 for the copper losses in a transformer, the drop is seen to be proportional to the load. In the present case the drop due to magnetic leakage was very small, but in all cases the loss of voltage due to this cause would be approximately proportional to the load, and the curve showing the voltage drop would be practically a straight line. This is

owing to the fact that at the comparatively low saturation employed in the cores of transformers, both induction and leakage field increase in almost direct proportion to the magnetising current.

Fig. 63 should be compared with Fig. 61, as both curves were taken on the same transformer. Full load corresponds to 20 amperes in Fig. 61, since the current is measured on the low-pressure winding, whereas in Fig. 63 the current is measured on the high-pressure winding, and full load is 10 amperes.

Employing the formula for drop in the windings in the same way as on page 132, k = ratio of transformation now becomes $\frac{T_2}{T_1}$ instead of $\frac{T_1}{T_2}$.

Thus we have in this case the following expression for the drop at full load instead of that given on page 132 :—

$$\begin{aligned} \text{Drop} &= 10\,[\cdot 685 + (\cdot 183 \times 1\cdot 935^2)] \\ &= 10\,(\cdot 685 + \cdot 685) \\ &= 13\cdot 7 \text{ volts.} \end{aligned}$$

From the curve Fig. 63, the drop at 10 amps. is seen to be 16·6 volts, *i.e.*, 2·9 volts greater than the calculated drop, exactly as in the case of the previous experiment.

It is to be noticed that the voltage drop when transforming up and when transforming down is not the same.

The ratio of the resistances of the two windings is generally approximately that of the square of the number of turns, as in the present case.

The same method may be carried out, using two windings on the same transformer, instead of two separate transformers.

Measurement of Ratio of Transformation.—The following is a simple method of comparing the number of windings of two coils of a transformer not requiring a supply of alternating current. It may be used to determine the ratio of transformation of a transformer.

The two windings to be compared are connected in parallel to a couple of cells through variable resistances of known value ; for instance, through two arms of a post-office Wheatstone's bridge. A galvanometer or milli-voltmeter is connected to another winding. The resistances are then so adjusted that no deflection of the galvanometer is produced on closing the switch in the battery circuit. The ratio of the windings will then be equal to the ratio of the

two resistances. The resistances of the windings must be small compared with the resistances in series with the cell.

EXPERIMENT XXIV.—DETERMINATION OF RATIO OF WINDINGS OF A TRANSFORMER (WITHOUT ALTERNATING CURRENT).

DIAGRAM OF CONNECTIONS.

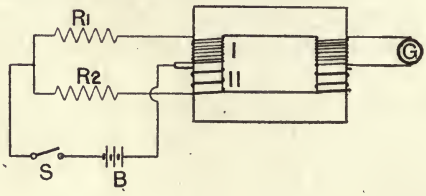


FIG. 64.

I., *II.* Windings whose ratio is to be determined.
R_1, R_2 Adjustable known resistances (*e.g.*, the arms of a P.O. Wheatstone's bridge).
B Battery of a few cells.
G Galvanometer or milli-voltmeter.
S Switch.

Instructions.—Connect the two windings to be compared in parallel to a battery consisting of two or three cells in series with a key-switch, inserting in series with each winding a variable known resistance, *e.g.*, a plug resistance box. Care must be taken that the windings are connected in such a way that the field due to the currents in them are oppositely directed. Connect a third winding on the same magnetic circuit to a galvanometer or milli-voltmeter. Adjust the resistances in series with the coils until no throw of the galvanometer is produced on breaking the battery circuit. When this has been done, the ratio of the windings will be the same as the ratio of the resistances in series with them. The resistances employed should be large enough to make the resistance of the transformer windings negligible in comparison with them. If this is not the case, the resistance of the windings must be included in the calculation, the ratio of the number of turns in the windings being equal to the ratio of the total resistances of the two branch circuits.

The principle of the method is as follows :—

The magnetising effect of each winding is proportional to the number of turns × current sent through it by the battery.

Or lines due to one coil $= Z_1 = k\ N_1 C_1$, k being an unknown constant.

Also the current is equal to the voltage of the battery divided by the resistance of the branch circuit or $C_1 = \frac{E}{R_1}$, E being voltage of battery.

$$\therefore Z_1 = k\frac{N_1 E}{R_1}.$$

Similarly, the lines due to the current in the secondary coil

$$= Z_2 = k\frac{N_2 E}{R_2}.$$

When the galvanometer is unaffected by the starting or stopping of the current, the fields due to the two windings exactly neutralise each other, *i.e.*, $Z_1 = Z_2$

$$\therefore \frac{N_1 E}{R_1} = \frac{N_2 E}{R_2} \text{ or } \frac{N_1}{N_2} = \frac{R_1}{R_2}.$$

The method of measurement just given is to be considered as a convenient approximate test for use on occasions when an alternating supply is not readily available rather than a reliable method of determination.

No-load Current of a Transformer.—In order to produce the magnetic field in the core of a transformer, a certain amount of *magnetising current* is necessary. The amount of this current is determined in the same way as the magnetising current of a direct-current electromagnet. At any instant the value of the current is given by the formula generally used for direct-current calculations :

$$C t = \frac{l B''}{\mu} \times \cdot 313$$

or

$$C t = \frac{l B}{\mu} \times 2 \cdot 02$$

or,

$$C t = \frac{\lambda B}{\mu} \times \cdot 8$$

where

C = current in amperes.
t = number of windings.
l = length of path in inches.
λ = length of path in cms.
B'' = induction in lines per square inch.
B = induction in lines per square cm.
μ = permeability of core.

In an alternating circuit the values of C and B are, however, constantly changing. Hence if B is the *maximum* value of the induction, C will be the maximum value of the current. Thus a virtual value of $c = \frac{C}{\sqrt{2}}$ will correspond to a maximum induction of B lines as given by the formula above.

Hence if we use the virtual value of the current which would be read on an ammeter, we must write the formula thus :—

$$c\,t = \frac{l\,B''}{\sqrt{2}\,\mu} \times \cdot 313$$

i.e.,

$$c\,t = \frac{l\,B''}{\mu} \times \cdot 221$$

or,

$$c\,t = \frac{l\,B}{\mu} \times 1 \cdot 435$$

or,

$$c\,t = \frac{\lambda\,B}{\mu} \times \cdot 565$$

Thus in order to produce a given alternating flux in the magnetic circuit of a transformer there must be supplied a definite magnetising current, the actual value of which will depend on the number of magnetic lines and upon the permeability of the core. Actually the value of the permeability will not be constant, since it will vary during each period as the instantaneous values of the current and field fluctuate. For low values of B, such as are used in practice for transformers, no great error is introduced by taking the permeability as constant, and equal to its value at the maximum induction due to the current employed.

As in the case of a continuous-current electromagnet, no power is spent in the production of the magnetic field directly, although in both cases a loss necessarily occurs in the resistance of the windings. An additional source of loss exists with an alternating field, owing to the imperfect nature of the iron, which absorbs power in overcoming its molecular friction or hysteresis, and also has local eddy currents produced in it, which entail a loss of power. The power, which is thus always required in order to produce an alternating field in the core of a transformer, is really spent in overcoming incidental losses, and not in producing the alternating flux, which, as already stated, does not require an expenditure of energy. The magnetising current, which is given by the formula above, is an idle current, not requiring any power

for its production. The no-load current of a transformer consists, consequently, of two portions, namely, the true magnetising current, which being an idle current lags quarter of a period behind the applied voltage, and the energy current required to overcome the hysteresis and eddy current losses as well as the losses in the primary winding. This current is in phase with the primary voltage.

As usual, the total current may be obtained graphically from these two component currents, by representing them as two sides of a right-angled triangle, of which the no-load current is the third side. The value of the no-load current may consequently be expressed thus :—

No load current

$$= \sqrt{(\text{magnetising current})^2 + (\text{energy current})^2}.$$

The composition of the no-load current will be rendered more clear by the following experiment :—

EXPERIMENT XXV.—DETERMINATION OF MAGNETISING CURRENT OF A TRANSFORMER AT VARIOUS VOLTAGES.

DIAGRAM OF CONNECTIONS.

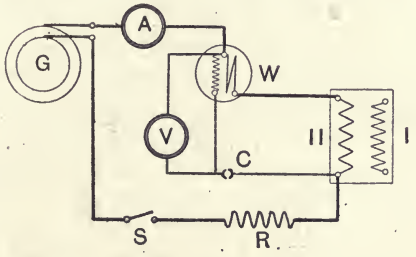


FIG. 65.

G Alternator supplying current.
I, II Primary and secondary of transformer.
V Voltmeter.
A Ammeter.
W Wattmeter.
R Variable resistance for altering voltage at transformer terminals.
C Voltmeter key.
S Switch.

Instructions.—Connect one coil of the transformer to a supply of alternating current. It will generally be found advisable to choose the low-tension winding or a part of this winding for the purpose, in order to get a magnetising current sufficiently large to be conveniently read. Connect a low-reading ammeter in series with the supply, and a wattmeter to read the power supplied. Connect a voltmeter to the same winding.

Keeping the periodicity of the current constant, vary the voltage at the terminals of the transformer connected to the supply, from a low value upwards. This may be done, either by varying the excitation of the alternator supplying the transformer, or by gradually cutting out a resistance inserted in series with the transformer winding. For each value of the voltage take readings of the voltmeter, ammeter, and wattmeter. Enter the results in tabular form as shown below. The coil supplied with current is here called the "primary," although it may be intended to be used as secondary under normal conditions.

If the secondary winding is used in the experiment, the value of the no-load current which would be supplied to the primary when connected to the supply may be calculated by dividing by the ratio of primary to secondary windings.

DETERMINATION OF TRANSFORMER MAGNETISING CURRENT AT VARIOUS VOLTAGES.

Transformer No. ..... Type .....
Output ....kw. ....periods per second.
Ratio of Transformation ....to ....volts.

Primary.				Energy Current, $\frac{W_1}{V_1}$.	Magnetising Current. $\sqrt{C_1^2 - \left(\frac{W_1}{V_1}\right)^2}$
Volts. V_1.	Amperes. C_1.	Watts. W_1.	Back Volts. $= V_1 - C_1 R_1$.		
15	·20	1	15	·067	·19
25	·31	3	25	·12	·286
35	·46	7	35	·20	·414
50	·86	15	50	·30	·807

From the results of this experiment a curve should be plotted, comparing the primary voltage with the magnetising current. Usually the volts lost in the winding at no-load

will be negligible compared with the applied voltage. If this is the case, column 4 need not be filled in, and the back voltage may be considered equal to the applied voltage. If an appreciable drop of voltage occurs in the windings, a second curve should be plotted, comparing back volts (the vector difference between V_1 and $C_1 R_1$) and current.

In addition to the total no-load current, the energy and idle currents should be plotted separately. The energy component is obtained by dividing the power supplied to the transformer (as read on the wattmeter) by the voltage of supply. The idle or magnetising current is then obtained by calculation, or from a diagram in which the total no-load current is drawn as the hypotenuse of a right-angle triangle, and the energy and idle currents are represented by the sides. The method of drawing this triangle is to describe a semi-circle on a line A B representing the total current to a scale of amperes, and then to draw a circle from one end as centre, with radius equal to the energy component. Joining the intersection of the circles to A and B, the triangle is complete.

Plotting both currents on a voltage base, we obtain curves showing the dependence of both magnetising current and energy loss current upon the saturation of the transformer core, since the back voltage of the transformer at constant frequency depends only on the magnetic flux, *i.e.*, on the density of magnetic lines in the core.

Thus the curve of magnetising current gives a magnetisation curve for the transformer analogous to the magnetisation curve which is of such great importance in the case of direct and alternating-current generators.

If the number of turns in the winding to which the voltage is applied is known, the curve may be plotted with " Number of lines " instead of " volts " as the base, since the lines can be calculated from the formula already given.

$$V = n\ t\ Z\ 4{\cdot}44 \times 10^{-8},$$

when V = voltage.
n = periodicity.
t = number of turns.
Z = maximum number of lines.

The power lost in the windings and recorded by the wattmeter can be separated readily from the watts spent in overcoming the iron losses, if the resistance of the winding is known, since watts lost in winding $= C_1{}^2 R_1$.

where C_1 = current measured.
R_1 = resistance of winding.

The whole of the power spent in magnetisation is actually converted into heat in the core, and consequently plays an important part in the heating of the transformer.

In Fig. 66 are shown the results of an experiment carried out upon a small arc lamp transformer designed for a secondary voltage of 50. The thick line curve I. shows the readings of the no-load current actually observed. Curve III. is the curve of iron loss obtained by dividing the watts by the voltage of the supply (the copper losses were too small to be taken into account). Curve II. shows the magnetising current calculated in the manner just explained.

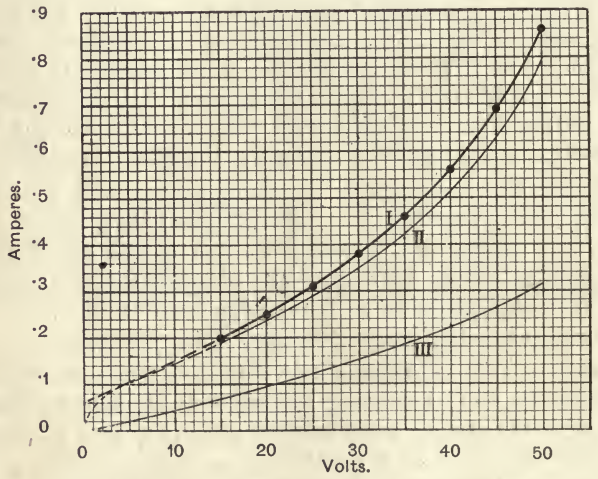


Fig. 66.—No-load Currents of a Transformer.
I.—No-load current.
II.—Magnetising current.
III.—Iron loss current.

It will be noticed that the curves I. and II. do not appear to pass through zero, although obviously the voltage and current must become zero together. The explanation is that the curve would bend downwards at the lower end, as shown by a thin dotted line, although the instruments available would not read sufficiently low to enable readings to be taken at this part of the curve. This bend is owing to the fact (referred to under Experiment VI.) that at very low saturation the permeability of iron is lower than at

rather higher values of the saturation. At the beginning of the magnetisation curve, therefore, the increase of current is rapid compared with the magnetic field produced.

Curve II. is the true magnetisation curve for the transformer, and is similar in form to the magnetisation curve for iron, as will be at once recognised on viewing the curve with the figure turned so that the ampere scale becomes horizontal.

Transformers are always designed to work at low-core saturation (say 4,000—5,000 lines per cm.^{2}), compared with the density employed in direct-current magnetic circuits.

This is necessary on account of the high iron losses which would otherwise result from the rapid alternation of the field. The "knee" of the magnetisation curve, which is so important in direct-current work, is never reached in the magnetisation curve of a transformer.

The experiment just given has a further important application, owing to the fact that the core losses of a transformer are very nearly the same at all loads as they are at no-load. The experiment is therefore a determination of the core losses of the transformer when the induction in the magnetic circuit is such as to produce the measured voltage at the terminals of the winding connected to the voltmeter in the test. If the winding supplied with current is the low-tension winding, the wattmeter will indicate the core losses under working conditions, if the voltage applied to this coil is the working voltage of that winding.

From the results of this experiment it is consequently possible to calculate the efficiency of the transformer at all loads if the resistance of the windings is known, since the losses which occur in working consist of the iron losses and the C^2R losses in the windings.

A simple method of determining experimentally the copper losses (*i.e.*, the losses in the windings) is given later (Experiment XXVIII.), and may be considered in some respects as forming the complement of the experiment just described, although it is less interesting in other respects.

No-load Losses Due to Defective Insulation.—If the no-load losses of a transformer are found to be too high, and the core is of normal weight and has not been damaged, and if the ratio of transformation is also normal, the excessive loss may probably be traced to defective insulation, this is especially likely

if the losses have been measured on the high-tension side. Frequently the no-load losses are measured on the low-tension side for convenience. In this case, if a measurement with 100 volts shows an insulation resistance of 100,000 ohms, the loss amounting to $C^2R = \frac{e^2}{R} = \frac{100^2}{100{,}000} = \cdot 1$ watt would remain unnoticed when measured in the manner described in the last Experiment. If, however, with the same insulation resistance a measurement is made on the 2,000 volts side, the loss due to defective insulation becomes $\frac{2{,}000^2}{100{,}000} = 40$ watts, which would no longer be negligible.

On this account no transformer intended for a pressure of 200 volts should leave the testing house with a lower insulation resistance than 2 megohms from primary to secondary and iron. These values will be usually considerably exceeded. If a newly-constructed transformer shows an unsatisfactory insulation resistance, it should be heated continuously for some time by having its primary connected to a low voltage while its secondary is short-circuited, so that its windings become heated, and the moisture in the insulation of the windings is expelled.

The following modification of the experiment just given is interesting as a further illustration of the relation between the no-load current and periodicity of a transformer.

Experiment XXVI.—Determination of No-load Current and Watts with Varying Frequency.

Diagram of Connections.

As for Experiment XXV., page 141.

Instructions.—Make connections as for Experiment XXV., connecting the circuit to an alternator, which can be made to supply current of varying frequencies, while maintaining the terminal voltage of the transformer constant, either by adjustment of a resistance in series with it or by regulation of the alternator fields.

Measure the current and power supplied to the transformer at various frequencies on open circuit at constant voltage. Enter the results in columns with suitable headings, and plot on a frequency base curves of current and power supplied.

The curves shown in Fig. 67 were obtained in this manner, and show that both current and watts increase with a decrease of periodicity. This is owing to the fact that the induction in the transformer core must be increased at low periodicity for the terminal voltage to remain constant. The magnetising current increases with a decrease of periodicity in consequence of this. The current spent in overcoming eddy-current losses in the core remains practically constant in amount, since these currents are proportional to the square of the periodicity and the square of the induction, and these two quantities vary in an inverse ratio in the experiment. The hysteresis losses, however, decrease with an increase of periodicity, since they are proportional to the product of periodicity and of the induction raised to the power 1·6. Consequently an increase of induction will more than counterbalance a proportionate decrease in periodicity. The fall of power at increased periodicities is, therefore, chiefly owing to a decrease in the hysteresis losses.

Perhaps the most interesting point illustrated by the curves is the decrease in both current and power brought about by the higher frequency. It is easy to see from these results that an increase of frequency would enable a smaller core, and consequently a less expensive transformer, to serve a given purpose.

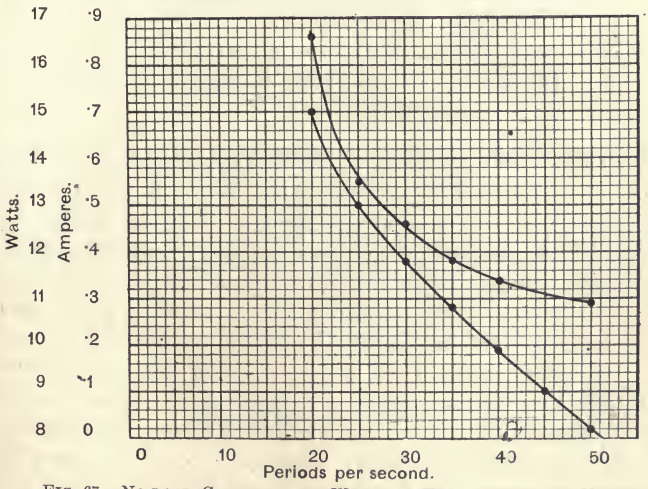


FIG. 67.—NO-LOAD CURRENT AND WATTS IN TRANSFORMER AT VARIOUS FREQUENCIES AND CONSTANT VOLTAGE.
Upper Curve – Current.
Lower Curve – Watts.

Iron Losses.—The iron losses occurring in the core of the transformer are similar to those which take place in the armature core of a direct-current dynamo or motor, and are due to the same two causes.

The power spent in hysteresis increases with the intensity of magnetisation of the core, but with a constant magnetic flux is proportional to the number of reversals in the polarity, *i.e.*, it is proportional to the periodicity of the current.

The eddy currents in the core, being induced in the conducting plates of the core by the varying flux across them, will also increase in direct proportion to the rate of alternation of the flux, obeying the usual law of induction in conductors by magnetic lines. The watts spent in producing the eddy currents will, however, vary as the square of these currents (since watts $= C^2R$). Hence the watts lost in producing eddy currents will vary in proportion to the square of the periodicity of the current supplying the transformer.

Separation of Iron Losses.—The usual formulæ employed for the iron losses in the core of a transformer are as follows :—

Watts lost in hysteresis $= K\ n\ B^{1\cdot6}.\ 10^{-7}$ per cub. cm. of core.

K = constant between ·002 and ·004.
n = periodicity of current.
B = max. induction in lines per sq. cm.
t = thickness of core plates in mils. (thousandths of an inch).

Watts lost in eddy currents per cub. cm. of core $= n^2\ t^2\ B^2.\ 10^{-16}$

From these expressions it appears that it would be possible, but somewhat difficult, to separate the two losses from observations made on a transformer at various magnetic densities, since the eddy current losses increase as the square of the induction B, while the hysteresis losses increase as the 1·6th power of B. An easier way of experimentally separating them is to keep the induction constant, and to measure the watts lost in the iron at different frequencies. Thus if two measurements are made, one with a current of double the frequency of the first, it will be found that the power spent on the core has more than doubled, since the eddy current losses increase as the square of the frequency.

By making a series of such observations, and plotting the results on a curve, as shown in Fig. 68, it will be found that at low frequencies the effects of the eddy currents are hardly seen, and loss and frequency are proportional, *i.e.*, the curve is straight. If there were no eddy current losses the curve

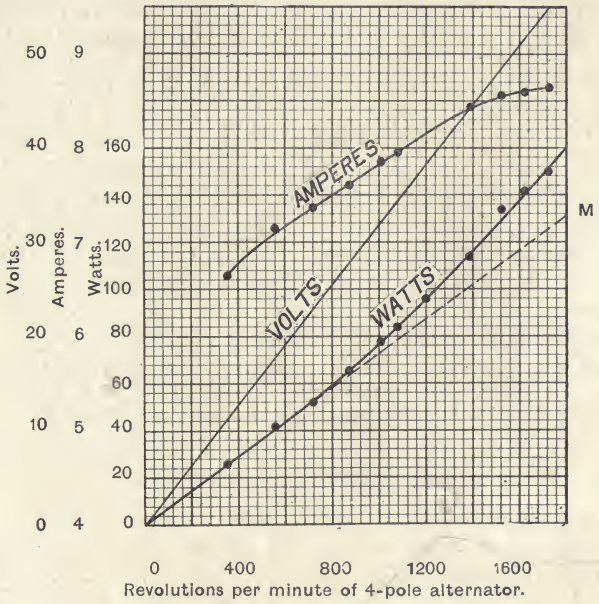


FIG. 68.—SEPARATION OF IRON LOSSES IN TRANSFORMER.

would continue straight, *i.e.*, along the line *O M*, which is drawn as a tangent to the lower part of the curve.

The ordinates of the curve above the line *O M* represent the losses due to eddy currents alone, and the ordinates of the straight line represent the power spent in hysteresis losses.

EXPERIMENT XXVII.—SEPARATION OF LOSSES IN A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

(Same as for Experiment XXV., page 141.)

Instructions.—Connect the transformer to a source of alternating current in series with a low-reading ammeter.

Connect a wattmeter to this circuit so as to read the power supplied. Connect a voltmeter to the terminals of the same winding. Supply to the transformer current of different frequencies, commencing with the lowest periodicity, for which satisfactory readings can be taken, taking care that the induction produced in the transformer remains constant. This can be most simply insured by adjusting the voltage in each case to the value of the first reading, increased in the proportion of the frequency. Thus the quantity $\frac{V}{n}$ should be kept constant where n = frequency and V = voltage.

Take readings on the wattmeter and ammeter after adjusting the voltage to the value required to correspond to each value of the frequency. The readings should be entered in tabular form, and two curves plotted with frequency measured horizontally, and wattmeter readings and current vertically. A tangent drawn to the curve of watts at its lowest point and passing through zero divides the losses according to their causes as already explained.

Results should be entered as in the following example.

SEPARATION OF IRON LOSSES IN A TRANSFORMER.

Transformer No. ..... Type .....
Output ....kw. ....cycles per second.
Ratio of transformation ....volts to ....volts.

Volts.	Amperes.	Watts.	Speed. Revs. per min.	Frequency. Cycles per sec.
55	8·65	150	1725	57·5
44·2	8·45	114	1385	46·2
32·2	7·85	78	1010	33·7
11·1	6·65	26	348	11·6

The curve shown in Fig. 68 gives the results obtained from an experiment carried out not on a transformer, but on a choking coil. The coil was chosen in preference to a transformer, as it was found that owing to the larger currents taken, and the less perfect lamination of the magnetic circuit, much more marked variations in the power could be obtained than with the transformers which were available. When

carried out for the sake of an experiment rather than to obtain information about a special transformer, this course may be recommended to the student.

The curve of voltage has been added in Fig. 68, and is necessarily a straight line passing through zero, since the magnetic flux is maintained constant. In carrying out the experiment it is convenient to obtain this voltage curve first, after ascertaining the range of speeds available, but before taking readings. At any given speed the voltage of the transformer must afterwards be regulated to agree with the corresponding point on the voltage line.

The effect of eddy currents becomes more marked at frequencies higher than those shown on the curve, and the losses become very serious at frequencies above 100 if not specially guarded against by very thin lamination and satisfactory insulation between the core plates.

This method of determining the iron losses may be applied to samples of iron in order to find the loss per cubic centimetre, and thus to judge of the quality. For this purpose samples are made up into a ring, or other form of magnetic circuit, and a number of turns of wire are wound on the circuit. The test is then carried out exactly as above described.

If the losses in the resistance of the winding employed in this experiment are appreciable, they must be calculated, and subtracted from the wattmeter reading, as explained in Experiment XXV.

Separate Determination of Copper Losses.—The method of calculating the voltage drop due to the resistance of the windings of a transformer has already been given. The power lost in the windings can be simply calculated from the voltage drop by multiplying by the output current. Thus, if v is found to be the drop in volts due to the resistance of the windings at any load current C_2, then the watts spent in heating the windings are equal to $v \times C_2$.

The following experiment is a method for rapidly determining the copper losses by direct measurement, and depends on the fact that the iron losses in a transformer are very small when the magnetic flux in the core is low, since both eddy current and hysteresis losses decrease rapidly with decreased magnetic densities.

EXPERIMENT XXVIII.—DETERMINATION OF COPPER LOSSES IN A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

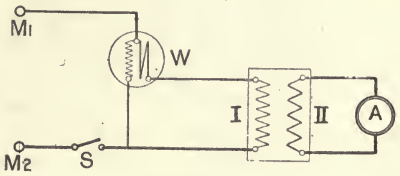


FIG. 69.

M_1, M_2 Source of alternating current.
I, II Transformer windings.
W Wattmeter.
A Ammeter.
S Switch.

Instructions.—Connect either primary or secondary winding of the transformer to the source of alternating current through a switch, inserting a wattmeter to measure the power supplied. Connect an ammeter directly to the terminals of the other winding.

Arrangements must be made for varying the voltage supplied to the transformer, which throughout the experiment will receive only a few volts. The variation may be accomplished either by varying the excitation of the alternator supplying the current, or by a resistance in series with the transformer winding.

Commencing with only a volt or two, gradually increase the applied voltage until the current read on the ammeter is the full-load current of the winding to which the ammeter is connected. For each value of the voltage applied take readings on the wattmeter and ammeter.

On account of the very low induction used, the losses due to magnetisation of the iron may usually be neglected, and the wattmeter reading may then be taken to be equal to the power lost in the resistance of the windings. If it is found that the wattmeter gives a reading when the secondary circuit is open, then this reading should in each case be subtracted from the reading taken with the circuit closed for the same applied voltage. The power supplied to the transformer on open circuit has already been discussed and shown to be due to the iron losses.

It must be remembered that in this experiment the losses which are being measured are due to a high current flowing through a low resistance. Consequently the resistance of leads or of the ammeter in the secondary circuit may be quite appreciable compared with the resistance of the transformer windings, and if not allowed for may introduce serious errors. The watts supplied to the transformer should consequently be measured at the terminals of the transformer itself, so as to eliminate as far as possible losses in the leads. The resistance of the leads forming the secondary circuit, including the ammeter, should be measured, and the watts spent in them subtracted from the watts supplied in order to obtain the watts actually spent in the windings alone.

In order to illustrate what has just been said, the following figures, taken from an actual test, are given, together with the curve showing the results of the experiment in Fig. 70. The

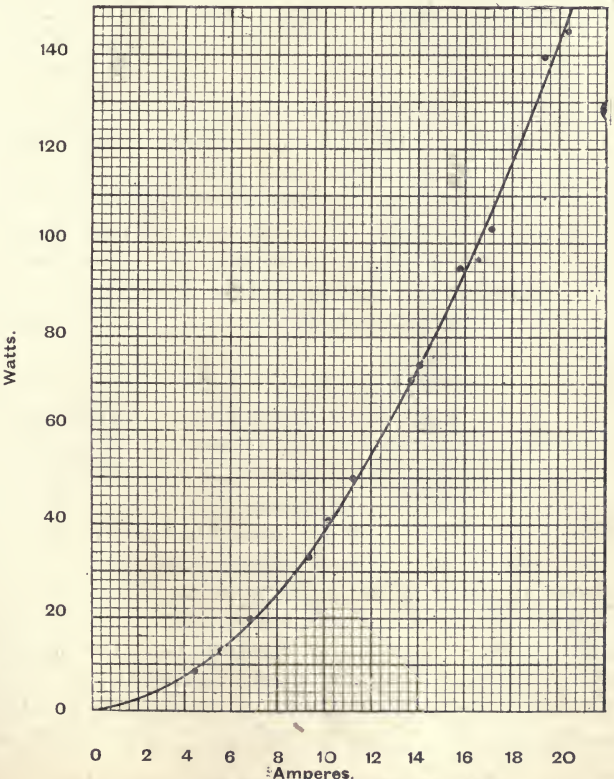


FIG. 70.—COPPER LOSSES IN A TRANSFORMER.

curve is seen to be part of a parabola. This is due to the fact that the voltage drop due to resistance is proportional to the load current, and the watts spent in the losses are the product of current and voltage drop. Thus the watts are proportional to the square of the load current.

DETERMINATION OF COPPER LOSSES IN A TRANSFORMER.

Transformer No. .....Type.....
Output ....kw.....cycles per second.
Transformation .....volts to..... ..volts.

Secondary Current.	Losses in Ammeter and Leads ($R = \cdot 065\omega$).	Wattmeter Reading.	Losses in Transformer.
4·55	1·35	10	8·6
6·8	3·0	23	20·0
10·2	6·8	48	41·3
13·7	12·2	83	70·8
15·8	16·3	111	94·7
20·45	27·2	172	144·8

A slight modification of the method just given avoids the measurement of the loss in instruments. By putting an ammeter in the *primary* circuit between the supply terminals and the wattmeter, the observations of the power supplied at a given primary current may be observed. The secondary winding is then short-circuited by a stout conductor of negligible resistance. The ratio of the primary to secondary currents may be determined by calculation or by a separate measurement.

Effect of Magnetic Leakage.—The alternating field produced by the current in the primary winding will only produce the voltage given by the formulæ on page 126, if the whole of the magnetic lines follow the magnetic path formed by the core, so as to pass through both primary and secondary windings. Any lines which are formed in the primary which do not pass through the secondary will produce a back electromotive force in the primary in the same way as the remainder of the field, but will not produce any effect on the secondary voltage. Similarly, the current in the secondary may produce some magnetic lines which do not pass through the primary. These lines will act in opposition to the main primary field in the secondary winding, lessening the flux

and the consequent voltage produced, but will not affect the flux and back voltage of the primary.

The lines which penetrate either winding without affecting the other are called the leakage field. There may be thus in a transformer both primary and secondary leakage fields.

The effect of the primary leakage field is to increase uselessly the number of lines to be produced, exactly as if an inductive resistance had been put in series with the winding.

The effect of the secondary leakage field (which, of course, only occurs when there is a secondary current) is that the secondary current has to overcome the back electromotive force produced by the alternating flux thus set up, which is not counteracted by the primary flux, like the other lines formed by the secondary current. In this case also the result is similar to that of an inductive coil put in series with the winding.

In either case the result of magnetic leakage is to reduce the useful effect of the voltage of the winding in which it occurs, and to produce the result of an added self-induction in the coil. Leakage consequently produces a lag in the current of the secondary circuit, and increases the lag of the current in the primary.

It is an important fact that the magnetic leakage increases with the load on the transformer. In order to explain the reason of this Fig. 71 is drawn to represent the primary and

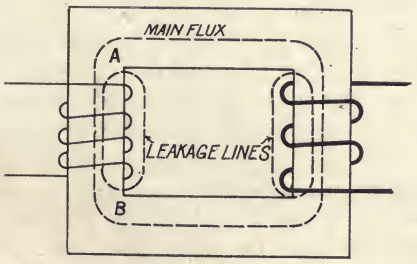


FIG. 71.—DIAGRAM SHOWING LEAKAGE IN TRANSFORMER.

secondary coils of a transformer wound in a way to produce considerable leakage. At no load, the primary receives magnetising current and produces an alternating field most of which will follow the magnetic circuit of the iron core because this path has the least reluctance or magnetic resistance.

If the reluctance of the iron path is $\frac{1}{99}$th of the reluctance of the leakage path A B through the air, then $\frac{1}{100}$th of the lines will pass through the air from A to B, the remainder following the iron path. If the secondary coil carries current, its effect is to oppose the flux produced by the primary, and the current in the primary has to increase in order to force the same number of lines round the circuit. The primary ampere-turns acting on the circuit are thus increased, while the difficulty of sending lines from A to B on account of the secondary reverse ampere turns, through the *iron* path increases in the same proportion. The reluctance of the *air* path from A to B remains, however, as before, so that the increased magnetising force applied will send a correspondingly increased leakage field through this path. Similar reasoning may be applied to trace the effect of load on the secondary leakage field. In either case the amount of leakage increases in practically direct proportion with the load.

Since both primary and secondary leakage are practically equivalent to the addition of an inductance in these circuits, it is evident that they both produce the effect of lessening the secondary terminal voltage, and thus increase the ratio of transformation of a step-down transformer, or decrease the ratio of transformation if the transformer is used for transforming up.

Effect of Lag in Secondary Circuit.—It has already been shown that any current in the secondary circuit must correspond to a current in the primary, whose value is equal to the secondary current × by the ratio of transformation. This is due to the fact that the secondary ampere-turns must always be neutralised by an equal number of primary ampere-turns.

If the secondary circuit is inductive, the secondary current will lag behind the secondary voltage, and, consequently, the secondary ampere-turns will tend to produce a magnetic flux later in phase than the main flux, due to the primary magnetising current. This lagging field must be counteracted by a corresponding extra current in the primary circuit. This primary current will be in phase with the field which it overcomes, and will lag behind the primary voltage. The effect on the primary circuit will be the same as if the secondary current had been in phase with its voltage, producing a non-lagging current in the primary, and an

inductance had been added in the primary circuit. The effect of a lagging secondary current is, consequently, to produce a lagging primary current, and to reduce the ratio of secondary to primary voltage. An inductive secondary load produces, therefore, similar results to the magnetic leakage just discussed.

Efficiency Tests of a Transformer.—The most direct method of measuring the efficiency of a transformer is to measure the power supplied by one wattmeter, and the power given out by a second wattmeter. The ratio of the two quantities measured, then gives the efficiency, since

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

If the load on the secondary circuit is non-inductive, readings on a voltmeter and ammeter will be sufficient to give the output, since there will be no phase difference in the circuit, and consequently watts output = volts × amperes.

The properties of the transformer are in general more easily studied when the load is non-inductive, and consequently tests of efficiency are nearly always made on non-inductive loads. Transformers are not generally tested on inductive loads except to determine whether they are suitable for working under certain given conditions, or in accordance with a specification.

EXPERIMENT XXIX.—EFFICIENCY TEST OF A TRANSFORMER ON NON-INDUCTIVE LOAD.

DIAGRAM OF CONNECTIONS.

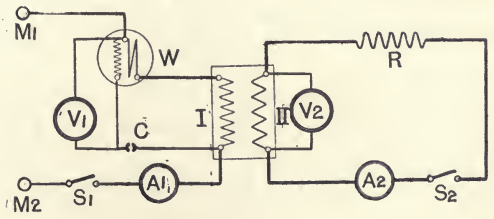


FIG. 72.

M_1, M_2	Source of alternating current.
I, *II*	Primary and secondary windings of transformer.
R	Variable non-inductive resistance for altering load.
V_1	Voltmeter for reading primary voltage.
V_2	Voltmeter for reading secondary voltage.
A_1	Ammeter for reading primary current.

A_2	Ammeter for reading secondary current.
W	Wattmeter for reading primary input.
C	Voltmeter switch or plug.
S	Switch for breaking secondary circuit.

Instructions. — Connect the primary of the transformer to a source of current, having suitable voltage and frequency, in series with an ammeter.* Connect a wattmeter in the primary circuit to read the watts supplied to the winding and join a voltmeter to the primary terminals.

Connect the secondary winding to a variable non-inductive load resistance in series with an ammeter and switch. Join a voltmeter to the secondary terminals of the transformer.

It is well to insert a voltmeter switch, as shown at C, in the circuit of the voltmeter and volt coil of the wattmeter, so that these instruments are only in the circuit when their readings are being taken. Errors due to the current taken by these instruments may not be negligible at small loads.

Maintain the applied voltage constant during the measurement, and take readings of primary watts and primary and secondary current and voltage, first on open circuit and then with gradually increased loads.

Tabulate the readings as in the following table :—

LOAD TEST OF A TRANSFORMER.

Transformer No. .....Type.....
Output.....kw.....cycles per second.
Transformation .....volts to .....volts.

Primary.					Secondary.			Efficiency.	Ratio of Transformation.
Current.	Voltage.	Watts.	Volt-amperes.	Power Factor.	Current.	Voltage.	Watts.		

* The primary current need not necessarily be measured if the *efficiency* only is required. It is, however, desirable to put an ammeter in the primary circuit to enable the no-load current and the power factor of the primary circuit to be observed. Both these quantities are of great importance.

The following curves should be plotted on a load base, either of secondary current or watts.

(1) Efficiency.
(2) Secondary voltage.
(3) Power-factor.

Fig. 73 shows the results of an experiment carried out on a small 1 kw. transformer having a transformation ratio of 100 to 50 volts. The efficiency curve shows a characteristic which will be generally noticed in lighting transformers which are only occasionally required to work at full load, and

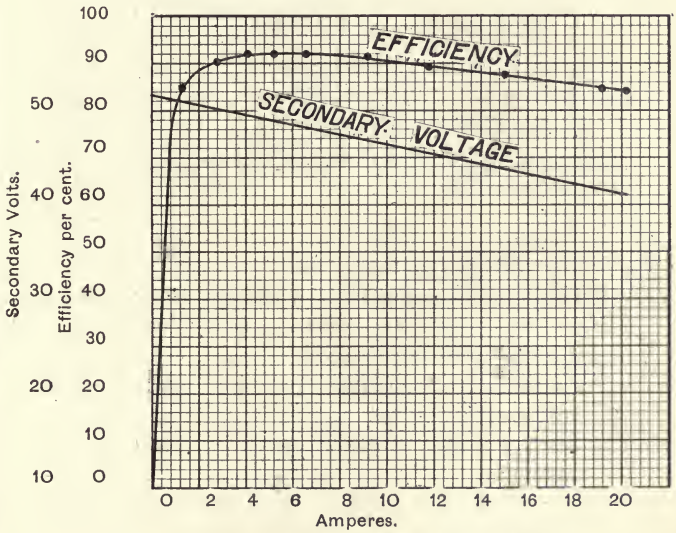


FIG. 73.—EFFICIENCY OF A TRANSFORMER.

are often connected to the mains for many hours to supply a light load only. This feature is the comparatively high efficiency at light loads, where the iron losses are almost the only ones felt, with a decided fall of efficiency towards full load on account of the comparatively large copper losses.

A transformer designed to have low iron losses (and consequently a high efficiency at light loads) and heavier copper losses will show a higher *annual* efficiency if working on a circuit which is only occasionally fully loaded, than a transformer designed with smaller copper losses and greater iron losses. A transformer intended to work usually at or near full load should, of course, have low copper losses.

In order to make a complete test of a transformer on inductive or capacity load, the method just described must be extended by measuring the output as well as the input by means of a wattmeter. In order to determine the power-factor of both circuits and the ratio of transformation an ammeter and voltmeter should still be put in both circuits.

EXPERIMENT XXX.—DETERMINATION OF EFFICIENCY OF A TRANSFORMER ON INDUCTIVE LOAD.

DIAGRAM OF CONNECTIONS.

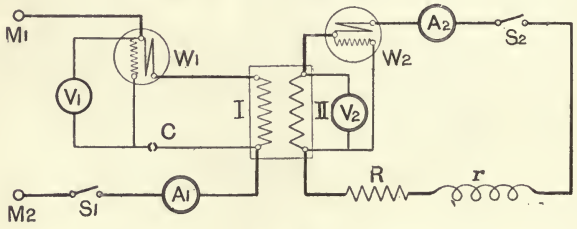


FIG. 74.

M_1, M_2 Source of alternating current.
$I, II.$ Primary and secondary windings of transformer.
R Variable resistance composed of inductive and non-inductive resistances.
V_1, V_2 Voltmeters for reading primary and secondary voltage.
A_1, A_2 Ammeters for reading primary and secondary current.
W_1, W_2 Wattmeters for reading primary and secondary power.
C Voltmeter switch or plug.
S_1, S_2 Switches for breaking primary and secondary circuit.

The method of carrying out this experiment is practically identical with that given for the preceding test, except that the secondary wattmeter must be read and the power-factor for the secondary circuit calculated. It is usually not possible in practice to obtain complete curves of efficiency and ratio of transformation at different power-factors, but isolated points are obtained at such power-factors as are required, and the curve can then be drawn in the neighbour hood of these points by comparison with the complete curve already taken on non-inductive load.

If a complete set of experimental curves were required for inductive loads, they might be obtained in a manner similar to that described in connection with Experiment XXXVI. for the characteristic of an alternator.

As already stated, transformers are seldom tested on inductive loads, except to obtain the secondary voltage at certain loads to fulfil the conditions of a specification.

Indirect Methods of Measuring Efficiency.—It is frequently more convenient to measure separately the losses occurring in a transformer, and then to state the efficiency as being

$$\frac{\text{power given out}}{\text{power supplied}} = \frac{\text{useful output}}{\text{output + losses.}}$$

This system of measurement has two important advantages, namely, that the power required to carry out the test is small, and that the losses are measured directly and consequently with greater accuracy than when they are derived from measurements made upon the total input and output of the transformer.

Method I.—The efficiency may be calculated directly from the measurements made in Experiment XXV., page 141, together with separate measurements of the resistances of the windings.

Let W_i = watts spent in iron losses—determined as in Experiment XXV.

R_1, R_2 = resistances of primary and secondary windings.

C_1, C_2 = primary and secondary currents.

W = output.

Then total losses = $W_i + C_1^2 R_1 + C_2^2 R_2$

and efficiency = $\dfrac{W}{W + W_i + C_1^2 R_1 + C_2^2 R_2}$.

This may be written $\dfrac{W}{W + W_i + C_2^2 \left(\dfrac{R_1}{k^2} + R_2\right)}$ where k is the ratio of transformation (see page 130).

The value of W_i, the watts spent in iron losses, is the value of the watts taken by the transformer on no-load at normal voltage as read on a wattmeter connected in the primary circuit. The iron losses are constant at all loads, and the copper losses are calculated for each load for which the efficiency is required. The efficiency is thus determined by calculation from a single measurement made at no-load.

Method II.—The approximate method for rapidly measuring the copper losses, if the resistance of the windings has not been previously determined, has already been given as Experiment XXVIII., and is very useful as a rapid workshop test.

The complete determination of the efficiency is then carried out as follows :—

The transformer is supplied with current, the watts taken at no-load being measured on a wattmeter connected in the supply circuit. For this purpose it is usually most convenient to supply the secondary instead of the primary on account of the more convenient voltage. The no-load current being very small, the copper losses under these conditions are very small and the power registered on the wattmeter is almost entirely due to losses in the iron. For greater accuracy the small copper losses may be calculated and subtracted, if thought necessary.

The free winding of the transformer is then short circuited through an ammeter and a low voltage applied to the same winding as before, through the wattmeter.

The voltage applied is so regulated as to give the full range of current which the transformer will carry under working conditions, and for each value of the current the wattmeter reading is noted. The power registered is now practically wholly due to copper losses, since the induction in the core is so low that the hysteresis and eddy current losses are quite small.

From the two sets of readings taken in this way the efficiency of the transformer may be obtained at any load.

$$\text{Efficiency} = \frac{\text{Watts output}}{\text{Watts output} + \text{Iron losses and copper losses.}}$$

Determination of the Drop at any Power-factor.—By a slight extension of Experiment XXVIII. for the determination of the copper losses in a transformer, the terminal voltage of the transformer at any load and any power-factor may be approximately calculated.

If in Experiment XXVIII. the transformer is entirely without leakage, and if the small magnetising current can be neglected (which is always permissible at the low induction employed), the whole of the voltage, applied to the primary winding would be spent in sending current through the resistance of the windings.

The value of the applied voltage would then be $c_2\left(r_2+\frac{r_1}{k^2}\right)$ *i.e.*, the drop due to the resistance of the windings as calculated on page 130.

The magnetic leakage is equivalent to a self-induction in the transformer windings. The ratio of applied voltage to current produced in Experiment XVIII., will, consequently, not be a function of the resistance only, but of the resistance and inductance together.

A measurement is made of the voltage applied to the primary winding, by a voltmeter connected to the primary terminals, and the values corresponding to the simultaneous readings of secondary current are noted.

The quotient $\frac{\text{applied voltage}}{\text{current}}$ will give a measure of the *impedance* of the windings. The voltage is measured on the high-tension winding, and the current is measured on the low-tension winding. In order to calculate the value of the impedance which would produce the actual drop of voltage in the transformer if placed in the secondary circuit, we must divide the fraction just given by the ratio of transformation. Thus impedance equals

$$\frac{1}{k} \times \frac{\text{voltage at primary terminals}}{\text{current in secondary circuit}}.$$

The relation between the quantities under consideration is shown in the triangle, Fig. 75, where the side *E D* repre-

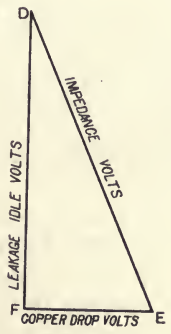


FIG. 75.—TRANSFORMER VOLTAGES ON SHORT CIRCUIT.

sents the impedance voltage of the windings, the side *F E* represents the voltage overcoming resistance, and the side

F D gives the voltage overcoming reactance due to leakage.

In any loaded transformer, the secondary voltage will be diminished by the impedance voltage and will be the resultant of the $\frac{\text{primary voltage}}{\text{ratio of transformation}}$ and of the impedance voltage. Consequently, in a transformer working on non-inductive load, we shall have the conditions represented

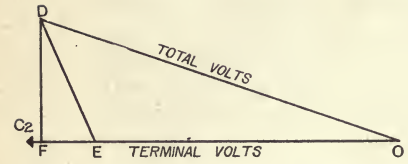


FIG. 76.—SECONDARY VOLTAGES. NON-INDUCTIVE LOAD.

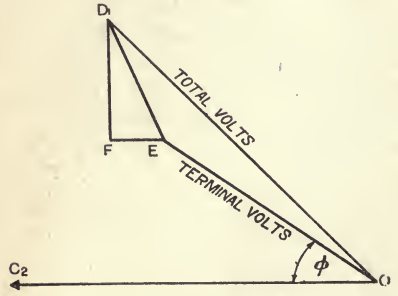


FIG. 77.—SECONDARY VOLTAGES. INDUCTIVE LOAD.

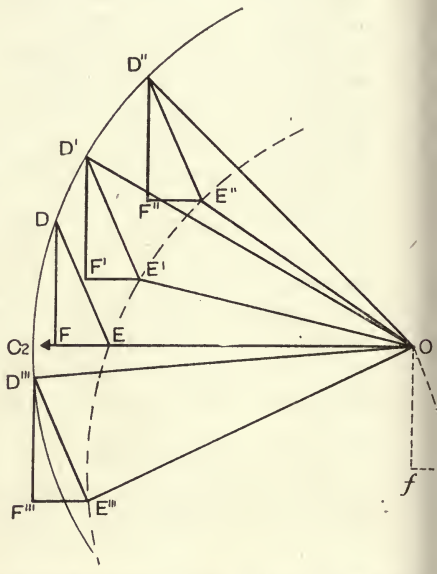
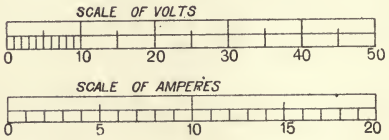


FIG. 78.—SECONDARY VOLTAGES. VARIOUS POWER FACTO



in Fig. 76, where *O D* represents the total induced secondary voltage, and *O E* is the terminal voltage, the line *D E* representing the impedance voltage determined by experiment as stated above. *D F* is the idle voltage introduced owing to the magnetic leakage of the transformer.

The drop in the transformer under these conditions is the difference between the length of *O D*, the no-load secondary voltage, and *O E*.

The actual figures taken in Fig. 76 correspond to readings taken on the transformer, whose drop of voltage is shown in Fig. 61. *O D* represents 51·4 volts, the no-load voltage. The volts at the primary terminals were 35, consequently *D E* is taken as $\frac{35 \times 128}{248} = 18{\cdot}1$ volts, $\frac{128}{248}$ being the ratio of the windings. The copper drop is taken as 7·3 volts, as already calculated on page 132. This is represented by *F E*. The terminal voltage *E O* is thus seen to be $41\frac{1}{4}$ volts, which agrees with the value shown from direct measurement in Fig. 61.

When working on inductive load, the diagram becomes that shown in Fig. 77, where the current lags behind the terminal volts by an angle ϕ.

In this case the terminal voltage is again obtained from the no-load voltage by constructing the triangle *D F E*, the side *F E* of which must be parallel to the current line, since the ohmic drop must be in phase with the current.

Fig. 77 has been drawn for the same impedance voltage, and the full-load current of 20 amps., as in the previous figure. The angle of lag is taken at about 34°, so that the value of the power-factor = cos 34° = ·83. At this power-factor, it will be seen that the terminal voltage is only 35 volts.

By drawing the diagram, Fig. 77, for a number of different power-factors, the regulation at any value of cos ϕ can be obtained.

Fig. 78 shows a diagram constructed in this way for a constant primary voltage (*i.e.*, a constant induced secondary voltage of 51·4 volts), and constant secondary load of 20 amps., but for four different values of the power-factor. It may be noticed that with a leading current the value of the terminal voltage *O* E^{III} is greater than the voltage *O E* on non-inductive load.

This diagram illustrates a useful approximate method of rapidly determining the drop in volts at any power-factor. By constructing the triangle *D F E* in the position shown dotted as *O f e* in Fig. 78, we obtain a point *e* such that a circle drawn from this point with the same radius as *O D* will pass through all the points *E*. This circle is shown as a dotted line. Thus any line *O* E^{I} drawn to cut the dotted circle and the full line circle drawn from *O* with radius equal to the no-load voltage, will show the terminal

voltage as $O\ E^1$ and the drop in volts as the length cut off between the two circles.

Fig. 79 shows the construction just described, the method of drawing it is as follows: From measurements made on the transformer with short-circuited secondary construct the triangle $O\ f\ e$, corresponding to full load, as

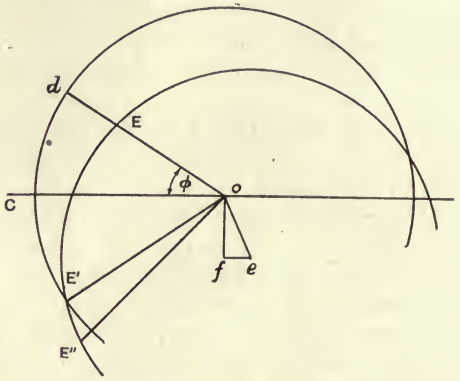


Fig. 79.—Construction for Finding Terminal Volts at Any Power Factor.

explained for Fig. 75. From O and e describe circles of radius equal to the no-load secondary voltage. For any value of the power-factor, draw from O a line $O\ E\ d$, making the angle $d\ O\ c$ equal to the angle of lag or lead. $O\ E$ then gives the terminal voltage and $E\ d$ the drop corresponding to that power-factor.

Efficiency of Two Transformers by Double Conversion.—Sometimes it is found desirable to measure both input and output of transformers on the low-pressure side, so that both quantities can be measured with equal accuracy and possibly on the same instruments. In this case two similar transformers may be connected with their high-tension windings together. The low-tension winding of one is then supplied with power and the low-tension side of the other is loaded. No measurement is made on the high-tension windings, and the ratio of low-tension input to low-tension output gives the joint efficiency of the two transformers. The efficiency of either is taken as the square root of this.

Split Dynamometer Method of Measuring Efficiency.—A dynamometer consists of a fixed and movable coil through

which the current to be measured passes successively, when the dynamometer is used in the usual manner. It is, however, usually possible to disconnect these coils, and different currents can then be sent through each. In any case the reading of the instrument is proportional to the mean product of the currents in each coil. Thus, when reading a single current, the deflection $= k\,\overline{c^2}$ where k is a constant, and $\overline{c^2}$ is the average value of the square of the current. If the same instrument has two different currents, c_1 and c_2, flowing through the two coils, its deflection will be $k\,\overline{c_1\,c_2}$, where $\overline{c_1\,c_2}$ is the average value of the product of the two currents. If these two currents are not in phase with each other, but differ by an angle α, we may consider one current c_2 as equivalent to two components 90° apart in phase, one component $c_2 \cos \alpha$ being in phase with c_1 and capable of producing a deflection of the dynamometer. The other component will be $c_2 \sin \alpha$, exactly a quarter period out of phase with c_1 and incapable, therefore, of producing any resultant action on the dynamometer.

In such a case the deflection of the instrument will be entirely due to c_1 and the component of c_2, which is in phase with c_1. Hence deflection $= k$ (mean value of $c_1\ c_2 \cos \alpha$). The principle of the behaviour of the dynamometer is in this case exactly similar to that of the wattmeter already discussed on page 57.

Suppose a dynamometer to have one coil inserted in the primary circuit of a transformer, and the other coil in the secondary circuit, the latter being non-inductive. Then, if $c_1\ c_2$ are the instantaneous values of primary and secondary current, and α is the angle of phase difference between the two currents, the reading of the dynamometer will be the average value of $c_1\ c_2 \cos \alpha$, which may be written for convenience $\overline{c_1\ c_2} \cos \alpha$.

As will be explained more fully later in connection with transformer diagrams, in a transformer with a non-inductive secondary load, the phase angle between the primary and secondary currents is (180°— angle of lag of primary circuit) $= 180° - \phi$, where ϕ is the angle of lag of primary circuit. Hence the reading of the dynamometer when connected so as to have the primary current in one coil and secondary current in the other, will be

$$k.\,\overline{c_1\ c_2} \cos\,(180° - \phi) = -\,k\,\overline{c_1\ c_2} \cos \phi.$$

Now, $c_2 R_2 = e_2$ = instantaneous value of secondary voltage if R_2 = total resistance of secondary circuit (including the secondary winding of the transformer).

Also $e_1 = \frac{N_1}{N_2} e_2$

where e_1 = instantaneous value of primary back voltage

and $\frac{N_1}{N_2}$ = ratio of transformation.

Hence $e_1 = \frac{N_1}{N_2} c_2 R_2$

or $c_2 = e_1 \frac{N_2}{N_1 R_2}$

Consequently the reading of the dynamometer

$$= \frac{c_1 \overline{c_2}}{K} \cos \phi$$

$$= \frac{N_2}{K^2 N_1 R_2} \overline{c_1 e_1} \cos \phi$$

Where K = the constant of the dynamometer, so that when used in the usual way $c = K \sqrt{\delta}$.

But $c_1 e_1 \cos \phi$ is the power given to the primary winding of the transformer exclusive of the copper losses in the coil. Let this $= W_1$

$\therefore$ The deflection of the dynamometer $= \delta = \frac{N_2}{K^2 N_1 R_2} W$

$$\text{or } W_1 = K^2 \delta \frac{N_1}{N_2} R_2$$

The total power supplied to the transformer

$= W_1 +$ watts lost in resistance of winding

$$= W_1 + c_1^2 r_1,$$

where c_1 = virtual amperes as read on an ammeter.

Also t e useful output

$$= c_2^2 (R_2 - r_2).$$

Where R_2 = total resistance of secondary circuit.

r_2 = resistance of secondary winding.

Hence efficiency of transformer $= \frac{\text{output}}{\text{input}}$

$$= \frac{c_2^2 (R_2 - r_2)}{K \delta \frac{N_1}{N_2} R_2 + c_1^2 r_1}$$

This result forms the basis of the next experiment.

EXPERIMENT XXXI.—DETERMINATION OF EFFICIENCY OF A TRANSFORMER BY SPLIT DYNAMOMETER.

DIAGRAM OF CONNECTIONS.

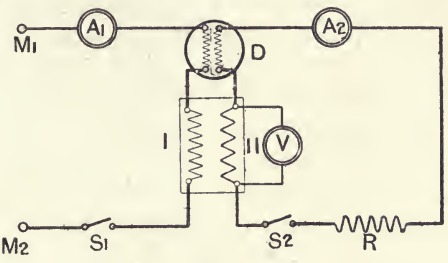


FIG. 80.

M_1 M_2 Source of alternating current.
I, II. Primary and secondary of transformer.
D Split dynamometer.
A_1, A_2 Ammeters or dynamometers reading primary and secondary currents.
R Non-inductive variable load resistance.
V_1, V_2 Voltmeters reading primary and secondary voltage.*
S_1, S_2 Switches.

Instructions.—Connect the primary of the transformer to be tested to a suitable source of alternating voltage in series with an ammeter and one coil of the split dynamometer. (It is preferable to put the swinging coil in the high tension side.) Connect the secondary of the transformer to a variable non-inductive load resistance in series with a second ammeter and switch. Throughout the experiment maintain the primary voltage constant.

Commencing with small values of the secondary current, gradually increase the load to the maximum value desired, and for each value of the current take readings on the two ammeters, the dynamometer, and the voltmeters. The no-load losses can obviously not be measured by this method, since the dynamometer will give no deflection without the secondary current.

While still hot, measure the resistance of primary and secondary windings by sending a direct current through

* In the diagram only a single voltmeter is shown connected to the secondary of the transformer, the primary voltage being assumed to be measured and kept constant at the supply mains.

them and measuring the drop of volts in the windings, or by a potentiometer if the resistance is very low.

Tabulate the results, calculating the efficiency in each case from the formula given above.

The following table shows a convenient form for entering up the results of observation and calculation. Two sets of readings are given as specimens. The figures employed in calculating the results from the readings were as follows :—

Resistance of windings $r_1 = \cdot 685$ ohm $r_2 = \cdot 183$ ohm.
Constant of dynamometer $= K = \cdot 975$ $K^2 = \cdot 95$.
Ratio of windings $= 1 \cdot 935$ $N_1 = 248$ turns $N_2 = 128$.

DETERMINATION OF TRANSFORMER EFFICIENCY BY SPLIT DYNAMOMETER.

Transformer No. .....Type.....
Output.....kw..... ..cycles per second.
Transformation.....volts to.....volts.

Current.				Voltage.		Resistance, Secondary Circuit $= R_2 = \frac{V_2}{C_2}$.	Dynamomet'r.		Total Power.		Efficiency, Per Cent.
Primary.		Secondary.		Primary.	Secondary.		Reading $= d$.	Power $= W_1 = K^2 d \frac{N_1}{N_2} R_2$	Input $= W_1 + C_1^2 r_1$	Output $C_2^2 (R_2 - r_2)$.	
C_1.	C_1^2.	C_2.	C_2^2.								
4·85	23·5	9·0	81	100	44·2	4·9	43·8	395	411	382	93·0
7·84	60·3	14·8	219·3	100	40·6	2·74	111·0	588	629	560	89

The curves to be plotted from this experiment are the same as those shown in Fig. 73, for the direct reading measurement of efficiency. In performing the experiment for the first time it is advisable to insert a wattmeter in the primary circuit and to check the accuracy of the calculated results by the wattmeter readings. When carefully carried out, the results may be made to agree very closely.

If sufficient instruments are available, it is convenient to employ dynamometers for measuring the primary and secondary currents, since the value of the mean squares of these currents is required, and is given directly by the dynamometers.

Sumpner's Method of Testing Two Transformers.—This is a method of testing transformers which is analogous to the Hopkinson method of testing dynamos, and founded

on Hopkinson's suggestions for testing transformers. It has the advantage that each of the two transformers being tested supplies power to the other, so that each transformer, although fully loaded, derives its power from the other and at the same time supplies power to it. Consequently the energy required from the supply circuit is only small, being that necessary to make up the losses in the two transformers. Also the losses are measured directly and are not obtained by subtracting the output from the input. In this way the result of a small percentage error in measurement affects the total efficiency to a much smaller extent.

The method is an indirect one, *i.e.*, the input and output are not measured directly, but the losses in the transformers are measured and the output of one of them is measured; the input is then calculated thus—

power supplied = output + losses in transformers.

The joint efficiency of the transformers is then

$$\frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

Since the two transformers are taken to be similar, they may be assumed to have the same losses, and

$$\text{the efficiency of each} = \frac{\text{output}}{\text{output} + \frac{1}{2}\text{ total losses}} \text{ or}$$

$$\text{efficiency of each} = \sqrt{\text{joint efficiency}}.$$

By the use of two wattmeters suitably connected it is further possible to measure the power spent in overcoming the iron and copper losses separately.

The method of carrying out the test is in outline as follows. Two similar transformers are both connected to the same alternating circuit—usually the low-pressure winding is the one supplied with current, but this is only a matter of convenience. For the purposes of explanation the winding supplied with current will be called the primary. The other windings of the transformers are connected so as to be in opposition, *i.e.*, the pressure generated in each will tend to send a reverse current through the other. If the pressure applied to both transformers is the same, no current will flow in the circuit formed by the windings, which are connected together. In order that a current may flow through these windings, an auxiliary transformer is put in series with the mains and the primary of one transformer, so as to make its voltage either higher or lower than that given

to the other. The secondary voltages will consequently be unequal also under these conditions, and a current will be produced, the amount depending on the extent of the out-of-balance voltage. By varying the voltage of the auxiliary transformer any required current may be obtained in the secondary windings. A current in the secondary winding must, however, necessarily correspond to a current in the primary, and consequently both transformers will carry a primary current, but *in opposite directions* relative to the supply circuit. If the auxiliary transformer assists the transformer to which it is connected, the secondary of this transformer will overcome the secondary of the other, and its primary will take current *from the mains.* The current in the secondary of the other transformer will be the reverse of that which its primary tends to send, and will consequently produce a reverse current in the primary, and this transformer will supply current *to the mains.* If the auxiliary transformer opposes the voltage of the first transformer, exactly the reverse conditions will arise. In either case the power supplied to the mains is nearly the equivalent of the power taken from them, the difference depending on the efficiency of the transformers.

Experiment XXXII.—Determination of Efficiency of a Pair of Transformers by Sumpner's Method.

Diagram of Connections.

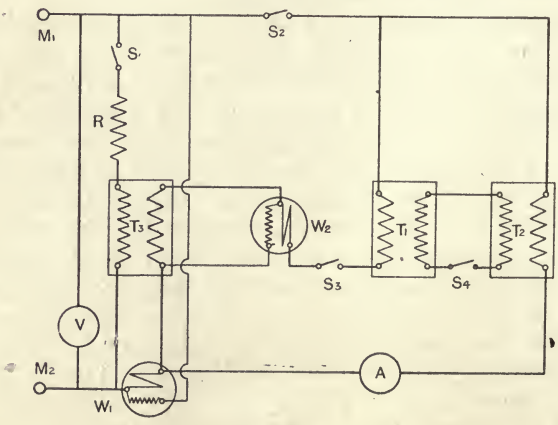


Fig. 81.

M_1, M_2 Source of alternating current.
T_1, T_2 Transformers to be tested.
T_3 Auxiliary transformer.
W_1 Wattmeter reading power taken from line exclusive of power taken by primary of auxiliary transformer.
W_2 Wattmeter reading output of auxiliary transformer.
V Voltmeter reading supply voltage.
A Ammeter reading current of T_2.
S_1, S_2, S_3, S_4 Switches.
R Variable resistance.

Instructions.—Connections.—Connect the primary winding of the auxiliary transformer to the supply mains through a variable resistance and switch. The resistance should be sufficient to reduce the secondary voltage of the transformer to a few volts only.

Connect one terminal of the secondary winding to one supply main, and the other through the series coil of a wattmeter and a switch to that winding of one of the test transformers which is to act during the test as primary. The other end of this winding of the transformer under test is to be connected to the opposite supply main.

Connect the primary* of the second test transformer to the supply mains through an ammeter, and insert a switch where shown in the diagram at S_2, so as to break the supply to both test transformers independently of the auxiliary transformer.

Connect a voltmeter to measure the voltage supplied, and put in two wattmeters as shown, W_2, to read the power in the secondary winding of the auxiliary transformer, and W_1, giving the total power supplied to the transformers exclusive of that passing into the primary of the auxiliary transformer.

The secondary windings of the test transformers must now be joined together through a switch. If this connection is made correctly, the two primary windings will take current from the supply mains, and will generate approximately equal voltages in their secondaries which will act in *opposition*, each tending to send current through the other. If the connections between the secondaries are the reverse,

* Throughout the description of this experiment the winding which is supplied from the mains is called the primary.

both will tend to send current round the circuit in the same direction, and a short circuit will result.

In order to test whether the connections have been correctly made, before commencing the test close only the switches S_4, S_2. Under these conditions transformer T_2 will receive current, and its secondary will excite the secondary of T_1, inducing a voltage in the primary of T_1 equal to that of the mains. If the connections are correct, the phase of the voltage will make the voltage always *opposite* to that of the mains.

In order to test if this is so connect a voltmeter or incandescent lamp to the terminals of the open switch S_3. If no voltage is found to exist between them the connections are correct, and the measurements may be proceeded with. If the connections have been reversed, there will be a voltage at the switch equal to twice the voltage of the mains. If this is found, the connections to the secondary of one of the test transformers must be reversed.

Method of Measurement.—After making the connections as above, connect to an alternating supply giving a constant voltage of the required value and frequency.

(1) Close all switches except S_1. Under these conditions T_2 will receive a magnetising current, and will in addition supply magnetising current to the secondary of T_1. The total current taken by T_2 will thus be the no-load current of the two transformers.

The transformer T_1 will not take its full current on account of the self-induction of the secondary winding of the auxiliary transformer which is in series with it. If this winding is short-circuited both transformers T_1 and T_2 will take equal currents, and there will be no current in their secondary windings (if their ratios of transformation are identical), and it will be immaterial whether the switch S_4 is open or closed.

In any case, the current taken from the mains will be equal to twice the no-load current of one transformer, and the reading of wattmeter W_1 will be twice the no-load losses. The no-load current of both transformers is shown on ammeter A if the secondary of the auxiliary transformer is not short-circuited ; if it is short-circuited the current of T_2 only will be indicated.

(2) After reading the no-load current and losses on A and W_1, close the switch S_1. This will either add to or subtract from the primary voltage of transformer T_1, the voltage of the secondary of the auxiliary transformer. In either case, the equality of the voltages in the closed circuit formed by the two secondary windings of the test transformers will be destroyed and a current will flow. By adjustment of the resistance R in series with the primary of the auxiliary transformer, vary the current passing through the ammeter A so that about six readings can be taken corresponding to loads varying by nearly equal amounts from light load to slightly above full load.

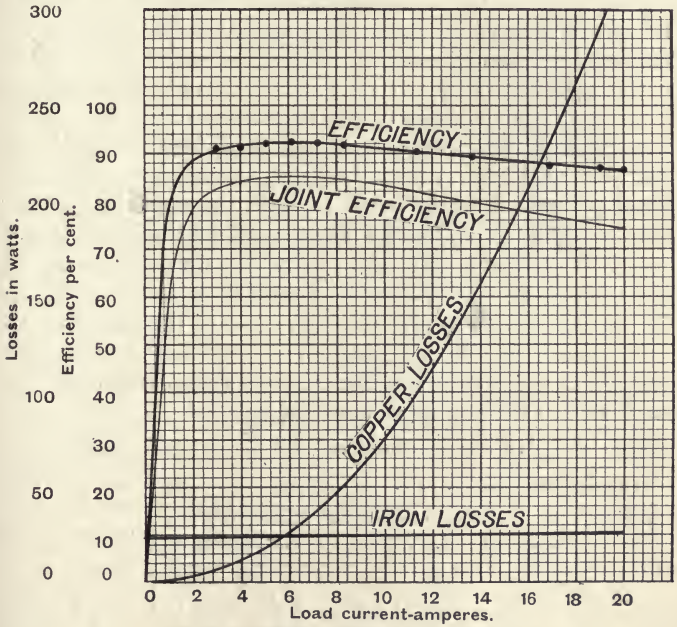


FIG. 82.—SUMPNER'S TEST OF TRANSFORMER.

The load on the transformer T_2 is the product of the reading of the ammeter A and voltmeter V. The losses in the two transformers are the sum of the watts read on the wattmeters W_1 and W_2.

The wattmeter W_1 carries the additional current supplied from the mains in order to supply the transformers with the magnetising and iron loss current. The readings of W_1 are thus the watts spent in overcoming the iron losses.

The action of the auxiliary transformer is to add to the primary voltage of transformer T_1 the voltage necessary to overcome the copper losses in both transformers. The power read on the wattmeter W_2 is consequently equal to the losses due to the resistance of the windings of both transformers. Included in the reading of W_2 will also be the losses in the resistance of the conductors joining the transformers and in the instruments (with the exception of W_1). To determine the losses in the leads and instruments alone, another measurement should be made.

(3) Open all the switches and short-circuit the primary windings of the test transformers. Close switches S_1 and S_3. The power supplied by the auxiliary transformer and measured by W_2 will now be only spent in overcoming the circuit and instrument resistance, and is therefore to be subtracted from the readings of W_2 in (2) in order that these readings may give the copper losses of the transformers.

This test of the instrument and lead losses should be carried out with the voltage of the auxiliary transformer adjusted to give a fairly large current through the instruments, in order to make the losses sufficiently large to be easily read. If the instruments or connections are changed during the test the losses should be read after each alteration.

In calculating the final results, the losses in the instruments for each value of the load should be calculated from the readings. It must be remembered that these losses will be proportional to the square of the load current.

DETERMINATION OF EFFICIENCY BY SUMPNER'S METHOD.

Transformer No..... Type .....
Output ....kw. ..... cycles per second.
Transformation ....volts to ....volts.

Current.	Output, Watts.	Wattmeter, W_2.	Iron Losses, Wattmeter W_1.	Instrument Losses $= W_3$.	Copper Losses, $W_2 - W_3$.	Total Losses, $W_1 + W_2 - W_3$.	Joint Efficiency, Per Cent.	Efficiency of each Transformer, Per Cent.
3·02	151	8·0	22	0	8·0	30·0	83·5	91·4
5·08	254	28·5	23	5·0	23·5	46·5	84·7	92·0
7·2	360	48·5	25	10·5	38·0	63·0	85·1	92·3
10·6	530	112·0	25	25·0	87·0	112·0	82·5	90·8

Supply voltage was maintained constant at 50.

The preceding table shows the method of entering up results, and gives a few readings taken from an actual test by way of illustration. The complete curves obtained in the experiment are shown in Fig. 82, which shows the following curves : (1) Joint efficiency of the two transformers ; (2) efficiency of each calculated from (1); (3) curve of copper losses ; (4) curve of iron losses.

The transformers experimented upon were the two 1-kw. $\frac{100}{50}$ volt transformers for which a number of curves have already been given.

Maker's Tests of Transformer.—The object of the most usual tests applied to transformers at the maker's works where they are made in large numbers, is not so much the determination in each case of the efficiency or quality of design, as to ascertain if each individual machine corresponds fairly to the standard transformer which was thoroughly tested and experimented upon at the time of adoption of that particular type.

Thus the transformers to be tested are mostly such as have a known ratio of transformation, efficiency, winding, &c., and the test is only to ascertain that no individual transformer is sufficiently unlike the rest through any accidental peculiarity, or error, as to render its return to the shops necessary.

It is to be expected that slight variations in the quality of the sheet iron forming the core will render unavoidable a certain want of uniformity in the no-load losses. A maximum value for the loss must be fixed, and transformers showing a greater loss must be rejected. Thus, for instance, if 2 per cent. at no-load is the normal value it may be decided to reject those giving more than 2·4 per cent. loss.

If a transformer with normal core is found to have 3 per cent. magnetising loss, it may be assumed that the windings are too few and the induction too high. This might be due either to too few windings having been wound on, or to a short-circuit amongst windings, so that some are cut out. Either of these faults would be detected on comparing the ratio of transformation with that of a normal transformer. A measurement of the resistance of the windings would probably point out the defect, but the difference in resistance might possibly be masked by the increase of the specific resistance of the wire, produced by the

hammering which it undergoes while being wound on to the former.

In consequence of the resulting uncertainty of the resistance of the windings, an agreement within 20 per cent. with the resistance of the standard transformer might be taken as sufficiently near. On the other hand, transformers intended to work in parallel must have the same ratio of transformation at all loads, and must consequently have practically equal resistances in the windings. In testing their suitability for working in parallel, use is made of the fact that a heavy circulating current will flow between the transformers when connected in parallel to the same circuit if their secondary voltages are not exactly equal. A rapid test of a series of transformers as regards equality of ratio of transformation may be made as described in the following experiment. This experiment is to be considered rather as a rough and practical test of the uniformity of a series of machines for use in the factory test-house, than as an experiment for giving information as to the characteristics of the machines individually

Experiment XXXIII.—Test of Uniformity of Ratio of Transformation in a Series of Similar Transformers.

Diagram of Connections.

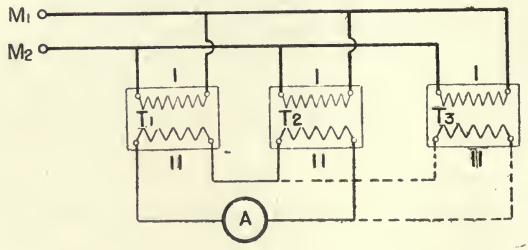


Fig. 83.

M_1, M_2. Source of alternating current.

T_1, T_2, T_3. Transformers to be tested for equality of transformation ratio.

A. Low reading ammeter.

T_1 is shown as the transformer taken as the standard of comparison. The leads from it are to be ultimately connected to all the other transformers in turn.

Instructions.—Place the transformers to be tested in a row with the high-tension terminals to one side and the low-tension terminals towards the operator. Connect the high-tension side of the transformers to the correct voltage derived from the mains or from an auxiliary transformer. Connect a pair of long leads in series with an ammeter to the low-tension side of the standard transformer. Connect these leads in turn with the secondaries of the other transformers, taking care to connect the left-hand terminal of each to the right-hand terminal of the standard, &c. When connection is made in this way, differences in secondary voltage of less than ·2 per cent. produce a readable deflection of the ammeter.

The necessary voltage may be obtained from one of the series of transformers; but, in this case, the voltage applied to its secondary winding must be slightly higher than the secondary voltage given out of the remaining transformers, since it has to supply the magnetising current for the whole series.

This method compares the ratio of transformation of the transformers at no-load only, but if the resistances of the windings do not vary by more than 20 per cent., the differences in the voltage drop will have a maximum variation of ·4 per cent. if a total copper loss of 2 per cent. is allowed.

A heating test under load should also be carried out to make sure that the normal load can be carried without excessive rise of temperature.

The Auto-Transformer.—Transformers, which are employed for low pressures only, sometimes have their primary and secondary windings connected in series.

Since the changes of voltage in the primary and secondary windings of a transformer go through their maximum and minimum values at the same time, the result of connecting the two windings in series is to produce a voltage which is either the sum or the difference of the voltages of the windings, according to the mode of joining them.

If the windings of a step-down transformer are joined in series so that their resultant voltage is the sum of the voltages of the two windings, the source of supply may be connected to the terminals of the composite winding, instead of to the terminals of what was originally the primary winding.

If this is done, the windings of the transformer may be reduced until the total voltage of the two windings equals the voltage of the original primary.

Thus, suppose a transformer with a ratio of transformation of 5 to 1 to be wound with separate primary and secondary coils of 500 and 100 turns, *i.e.*, 600 turns total. If the two windings are joined in series as shown in Fig. 84,

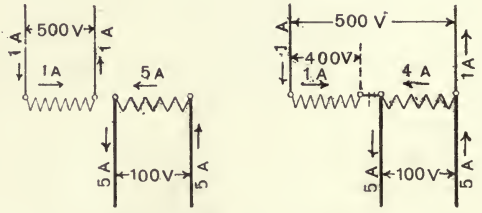


FIG. 84.—PRINCIPLE OF THE AUTO-TRANSFORMER.

the number of turns required is only a total of 500, if the voltage is applied to the ends of the whole winding, since the ratio of primary turns to secondary turns still remains 5 to 1, and with the same magnetic induction in the core the primary back voltage and the secondary voltage will both remain exactly as before. The ratio between primary and secondary currents remains also the same as before, but is not produced quite in the same manner, since the primary current will flow into the secondary winding, and take the place of some of the current which would have to be induced in the secondary winding if the two coils had remained separated.

This will be rendered clear by a consideration of the currents indicated by arrows in Fig. 84, together with the following explanation.

The primary and secondary coils still act in the same way as before they were joined, as regards their magnetising effect on the core. The windings are so connected that when joined to the source of supply on no-load, the magnetising current flows through them all, and in doing so produces a magnetic flux in the core, which in turn produces the back electromotive force in the windings, opposing the applied voltage. This back electromotive force is induced equally in all the windings, and is the voltage in virtue of which the secondary coil is able to supply current.

When the secondary of an ordinary separately wound transformer is loaded, the current in the windings composing it is in the direction of the induced (or "back") electromotive force and opposite to the direction of the voltage applied to the primary winding. Thus the load current acts as a *demagnetising* current on the core, and in order to maintain the magnetic flux constant, the current in the primary windings increases in direct proportion to the amount of current supplied by the secondary. Also, the ratio of the primary current to the secondary current must be inversely proportional to the number of turns in the two windings (neglecting the no-load current), so that the total magnetising effect on the core may remain constant.

The ratio of primary to secondary windings of a transformer whose transformation ratio is 5 to 1 is, however, 4 to 1 when the connections have been modified in the manner under discussion.

A demagnetising load current of *four* amperes in the secondary winding will consequently correspond to a magnetising current of *one* ampere in the primary turns; but this is directed in the *same* direction as regards the conductor leading from the junction of the two windings, and as shown in Fig. 84, the currents in the primary and secondary unite to supply a load current of five amperes for each ampere in the primary coil. Hence, the ratio of currents in the auto-transformer is inversely proportional to the voltages of the circuits and the general behaviour of transformers of both classes is the same as regards the circuits to which they are connected.

The method of connecting the windings of an auto-transformer is usually the one described above, and the transformer is then used for transforming *downwards*, in cases where the ratio of voltages is comparatively low. Obviously the supply might be taken to the winding termed the secondary above, and the end connections might give out current, in which case the transformer would transform *upwards*. It would also be possible to connect the windings so that their voltages are in opposition instead of in series. In this case the terminal voltage will be the difference between the voltages of the two sections, instead of their sum. For experimental purposes, auto-transformers are often wound so as to give a large range of voltages by different connections of the windings.

It will be seen that in an auto-transformer the current flowing in the primary circuit passes directly into the secondary circuit, and hence the amount of current to be generated in the secondary is less to this extent. This, together with the fact that the number of windings through which the currents flow is less than in an ordinary transformer, makes the losses in an auto-transformer less than in one of the ordinary type.

With high-tension currents there must be no connection between high and low tension circuits, and this prevents the use of auto-transformers in such cases.

Tests of auto-transformers are carried out as described for the usual types of transformers given above.

In order to familiarise himself with the action of an auto-transformer, the student should perform the following exercise on any low-tension transformer which may be available :—

Exercise.—(1) Supply the primary winding of a low-tension transformer with alternating current, and measure carefully the voltages of both windings.

(2) Connect primary and secondary windings in series, and supply the outer terminals with current. Allow each winding in turn to supply current to a secondary circuit, and note the current and voltage in each portion of the circuit.

(3) Reverse the connections between primary and secondary, and repeat the readings as before.

(4) Supply the primary circuit with current, and take current from the extreme terminals with the two windings connected first in series and then in opposition. Take readings of current and voltage in each portion as before.

(5) Repeat these readings with the secondary winding connected to the supply.

In each case the student should indicate his connections as shown in Fig. 84, and note in each diagram the current and voltage in each section in the manner there shown.

It is instructive to calculate the copper losses in each form of connection from the current and resistance of the windings, and to compare those with the copper losses to be expected from the usual type of transformer, if the resistance of the windings were increased in the same proportion as their number, in order to give the same ratio of transformation.

Graphic Representation of Phase-Relations in a Transformer.—It is important to form a clear idea of the relation existing between the several varying quantities in a transformer, and this may be best done by representing them graphically in vector diagrams.

Case I.—Transformer without load and without magnetic leakage. Let the line $O E_1$ (see Fig. 85) represent that

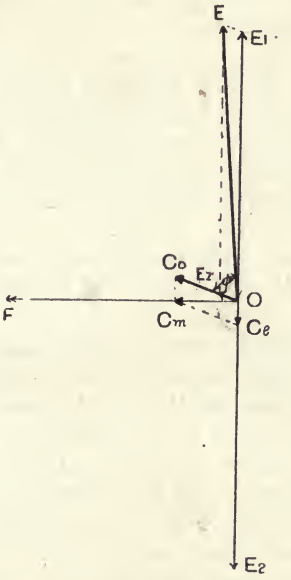


FIG. 85.—TRANSFORMER ON NO LOAD.

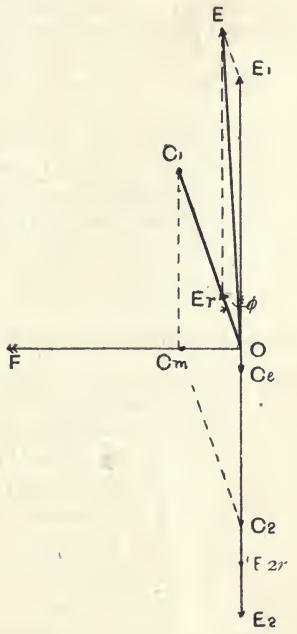
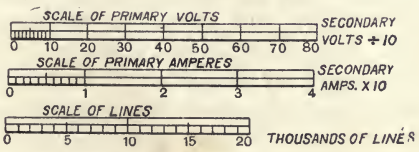


FIG. 86.—TRANSFORMER ON NON-INDUCTIVE LOAD.


SCALE OF PRIMARY VOLTS
0 10 20 30 40 50 60 70 80
SECONDARY VOLTS ÷ 10
SCALE OF PRIMARY AMPERES
0 1 2 3 4
SECONDARY AMPS. X 10
SCALE OF LINES
0 5 10 15 20
THOUSANDS OF LINES

part of the primary applied voltage which is spent in overcoming the induced back voltage, due to the alternating flux in the core.

The current taken by the transformer is partially magnetising current which is an idle current, and

lags 90° behind the voltage, and partially energy current necessary to overcome the iron losses and the small copper losses due to the no-load current.

In the case represented in Fig. 85 the magnetising current is ·8 amperes, and is represented by the line $O\,C_m$ drawn to a scale of amperes.

The current $O\,C_m$ is the portion of the current which alone acts as magnetising current and the magnetic flux produced in the transformer core is given by the formula on page 140, viz., $ct = \frac{B\,l}{\mu} \times 1{\cdot}435$.

Consequently, the number of lines actually formed in the core at any instant is proportional to the instantaneous value of the magnetising current at the same instant. Thus we may represent the field strength by a vector line drawn parallel to the magnetising current, to a scale of "number of lines." This is shown as the line $O\,F$.

In the secondary circuit there will be induced a voltage equal to the rate of change of lines × number of secondary turns ÷ 10^8.

Since the voltage depends upon the *rate of change*, and not on the *number* of lines, it will lag in phase $\frac{1}{4}$ period behind the line showing "number of lines." It must be remembered that the primary induced back voltage and the secondary induced voltage are in phase, being due to the same cause. Thus the secondary voltage will be represented by a line exactly opposite in direction to the line of primary applied volts overcoming the back electromotive force. The secondary volts would be represented by $O\,E_2$ one-tenth of the length of $O\,E_1$ if the ratio of transformation is 10 to 1.

For convenience in drawing, it is the usual plan to represent the primary and secondary voltages not of the actual relative magnitude, but to show the low-tension voltage as multiplied by the ratio of transformation, thus bringing it to the same length as the primary voltage on the diagram. In reading the results of a diagram, all lines representing voltages in the secondary circuit must be divided by the ratio of transformation, in order to give the actual number of volts. Adopting this device the line $O\,E_2$ is shown in Fig. 85 as being equal in length to $O\,E_1$.

The secondary voltage is seen from the figure to be always exactly opposite in phase to the applied voltage overcoming the

primary back voltage, *i.e.*, it passes through its maximum and minimum values at the same time, but the direction of the electromotive force is always opposite in the two coils.

This bears out the fact, which was stated earlier, that the secondary current acts as a *demagnetising* current on the transformer core.

The no-load current of the transformer will consist partly of the *idle* magnetising current and partly of the *energy* current, which is spent in overcoming the iron losses. The iron losses are equivalent to a demagnetising action on the magnetic circuit, in fact they are partly due to eddy currents, which are demagnetising currents in the same way as the load currents in the secondary winding. It is consequently convenient to consider the iron losses as equivalent to a current acting on the transformer as a load current, *i.e.*, as a demagnetising current. The value of this imaginary current is equal to $\frac{\text{watts spent in iron losses,}}{\text{primary voltage}}$ and is represented in Fig. 85 by the line $O\ C_e$. The actual primary current overcoming this demagnetising action is equal and opposite in phase to this. Thus the total no-load primary current is the resultant of the magnetising current $O\ C_m$ and of a current which is equal and opposite to $O\ C_e$. This resultant is shown as $O\ C_o$.

The iron losses in the transformer under consideration were 21 watts, which corresponds to an energy current of $\frac{21}{70} = \cdot 3$ amperes, since the secondary voltage is taken as 70. The loss current must be in phase with the secondary voltage and is accordingly represented by the line $O\ C_e$. The resultant of this current reversed and the current $O\ C_m$ is $O\ C_o$, which is the total no-load current of the transformer. To produce the current $O\ C_o$ requires a voltage $C_o \times R$, which is shown on the diagram by $O\ E_r$, R being 5·8 ohms, the resistance of the primary winding. The volts given to the transformer are the resultant of the back voltage $O\ E_1$ and the resistance electromotive force $O\ E_r$. Thus the total impressed voltage is shown in magnitude and phase by the line $O\ E$.

The angle of lag of the primary circuit is the angle $E\ O\ E_r$, since it is the angle between current and voltage.

Case II. Secondary Circuit loaded non-inductively. (See Fig. 86.)

The only difference between the diagram for this case and for the preceding is that the secondary current must be added to the current $O\,C_e$ drawn in the previous case to represent the iron-loss current.

In order that the scale of the current flowing in the secondary winding may be the same as that of current in the primary, it is convenient to divide the secondary current in all cases by the ratio of transformation. By this means all currents represented on the diagram can be considered equally effective as regards the magnetic circuit, since the secondary currents flow round the core in a less number of turns. Suppose that the current in the secondary circuit is 20 amperes. Since the ratio of transformation is 10 to 1, this must be represented on the diagram as 2 amperes.

The length $C_e\,C_2$ is taken equal to 2 amperes.

The magnetising current remains the same as before, and the resultant primary current becomes equal to $O\,C_1$, which is the resultant of $O\,C_m$, and of a current equal and opposite to $O\,C_2$. The applied voltage is obtained as in the previous case as the resultant of $O\,E_1$ and $O\,E_r$, where $O\,E_r$ is taken equal to $C_1 \times R$, on the scale of volts. The angle of lag in the primary circuit is in this case $E\,O\,C_1$. In the secondary circuit, current and voltage are in phase.

The voltage at the terminals of the secondary winding will not be $O\,E_2$, but will be less by $C_2\,R_2$ volts. This may be shown on the diagram by marking off E_2E_{2r} from E_2 equal to $C_2\,R_2\,K$

Where C_2 secondary current.
R_2 resistance of secondary winding.
K ratio of transformation.

Then $O\,E_{2r}$ gives the terminal voltage.

Case III. Secondary circuit loaded inductively.—(See Fig. 87.) If the secondary circuit is inductive, the current, instead of coinciding with the line $O\,E_2$, will lag behind this as shown in Fig. 87, where $C_e\,C_2$ is drawn of the same length as in the previous case, but has an angle of lag ϕ_2 of about 19°.

The remaining part of the construction is exactly the same as before, the primary current being again the resultant of a current equal and opposite to $O\,C_2$ and $O\,C_m$, and the

primary voltage $O\,E$ being the resultant of the voltage overcoming the back electromotive force $O\,E_1$, and the primary copper loss $O\,E_r$.

The angle of lag of the primary circuit is again the angle $E\,O\,C_1$, which in this case is greater than before on account of the lagging current in the secondary circuit.

The secondary current will produce a drop of voltage in phase with the current and equal to $C_2\,R_2$, so that the terminal voltage is the resultant of $O\,E_2$ and $E_2\,E_{2r}$ drawn parallel to the current $O\,C_2$. The terminal voltage is consequently represented by $O\,E_{2r}$, and the angle of lag is the angle between $O\,C_2$ and $O\,E_{2r}$.

Two important results can be seen to follow from an inductance in the secondary circuit. Firstly, the primary circuit receives a correspondingly increased current-lag, and secondly the ratio between the magnitude of the primary and secondary voltages is seen to be affected, so that the secondary voltage is always lowered by an inductive load. This is very clearly seen by comparing the two diagrams, Figs. 86 and 87, which have purposely been chosen with the same amount of secondary current, the angle of lag in the secondary circuit only being changed.

Case IV. Secondary circuit on load with leading current. (See Fig. 88.) This case arises when the transformer supplies a circuit having considerable capacity, or synchronous motors which are over-excited.

The method of drawing the diagram is the same as in the preceding case, the only difference being that the current line $C_e\,C_2$ lies on the opposite side of $O\,E_2$.

From the resultant voltage, it appears that the effect of a slightly leading current is to *increase* the transformer secondary voltage, so that a slightly leading current tends to counteract the effect of the resistance of the windings and maintain the secondary voltage more constant.

Case V. Transformer with leakage. (See Fig. 89.)—The effect of leakage in a transformer is similar to the insertion of an inductance in the circuit in which the leakage occurs. It will be remembered that a self-induction in a circuit gives rise to an electromotive force 90° in phase behind the current in the circuit.

The value of the electromotive force thus introduced by the leakage field is given by the same formula as is used for

the voltage of a transformer when the number of the lines forming the alternating field is known.

Thus, if the leakage flux in the transformer be z lines, the electromotive force of self-induction set up in a winding of t turns at a frequency of current $n = E = z\,t\,n\,4{\cdot}44 \times 10^{-8}$.

It has already been pointed out that the strength of the leakage field increases in proportion to the primary current. Its actual value can only be determined by experiment or previous experience of transformers of similar type. The value of the electromotive force due to self-induction in the primary and secondary windings respectively is approximately proportional to the number of turns in the windings as just stated.

Most simply the leakage effect may be shown as a single electromotive force acting in the primary circuit $\frac{1}{4}$ period out of phase with the resultant primary current, as shown at e in the diagram.

In Fig. 89 the resultant current and voltage obtained from the preceding case without leakage are thus shown for a leakage equivalent to a voltage of 10 volts.

A leakage field $\frac{1}{7}$ of the main field is assumed, and e is drawn to represent the electromotive force introduced thereby.

The output voltage of the transformer is $O\,E_2'$, the resultant of this voltage and the terminal voltage which would exist without the leakage, and obtained as in Fig. 87.

NOTE.—In the preceding diagrams, the angle of lag in the secondary circuit is shown as the angle between the phase of the current and the *total induced* secondary voltage. It must be remembered that the angle of lag between current and *terminal volts* is the angle between $O\,C_2$ and $O\,E_{2r}$

The following table referring to the figures just given may be found convenient for reference :—

$O\,E_1$	Voltage overcoming induced primary back volts.
$O\,E$	Primary terminal voltage.
$O\,C_1$	Primary current.
ϕ	Angle of lag of primary circuit.
ϕ_2	Angle of lag between current and total induced secondary volts.
$C_2\,O\,E_{2r}$	Angle of lag between current and secondary terminal volts.
$O\,C_2$	Secondary current.

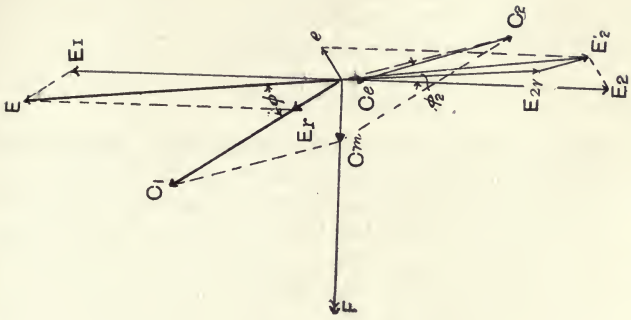


FIG. 89.—TRANSFORMER WITH LEAKAGE ON INDUCTIVE LOAD.

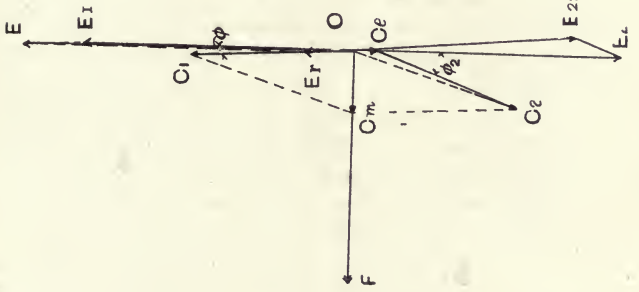


FIG. 88.—TRANSFORMER ON CAPACITY LOAD.

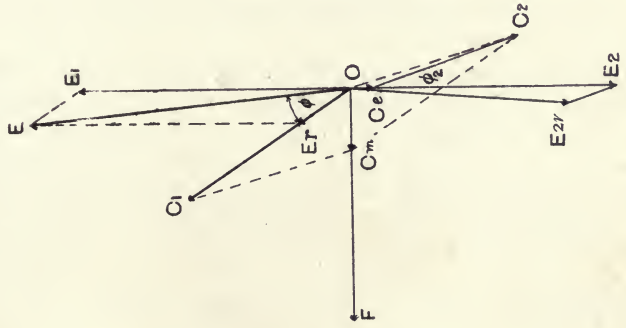
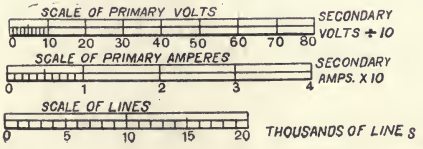


FIG. 87.—TRANSFORMER ON INDUCTIVE LOAD.


SCALE OF PRIMARY VOLTS
0 10 20 30 40 50 60 70 80
SECONDARY VOLTS ÷ 10
SCALE OF PRIMARY AMPERES
0 1 2 3 4
SECONDARY AMPS. X 10
SCALE OF LINES
0 5 10 15 20
THOUSANDS OF LINES

$O C_e$	Imaginary secondary current equivalent to iron losses.
$O E_2$	Secondary induced voltage.
$O E_{2r}$	Secondary terminal voltage.
$O C_m$	Magnetising current.
$O F$	Flux in core.

Tracing of Curves of Primary and Secondary Voltage.—The method of determining experimentally the wave form of the voltage of an alternator (see page 85) may be extended to the determination of the wave form of primary and secondary voltage of an alternator, either on open circuit, or when loaded.

The rotating contact must be driven synchronously by the generator supplying the current; the portion of the circuit which is connected through the contact to the voltmeter can be chosen at will. Thus the primary terminals of a transformer supplied by the alternator may be connected through the rotating contact to the voltmeter, and a curve of primary voltage obtained by moving the contact brush into a series of positions. Simultaneous readings may be taken on the secondary winding by throwing over the voltmeter connections to the secondary terminals for each position of the contact. The two curves of primary and secondary volts may then be traced on the same sheet of squared paper. Similar curves for a transformer loaded non-inductively and inductively should be obtained by the student.

General Note to Chapter VI.—In all the discussion given in this chapter, the wave form of current and voltage has been assumed to be sinusoidal. In the case of transformer work, this is never accurately true, on account of the variation in permeability of the core corresponding to the varying magnetising currents. It has been thought to be advisable, in order to obtain simplicity, not to go into the question of distortion of wave forms, but to refer the student who wishes to study the question more completely to the more mathematical treatises on the subject. The results obtained as described are in most cases sufficiently accurate for commercial purposes.

Chapter VII.

Alternators.

Generation of Electromotive Force in the Armature.—In an alternator electromotive force is generated in the conductors by causing them to move across the lines of a magnetic field.*

The value of the electromotive force induced in each conductor is equal to $\frac{\text{lines cut per second}}{10^8}$ volts. The voltage induced in the armature at any instant = (the voltage induced in each conductor) × (number of conductors in series), if the voltage in each conductor is equal and similarly directed to that of the others.

In a direct-current generator the number of conductors acting in series between the brushes is constant, and, owing to the action of the commutator, the conductors supplying current to the external circuit are always similarly situated with respect to the magnets of the machine. Consequently the voltage induced in the armature is constant, so long as the speed and strength of the field do not vary.

In an alternator, current is collected from the *same* and not from *similarly situated* conductors. Consequently the voltage varies with the strength of the field in which the conductors are moving at any moment, and reverses as the conductors move from a field of a given direction into an oppositely directed field.

Since the conductors move at a constant speed, the electromotive force induced in them will undergo exactly the same variations as the field strength exhibits when traced round the armature.

With rectangular poles it is not usually possible to get a field which varies exactly harmonically so as to give a true sine curve as the wave form of the electromotive force induced in the conductors. This may, however, be accomplished by using specially shaped pole tips, or setting the pole faces at an angle to the axis of the armature.

* For simplicity in explanation it is assumed that the conductors move and the field is fixed. The same reasoning exactly applies to machines in which the armature is fixed and the field rotates.

The exact wave form of the electromotive force can only be ascertained by some curve-tracing device such as that described on page 84, or by the use of an oscillograph.

Another factor affecting the voltage of the armature is the distribution of the conductors connected in series. If each winding consists of a number of conductors side by side, the individual conductors will not come under the influence of the poles simultaneously. The maximum and minimum voltage will be induced in the conductors successively and not simultaneously, thus tending to make the changes of voltage more gradual. A winding in which each coil consists of a number of conductors concentrated in a single pair of armature slots, will consequently tend to give a wave form passing

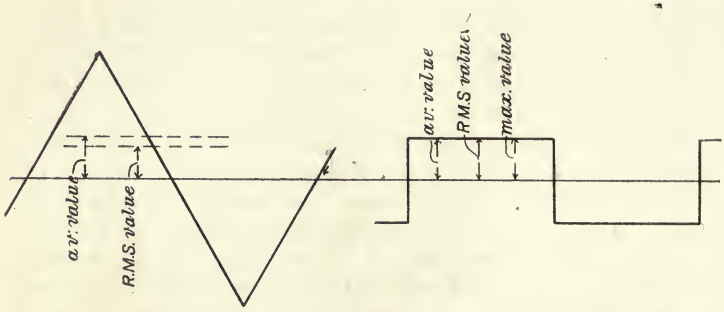


FIG. 90.—ILLUSTRATION OF WAVE FORMS WITH SAME AVERAGE VALUES AND DIFFERENT VIRTUAL VALUES.

steeply from zero to its maximum value, whereas a winding distributed in a number of slots would produce a voltage rising and falling more gradually.

As pointed out on page 89, the virtual or effective value of the alternating current depends upon the *average of the squares* of the instantaneous values, and not upon the *average* values. In consequence of this a voltage of a given *average* value will have a virtual value which is not constant for all wave forms, but will depend upon the wave form, and will in general be low for a flat wave and higher for a wave form possessing a high maximum value, or peak. As an example of this, two very simple wave forms are given in Fig. 90, drawn to such a scale that they both have the same average value. The virtual (root of mean squares) value of the rectangular curve in Fig. 90 is the same as its average value, and equal to its maximum value, while the virtual value of

the triangular curve is quarter of its maximum value, or three-quarters of its average value.

Thus, unlike the case of a direct-current generator, the voltage of an alternator depends not only upon the speed, field strength, and number of armature conductors, but also upon the *distribution* of field and grouping of the conductors, since these affect the wave form of the electromotive force generated, and consequently the virtual value of the voltage.

Voltage of an Alternator.—In the armature of an alternator with a winding so arranged that the conductors always act in series (*i.e.*, the conductors of one winding never simultaneously have oppositely directed electromotive forces induced in them) the *average* voltage with a closed armature winding will be the same as for a direct-current dynamo† *i.e.*,

$$E_{\text{av.}} = \frac{N\,n\,Z}{10^8 \times 60}$$

Hence for sine wave-form

$$E_{\text{virt.}} = \frac{N\,n\,Z}{10^8 \times 60} \times 1{\cdot}11$$

Where N = Number of armature conductors in series,

Z = total number of lines passing through armature, from the north poles to the south poles.

n = revolutions per minute.

1·11 = the ratio of the R.M.S. value to the average value of the ordinates of a sine curve (see page 90).

In a dynamo there are at least two parallel paths through the armature and the number of conductors acting in series is only half the total number of armature conductors.

In an alternator no commutator is used, and the conductors may be all connected in series, the ends of the winding not being closed, but being connected to the slip rings or terminals of the machine; in this case the voltage is doubled, since the number of conductors acting in series is doubled.

For an open winding with sine wave form the formula for the voltage of an alternator becomes consequently

$$E_{\text{virt.}} = \frac{N\,n\,Z}{10^8 \times 60} \times 2{\cdot}22$$

†See "Dynamo and Motor Testing," page 25. *Note.*—In the case of alternators the armature conductors usually are connected in series, consequently the formula is not affected by the number of poles if Z is the total number of lines passing from all the north to south poles through the armature.

The usual plan of stating the voltage of an alternator is to put it in the form

$$E = \frac{N\,n\,Z}{10^8 \times 60} \times k$$

k being a constant depending upon the nature of the wave form of the machine.

In a machine with uniformly distributed and closed winding, like a direct-current armature, the alternating virtual voltage is only ·707 of the continuous voltage which the same armature would give if provided with a commutator. This is due to the fact that only twice during each period will the alternating voltage reach its maximum value, and this voltage is the same as that continuously delivered to the brushes situated on the commutator, and placed in such a position as always to receive the maximum armature voltage when the machine is acting as a direct-current generator.

Thus the alternating voltage (which in this case is practically of sine wave form) is ·707 of the maximum voltage *i.e.*, ·707 of the direct-current voltage.

The ratio of the R.M.S. value to the mean value is usually known as the *form factor*. The form factor varies from about 1·1 to 1·4 for different types of machines.

Periodicity of an Alternator.—In most types of alternators the magnets are alternately north and south when counted in order round the armature. Thus each conductor passes under a north and south pole alternately, and, in doing so, experiences a complete cycle of changes in the electromotive force induced in it. In such a machine the periodicity of the current will be equal to the number of revolutions of the armature (or field) per second × the number of pairs of magnet poles or

$$\text{periodicity} = \frac{n\,p}{60}$$

n = revolutions per minute.

p = number of pairs of poles.

In certain special machines the poles on each side of the armature are all of the same polarity, and the electromotive force induced in the conductors is due to the passage of the conductors through alternately weak and strong fields *in the same direction*. In this case the periodicity equals the number of revolutions per second × the number of poles.

Many of the experiments which follow are almost identical with similar tests upon direct-current machines, described in the author's "Dynamo and Motor Testing." In such cases the experiments are only discussed briefly here, special attention being given to points of difference in the results or conclusions from those previously given.

Relation Between Speed and Voltage of an Alternator.—The formula for the voltage of an alternator

$$E = \frac{n\,N\,Z}{10^8 \times 60}\,k$$

shows that the voltage will vary in the same proportion as the speed if the remaining factors remain constant.

n = revolutions per minute.
N = number of armature conductors acting in series.
Z = number of magnetic lines entering the armature.
k = constant, depending upon wave form of electromotive force.

Hence, as in the case of a direct-current generator, if an alternator be constantly excited and driven at various speeds, the relation of speed to voltage will be constant. If plotted on squared paper with speed horizontal and voltage plotted vertically the relation would be shown as a straight line, exactly as in the case of a direct-current dynamo.

The Magnetic Circuit.—The magnetic circuit of the alternator is designed exactly as in the case of a direct-current dynamo. The magnetisation curve, showing the relation between excitation and magnetic flux, is also of the same importance in the case of alternating generators. This relation is obtained experimentally by running the alternator at a constant speed and varying the excitation. The armature voltage corresponding to each value of the excitation is then observed. The curve is either plotted to show the relation between "ampere-turns" on the magnets and magnetic flux through the armature, or, more usually, the curve shows the relation between the actual quantities observed, *viz.*, exciting current and armature voltage.

The ampere turns can be calculated by multiplying the exciting current by the number of magnet windings, while the useful magnetic flux can be calculated from the observed

voltage and speed by substituting in the following formula already given above :—

$$e = \frac{N\,n\,Z}{10^8 \times 60} \times k$$

$$\therefore Z = \frac{e \times 10^8 \times 60}{N\,n\,k}$$

k being 2·22 for an open circuit alternator giving a sine wave form.

EXPERIMENT XXXIV.—DETERMINATION OF MAGNETISATION CURVE OF AN ALTERNATOR AT NO-LOAD.

DIAGRAM OF CONNECTIONS.

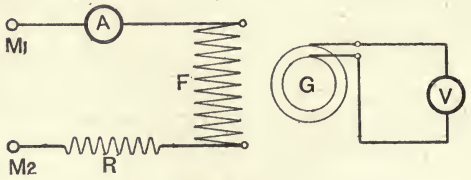


FIG. 91.

$M_1\ M_2$ Source of direct current.
G Alternator armature.
F Alternator field windings.
R Resistance for varying exciting current.
A Ammeter for measuring exciting current.
V Voltmeter for measuring alternator voltage.

Instructions.—Connect the field windings to a supply of direct current in series with a variable resistance and ammeter.

Connect the alternator armature terminals to a voltmeter.

Run the alternator at constant speed. Read the armature voltage first without excitation, and then with exciting current gradually increased to its full value. Then take a similar series of readings with gradually decreasing values of the current.

Enter the readings under headings as shown in the following table and plot a curve in which exciting current is plotted horizontally and armature voltage vertically.

If the speed cannot be kept absolutely constant a correction in the observed voltage must be made. If the speed is too low, the voltage observed will be too low in exactly the same proportion.

Let n = revolutions per minute at normal speed.
n^1 = revolutions per minute observed.
V = required voltage corresponding to normal speed.
V^1 = voltage actually observed at speed n^1.

$$\text{Then } V = V^1 \frac{n}{n^1}$$

DETERMINATION OF MAGNETISATION CURVE AT NO-LOAD.

Observer..... Date.....
Alternator No. ..... Type.....
Normal output.....volts, ...amps., at..... revs. per minute.

Exciting Current.		Revolutions per Minute.	Armature Voltage.		
Ammeter No. — Constant —			Voltmeter No. — Constant —		
Reading.	True Value.		Reading.	Actual Voltage.	Corrected. Voltage.

In the table of results given above, the column headed "Corrected Voltage" is obtained from the preceding column headed "Actual Voltage," by correcting for speed variations.

The readings obtained should form two curves, the curve with decreasing excitation being slightly higher than the curve with increasing excitation forming the first series of readings. This is due to the hysteresis of the iron forming the magnetic circuit. The final curve is the mean of the two, corresponding to the mean of the two voltages observed for each value of the exciting current.

The curve will probably not pass through zero on account of the residual magnetism which will remain in the magnets, and enables the machine to generate a low voltage without excitation.

The curve in Fig. 92 is the magnetisation curve of a 6 kw. 4-pole alternator. In this case the rotating field showed no

appreciable residual magnetism, and the alternator gave no voltage when unexcited, consequently the curve is made to pass through zero, and there are not two distinct branches of the curve as were obtained with the direct-current dynamo, whose magnetisation curve is shown in Fig. 12, page 39, of "Practical Testing of Dynamos and Motors."

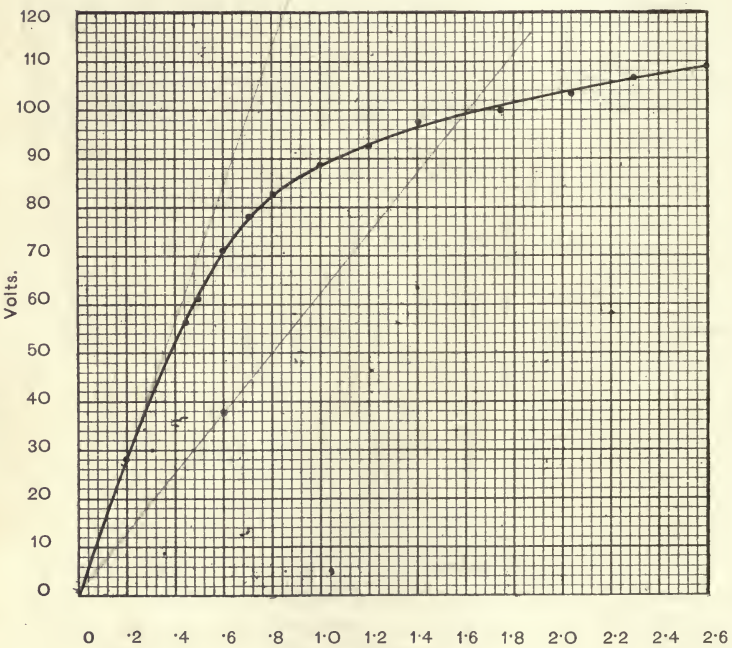


FIG. 92.—MAGNETISATION CURVE.
Speed 1,200 r.p.m.

The no-load magnetisation curve of an alternator does not differ in character from that of a direct-current dynamo. A brief summary only of the chief points is given here.

Hysteresis.—The ascending and descending curves do not usually coincide accurately on account of the hysteresis of the magnetic circuit, which is caused by some kind of molecular friction in the material, tending to maintain the particles of the metal in their positions of magnetisation or demagnetisation, when these positions are once assumed.

Soft wrought iron shows very little hysteresis effect, and the distance apart of the ascending and descending curves

consequently gives an indication of the nature of the material employed.

In the case of an alternator with rotating field, on account of the short magnetic circuit and the high quality of the material employed both for the poles and the whole magnet system, which has to be made structurally very strong and relatively light, the hysteresis and residual magnetism will be low compared with what will usually be found in the case of a machine with stationary field system formed of a massive casting.

Nature of Curve.—From the curve may be obtained directly the absolute value of the magnetic flux corresponding to any voltage, and the vertical scale may be graduated in terms of " lines " instead of volts, if desired.

This is illustrated by the following example in connection with the curve shown in Fig. 92.

The machine did not give a perfect sine wave form, but the virtual value of the curve only differed slightly from that of a sine curve. The total magnetic flux cut by the conductors at 100 volts was consequently approximately as follows, 80 being the number of conductors in series, and 1,200 revs. per minute being the speed (see formula above).

$$Z = \frac{e \times 10^8 \times 60}{N\,n\,k}$$

$$= \frac{100 \times 10^8 \times 60}{80 \times 1{,}200 \times 2{\cdot}22} = 2{,}815{,}000 \text{ lines},$$

or, since there are two pairs of poles,

$$\text{flux per pole} = 1{,}407{,}500 \text{ lines}.$$

It will be seen on reference to Fig. 92, that the first part of the curve is approximately straight. This is because the permeability of the iron of the circuit and of the air gap is fairly constant at low inductions, and the magnetic flux consequently increases in the same proportion as the magnetising force. The slope is determined by the ratio between magnetic flux produced and magnetising force applied. Hence, the tangent of the angle which the straight part of the curve makes with the horizontal will be inversely proportional to the reluctance of the magnetic circuit, since

$$\text{Flux} = \frac{\text{magneto-motive force}}{\text{reluctance}}$$

The steeper the inclination of the first portion of the curve, the less is the reluctance of the magnetic circuit. At a higher induction, the curve bends decidedly to the right, and then again becomes comparatively straight, making a much smaller inclination with the horizontal. When working upon this part of the curve, the effect of small variations in the excitation upon the voltage of the alternator is comparatively slight, and the ratio of magnetising current to magnetic flux is considerably greater, owing to the decreased permeability of the iron at the higher saturation.

Magnetic Leakage.—The magnetic leakage increases at higher magnetic densities in practically the same proportion as the excitation, since the reluctance of the leakage path is practically constant, and, consequently, the leakage flux varies directly with the field ampere turns. The effect of the leakage upon the curve will be to cause it to incline slightly to the right.

Magnetisation Curve under Load.—The effect of a current flowing in the armature upon the terminal voltage is to produce a loss of voltage due to the following causes :—

(1) Armature resistance.
(2) Armature self-induction.
(3) Eddy currents in pole faces.
(4) Demagnetising and distorting action of armature currents.

The first two causes produce a loss of voltage numerically equal to the product of armature current and armature impedance.

The voltage overcoming impedance will not usually be in phase with the total voltage generated. This must be borne in mind in subtracting the armature impedance volts from the total voltage.

The third cause is one depending largely on the construction of the machine and the method of winding. It should not contribute very much to the armature drop.

The last cause of loss of voltage might at first appear incongruous since the current in the armature is an alternating one, and it might be thought that it would consequently not affect the permanent field. It must be remembered, however, that the rate of alternation of the current is the same as the rate at which an armature coil moves from the centre of one pole to the centre of the next. Hence, if the

current attained its maximum value as the centre of the coil passed the centre of a pole, the armature currents would exert a direct strengthening or weakening effect on the main field. In a machine of the usual type, the maximum electromotive force is induced in a winding when it is situated mid-way between two poles, *i.e.*,when the conductors composing the winding are directly under the poles. If the current is in phase with the voltage, the maximum current will neither strengthen nor weaken the field. Before the coil gets to this symmetrical position the current will be smaller in value and will tend to strengthen the pole opposite to the coil. After passing the central position, the current will tend to weaken the next pole to the same extent. The result is, that with the current in phase with the voltage, neither strengthening nor weakening as a whole takes place. The result of a lagging or leading current is discussed later.

EXPERIMENT XXXV.—DETERMINATION OF MAGNETISATION CURVE OF AN ALTERNATOR AT FULL NON-INDUCTIVE LOAD.

DIAGRAM OF CONNECTIONS.

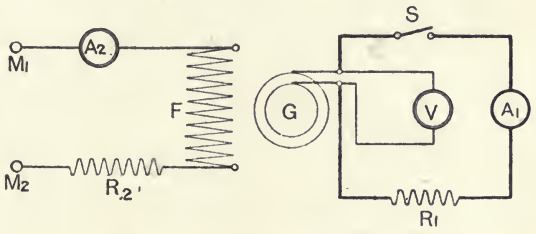


FIG. 93.

M_1, M_2 Source of direct current.
G Alternator armature.
F Alternator field windings.
R_1 Variable non-inductive load resistance.
R_2 Field regulating resistance.
A_1 Ammeter for measuring load current.
A_2 Ammeter for measuring exciting current.
V Voltmeter for measuring alternator voltage.
S Switch for breaking load circuit.

Instructions.—Connect the alternator field windings to a source of continuous current through an ammeter and

regulating resistance. Connect the armature to an adjusable load resistance in series with an ammeter and switch. Connect a voltmeter to the alternator terminals.

Before exciting the alternator, close the switch in the load circuit and make R_1 as low as possible. Run the alternator at normal speed and then increase the excitation slowly until the full load current is reached. Maintain the speed constant and increase the excitation step by step, at the

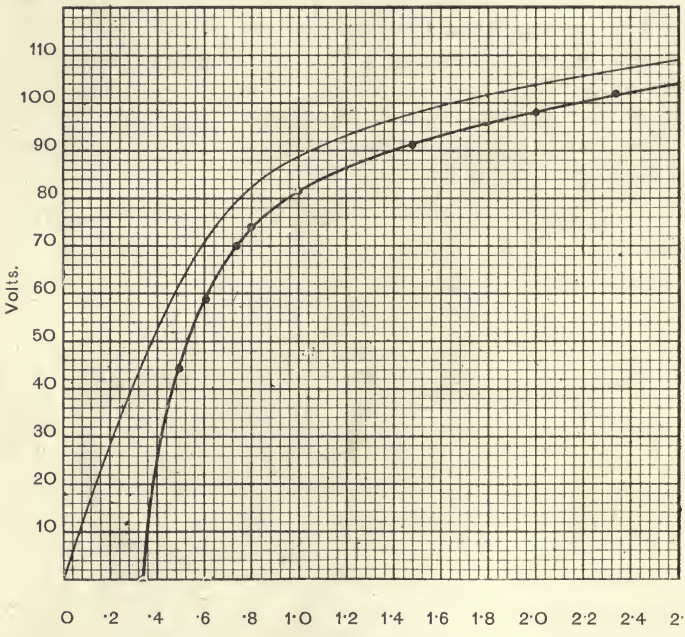


FIG. 94.—MAGNETISATION CURVE WITH LOAD. (NON-INDUCTIVE
Load current = 20 amps.

same time increasing the resistance in the load circuit so as to maintain the load current at the same value as before. In each case take simultaneous readings of voltage and exciting current. Enter the readings in tabular form as in the preceding experiment. The results are then to be plotted to give the full-load magnetisation curve with volts vertically and exciting current horizontally, as in the case of the no-load curve of the previous experiment.

The lower curve shown in Fig. 94 is a load magnetisation curve taken on the same alternator as the no-load curve shown in Fig. 92. In order to enable an easy comparison to be made between the two curves, the curve of Fig. 92 has been re-drawn in Fig. 94. The vertical distance between the two curves shows the amount of "armature drop" produced by the load, when measured on the vertical scale of volts. Thus with an excitation of 1·8 amps., the drop of volts is seen to be 6. Similarly, the horizontal distance between the curves, when measured on the horizontal scale of amperes, shows the amount by which the excitation would have to be increased in order to bring the voltage at full-load to its value at no-load. The direct experimental determination of this quantity is the subject of a later experiment.

The causes producing the armature voltage drop have been already enumerated. The most important of these are the armature resistance and the armature self-induction. The armature resistance forms the chief cause of loss of efficiency in the armature. It is readily obtained by sending a measured direct current through the armature and noting the voltage drop on a milli-voltmeter, or by one of the more sensitive galvanometer methods employed for measuring the resistance of direct-current armatures. The power lost in the armature at any current output is

$$W = C^2 R$$

where R is the armature resistance.

The self-induction does not produce an actual loss of power, although it does produce a larger drop of voltage, and consequently greater diminution in the output of the machine than the armature resistance.

The impedance of the armature is made up of the resistance and self-induction, and is given by the usual formula

$$\text{Impedance} = \sqrt{R^2 + (2\pi n L)^2}$$

where R = armature resistance.
L = armature self-induction.
n = periodicity.

The method of determining the armature impedance has already been given in Experiment VII., page 52. Another method, which is of considerable practical utility, is the following :—

Approximate Determination of Armature Self-induction.—The experiment just described suggests a simple approximate method for determining the armature self-induction of an alternator.

It has been stated that when an alternator is connected to a non-inductive circuit, the drop in voltage at its terminals is principally due to the voltage required to overcome the impedance of the armature. If it is assumed that the difference between the terminal voltage on load and on open circuit is wholly due to the armature impedance, it is easy to calculate the value of the self-induction. To make the measurement, run the alternator at normal speed and excitation and note the voltage on open circuit. Close the circuit switch, so that the machine supplies current to a non-inductive load, and read the current and voltage. The second voltage is equal to $C \times R$, when

C = measured current.

R = resistance in external circuit.

This voltage will be in phase with the current, since the circuit is non-inductive. The voltage spent in the armature will be partly energy voltage in phase with the current, its amount being $C \times r$, when r is the resistance of the armature. The remainder of the voltage lost in the armature will be idle voltage 90° out of phase with the current, and due entirely to armature self-induction. The total voltage generated by the alternator is assumed to be the same as before closing the switch and is therefore the first voltage noted. Hence this total voltage must be the resultant of the energy and idle voltages in the armature and external circuit together after the closing of the switch. The relation between the quantities is that indicated in the annexed diagram, Fig. 95, where

$AB = E$ = no-load voltage.

$DB = E_1$ = terminal voltage with load = $C\ R$.

$AC = E_s$ = idle voltage due to armature self-induction.

$AD = E_a$ = voltage overcoming armature impedance.

$CD = C \times r$ = voltage overcoming armature resistance.

The base of the triangle is the sum of the energy voltages for the whole circuit, which is partly in the armature and partly in the external circuit.

The value of E_s is obtained by construction or by calculation, since

$$E_s = \sqrt{E^2 - (E^1 + Cr)^2}.$$

E_1

The value of the self-induction is directly obtained from the value obtained for E_s, since $E_s = 2 \pi n L C$,

when n = periodicity of current,

and L = coefficient of self-induction of armature.

These results can be obtained directly from the readings shown in Fig. 94, and the following example, taken from that figure, is given to show the method of calculation.

With an excitation of 1·64 amperes, the total voltage generated at no-load is seen to be exactly 100 volts. With

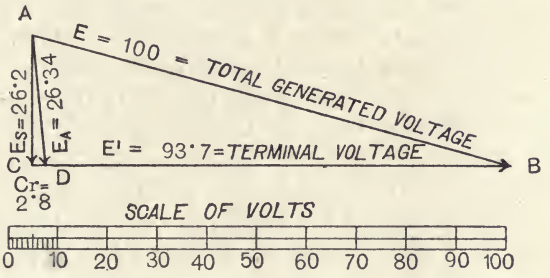


FIG. 95.—DIAGRAM OF VOLTAGES IN ARMATURE AND EXTERNAL CIRCUIT.

a load of 20 amperes the terminal voltage is 93·7. We have for the whole circuit, including external resistance and armature,

(Total volts generated)^2 = (idle volts)^2 + (energy volts)^2,

or 100^2 = (idle volts)^2 + $(93{\cdot}7 + 20 \times {\cdot}14)^2$,

since 93·7 is the energy voltage applied to the non-inductive external circuit, and the armature resistance was ·14 ohms.

Hence idle volts = $2 \pi n L C = \sqrt{100^2 - (96{\cdot}5)^2}$

$= \sqrt{10000 - 9312} = 26{\cdot}2.$

The periodicity was 40 and the value of the self-induction becomes

$$L = \frac{26{\cdot}2}{2 \pi n C} = \frac{26{\cdot}2}{2 \pi \times 40 \times 20} = {\cdot}0052 \text{ henry.}$$

= 5·2 millihenry at the particular excitation and current taken.

The value of $2 \pi n L$ at this excitation is $\frac{26{\cdot}2}{20} = 1{\cdot}31.$

Consequently the impedance of the armature

$= \sqrt{R^2 + (2 \pi n L)^2} = \sqrt{{\cdot}14^2 + 1{\cdot}31^2} = \sqrt{1{\cdot}736}$

= 1·317 apparent ohms.

This value will vary with the armature current and with the field excitation on account of the varying permeability of the iron.

It thus appears that the voltage drop due to armature impedance is about $20 \times 1{\cdot}317 = 26.34$ volts at normal excitation in the present case. At first, this appears to be contrary to the indications of the curves shown in Fig. 94 where the difference between the volts at the terminals of the armature at no-load and full-load is shown to be only $100 - 93.7 = 6{\cdot}3$ volts. The reason for the apparent difference between the two results is that the voltage lost in armature impedance is nearly all due to armature self-induction,and is consequently nearly $\frac{1}{4}$-period out of phase with the terminal volts, which are only slightly affected. The diagram, Fig. 95, shows the relation between the voltages of the example worked above.

The value of the self-induction obtained by this method will always be slightly too high, since the weakening of the field caused by the armature magnetic reaction is neglected.

For many practical purposes it is not important to distinguish between loss of voltage due to weakening of the field, and to self-induction. Neither cause produces loss of power; both diminish the available voltage.

A further point to be noticed is that as the resistance of the external circuit becomes lower, a greater proportion of the total voltage generated is spent in overcoming the armature impedance, and for this reason the distance between the curves becomes greater as this voltage becomes a more and more important factor of the total voltage and differs less and less in phase from it. Thus, even if the volts lost were the same for all excitations, this would cause the difference between total and terminal volts to increase at lower excitations. When the machine is short-circuited the voltage overcoming impedance is the total voltage, and its full value would therefore be seen on the curve. These changes should be followed out in connection with the changes occurring in the shape of the triangle, Fig. 95, when drawn for various current outputs.

The point where the load magnetisation curve cuts the horizontal axis is given by the value of the excitation required to send the load current through the armature when short-circuited, and where the terminal voltage is consequently

zero. In this case the whole of the voltage generated is spent in overcoming the armature impedance.

The voltage thus spent is apparently 46 volts in the case of the machine under consideration. The armature self-induction is lower at this low excitation, and consequently the voltage spent in overcoming the armature impedance is greater, but this is probably not sufficient to account even for the greater part of the difference between the voltage 46 and 26·3 obtained by calculation above. With very weak excitation and relatively strong armature current the magnetic re-actions are very strong and have a strongly demagnetising action, since the circuit (consisting of the armature alone) is highly inductive, and lagging currents tend to weaken the main field, as is explained below.

A comparison of the full-load and no-load magnetisation curves shows the loss in terminal voltage between no-load and full-load when the excitation remains at a constant value. It further enables a determination to be made of the change in excitation necessary to overcome this loss of voltage and to maintain the same voltage at full-load as at no-load. Thus, on referring to Fig. 94, if 93 volts is required, it will be seen that the difference in exciting current between the points corresponding to 93 volts on the two curves is ·38 amperes, so that the range of a field-regulating resistance to maintain this voltage at all non-inductive loads would have to be sufficient to produce this variation in the excitation. It must, however, be remembered that the loss of voltage will be greater on inductive loads, and consequently the curves shown in Fig. 97 would have to be employed in cases where the machine supplies inductive circuits.

A further important point, which is more likely to escape notice, is that the magnetisation curve enables the magnitude of the steps of the regulating resistance to be determined in order that regulation may be accomplished with any desired degree of sensitiveness. Thus, if it is desired to regulate the voltage within one volt, the variation of exciting current corresponding to the volt difference of pressure may be at once obtained by an inspection of the curve. From this the exact value of each step of the regulator can be calculated for a fixed voltage of excitation. For instance, at the upper part of the curve at 2·4 amperes excitation in Fig. 94 each volt corresponds to an increased excitation of ·092 amperes.

If the voltage of excitation is 100, the value of each step would be $\frac{100}{2{\cdot}4} - \frac{100}{2{\cdot}492} = \frac{9{\cdot}2}{5{\cdot}98} = 1{\cdot}53$ ohms.

Magnetisation Curves with Inductive Load.—The curve discussed in the foregoing paragraph is the magnetisation curve obtained with the machine supplying current to a non-inductive circuit. This curve is the most important of the load curves, which can be obtained from an alternator. In most cases, however, alternators have to supply circuits consisting partly of inductive resistances, and it is consequently important to determine in what way the self-induction of the external circuit affects the behaviour of the alternator. In order to do this, a partially-inductive resistance should be substituted for the non-inductive load resistance employed in Experiment XXXV., and the voltage corresponding to the full-load current at various power-factors and with various excitations should be determined.

Several curves obtained in this way have been drawn in Fig. 97, and show that the terminal voltage of the alternator is diminished by the lag of the current in an inductive circuit.

Before explaining the reason for this the method of obtaining the magnetisation curves for inductive loads must be described.

The difficulty of getting a complete magnetisation curve on an inductive load arises from the fact that it is in practice very difficult to vary the impedance of the external circuit, and to keep the power-factor of the circuit constant, on account of the difficulty of varying the inductive and non-inductive portion of the circuit simultaneously in the required proportion. In the case of large machines, a single reading often is all that can be obtained on full load at a given power-factor. The nature of the curve in the neighbourhood of the point can then be approximately traced by drawing a portion of the curve parallel to the non-inductive load curve. This often suffices for practical purposes, since the actual variation in excitation when running corresponds to a small portion only of the total magnetisation curve.

If more complete curves are desired corresponding to several values of the power-factor, the following method may be employed. The description may be most simply given by referring to the readings, actually taken in order to obtain the curves shown in Fig. 97.

It was desired to obtain magnetisation curves for the three power factors ·9, ·8, and ·7, the current being in all cases 20 amperes, as it was in the curve for non-inductive load.

From an inspection of the non-inductive curve it was considered that readings taken at voltages of about 100, 90, 80, 70, and 60 would be sufficient to enable the curves to be plotted. A preliminary curve was accordingly taken at each of these voltages in the following manner.

The alternator was connected to a circuit composed partly of non-inductive resistances and partly of an inductive resistance wound upon a magnetic circuit with a

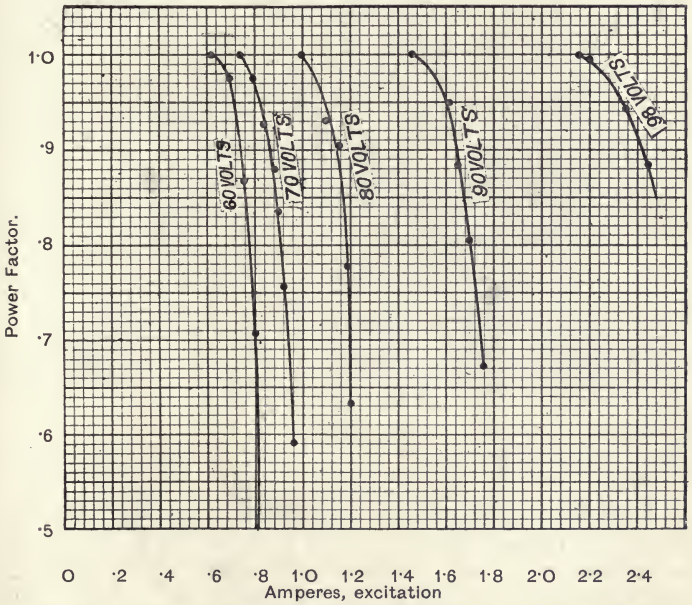


FIG. 96.—CURVES SHOWING VARIATION OF EXCITATION WITH POWER FACTOR.

variable air gap. (It will in some cases be found most convenient to connect the inductive and non-inductive resistances in parallel and in other cases in series.)

The inductive resistance was then given its maximum value, and the non-inductive part of the circuit was adjusted to give the correct current of 20 amperes. The excitation was then adjusted until the voltage had assumed each of the five values, 98, 90, 80, 70, and 60.

The inductive resistance was then made less inductive, and a similar series of readings taken. This was repeated for five different values of the inductive resistance, giving points corresponding to five different voltages on curves corresponding to five different values of the power-factor. These curves were plotted so as to show the variation of exciting current with power-factor. The curves actually obtained in this manner are shown on Fig. 96. From these curves points on the magnetisation curve corresponding to any required power-factor may be obtained, although actual readings for the exact value of the power-factor were not actually taken.

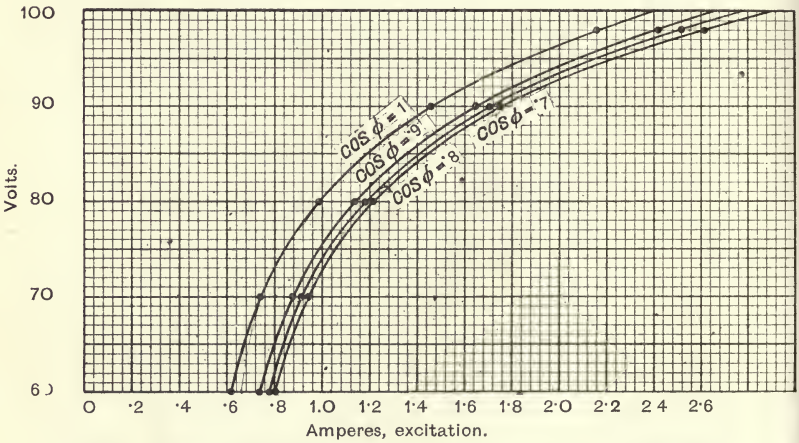


Fig. 97.—Magnetisation Curves at Full Load and Various Power Factors.

Thus the actual excitation required to give respectively 98, 90, 80, 70, and 60 volts at the three values of the power-factor were read off from the curves in Fig. 96 and the complete curves obtained from these points, as shown in Fig. 97.*

With a leading, instead of a lagging, current in the circuit supplied, it would be found that the terminal voltage of the alternator is *increased* instead of decreased. Curves illustrating this are given later in connection with the determination of the characteristic curve of an alternator.

* The speed was not exactly the same during the readings taken for Figs. 94 and 97, which accounts for the slight difference between the load curve in Fig. 94 and the curve marked cos $\phi = 1$ in Fig. 97.

It is now necessary to explain the action of a lagging or leading armature current in causing variation in the terminal voltage. The action depends on the magnetic re-action of the armature field upon the main field.

Effect of Lag or Lead of Current upon Armature Reaction.—As being the simplest case for purposes of explanation, the Fig. 98 has been drawn to represent part of a 2-pole alternator with ring-wound armature.

Case I.—Current and Voltage in Phase.—If current is taken from the armature and supplied to a non-inductive circuit, there will be no phase difference between current and volts, and these quantities will pass through their maximum

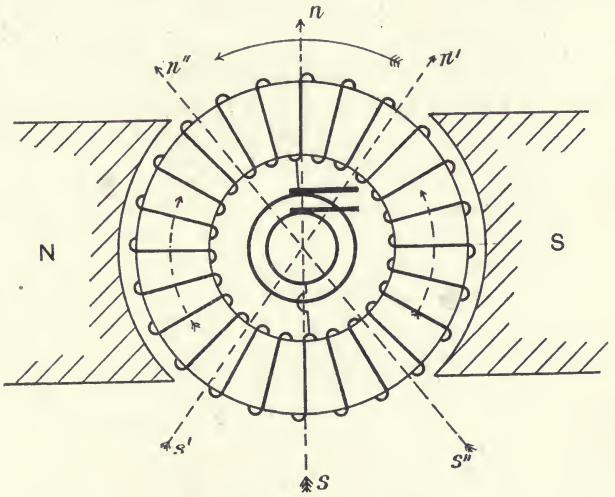


FIG. 98.—DIAGRAM ILLUSTRATING EFFECT OF ARMATURE REACTIONS.

and minimum values simultaneously if the effect of the self-induction of the armature is neglected for the present.

With the armature in the position shown, the voltage will be at its maximum value. If the current is also at its maximum value (*i.e.*, if there is no difference of phase between the current and volts) the current will produce a field perpendicular to the main field, as indicated by the symbols *n s*. This armature reaction field will distort but not strengthen or weaken the main field. Earlier in the revolution the field of the armature would be weaker, since the current would be weaker, and would be directed partly

assisting the main field as indicated by the axis n' s'. Later in the revolution the field will also be less, but will be directed so as to *oppose* the main field, its direction being, for instance, n'' s''.

The average effect of the armature currents is thus to distort, but neither to strengthen nor weaken the main field.*

Case II.—Current Lagging.—Suppose the current only attains its full value after the position of maximum voltage has been passed, as, for instance, after the conductors under s and n have moved into the position s'' n''. The maximum current will then produce a field in the direction s'' n''. Positions of the armature on either side of this will correspond to weaker fields, which will be of equal value at equal angles on either side of n'' s''. Hence the average effect of the armature currents will be the production of a field along n'' s'', which will partly weaken and partly distort the main field. This may be briefly stated thus: A lagging current tends on the whole to weaken the main field.

Case III.—Current Leading.—Suppose that the current leads, so that it attains its maximum value before the position of the conductors producing maximum voltage, for instance when the conductors shown under n s are at n' s'. Then the maximum armature field will be induced in the direction n' s', and the average magnetic reaction will be such that the main field is both strengthened and distorted. Hence a leading current will on the whole strengthen the main field of an alternator.

Exactly similar reasoning applies to the case of a multipolar alternator, whether with fixed or rotating armature. This action necessarily makes it more difficult to regulate an alternator with varying load when the power factor of the circuit is low, *i.e.*, when there is considerable phase difference between current and voltage. The amount of the effects to be expected may be found as a result of the following experiments.

The foregoing reasoning may be put into more exact form by the statement, which will be readily seen to be true, that if the ampere turns of the main magnetic circuit be N C and the product of current and windings in the armature

* The effect of decreased permeability owing to the concentration of the lines under one pole tip is here neglected.

be n c, the effective (R.M.S) value of the magnetising action of the armature currents will be $\cdot 7$ n c, and the total effective demagnetisation of the fields is due to

$$N\ C - \cdot 7\ n\ c \sin\ \phi \text{ ampere turns}\dagger,$$

i.e., the armature current diminishes (or increases if ϕ be negative) the effective ampere turns of the field by the amount $\cdot 7\ n\ c \sin \phi$ ‡, where ϕ is the angle of lag of the current in the armature.

Characteristic of the Alternator.—The determination of the loss of the voltage produced by a given current is a very important matter, and the result of such a determination is one of the most important records of the behaviour of the machine.

The curve comparing the voltage and load current is called the " characteristic curve " of the machine, and forms the subject of the next experiment.

After determining the variation in voltage produced by the armature current, it will be possible to partially separate the loss of voltage into that due to the various factors enumerated above.

---

† A multiplying factor must be introduced if the relative positions of windings and pole make the armature windings not all equally effective. The value of n may usually be taken to be

$$\frac{\text{total number of armature conductors}}{\text{number of poles}}$$

‡ This is obtained as follows :—

The actual current in the armature conductors may be written $c = C \sin (\theta - \phi)$ where ϕ is the angle of lag of the current behind the voltage, and θ is the angular distance of the conductors from the point corresponding to minimum electromotive force, C being here maximum value of current in conductor.

The demagnetising action of the current when the conductors are in any position is proportional to the resolved component of the current parallel to main field, *i.e.*, to $\cos \theta$, since the magnetic field produced is perpendicular to the main field. Thus the field due to the conductors opposes the main field, when $\theta = 90°$, and assists it when $\theta = -90°$.

Hence the demagnetising effect of the current in the conductors at any instant is

$$\begin{aligned} m &= n\ c \cos\theta = n\ C \sin(\theta - \phi) \cos\theta \\ &= n\ C (\sin\theta \cos\phi - \cos\theta \sin\phi) \cos\theta \\ &= n\ C (\sin\theta \cos\theta \cos\phi - \cos^2\theta \sin\phi) \end{aligned}$$

the average value of $\sin\theta \cos\theta = O$

,, ,, $\cos^2\theta = \frac{1}{2}$.

Hence the average value of $m = -\frac{1}{2}\ n\ C \sin\phi$

$$= -\frac{1}{2}\ n\ \sqrt{2}\cdot c \sin\phi$$

$$= -\cdot 7\ n\ c \sin\phi.$$

Experiment XXXVI.—Determination of the Characteristic of an Alternator.

Diagram of Connections.

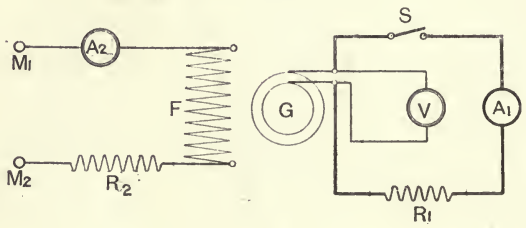


Fig. 99.

M_1 M_2 Source of direct current.
G Alternator armature.
F Alternator field windings.
R_1 Non-inductive resistance for varying load.
R_2 Resistance for maintaining excitation constant.
A_1 Ammeter for measuring load current.
A_2 Ammeter for measuring exciting current.
V Voltmeter for reading alternator voltage.
S Switch for breaking main circuit.

Instructions.—Excite the fields of the alternator from a source of direct current in series with a regulating resistance and ammeter.

Connect the armature to a variable non-inductive resistance in series with an ammeter and switch. Connect a voltmeter to the armature terminals.

Run the alternator at constant speed and maintain the excitation (read on ammeter A_2) constant throughout the experiment.

Take readings of armature voltage and current, first with the switch S open, and then for gradually increasing values of the load up to the maximum output. Repeat the readings while decreasing the current by similar steps.

Enter the results in tabular form as shown below, and plot a curve in which voltage is measured vertically and current horizontally.

Characteristic on Non-Inductive Load.

Alternator No..... Type ....., Periodicity .....
Normal output ....amperes ....volts at.... revs. per min.
Resistance of armature (hot) ....ohms.

Load Current.	Revolutions per Minute.	Voltage.

The voltage readings must be corrected if the speed does not remain constant.

A curve obtained in this way on a 6 kw. 3-phase alternator is shown in Fig. 100, at a constant excitation of 1·9 amps. The curve is here drawn for the full working range of the machine, which is rated to give a full-load current of 35

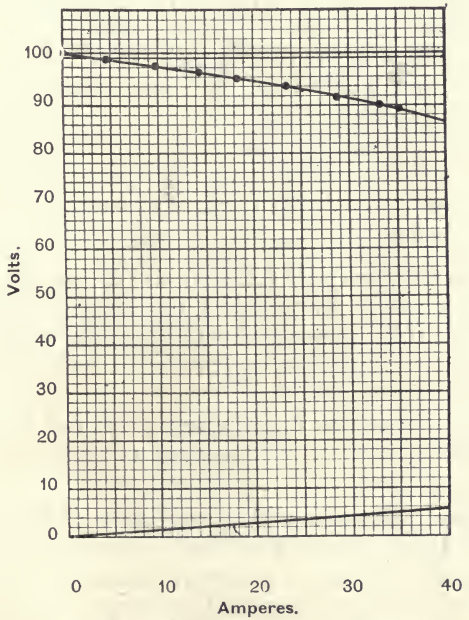


Fig. 100.—Characteristic of an Alternator.
Excitation 1·9 amps. Speed 1,200 r.p.m.

amps., at 100 volts, when the voltage is regulated by the field rheostat. The field was, of course, kept constant during the experiment.

The shape of the characteristic depends chiefly on the amount of self-induction in the armature, since this is usually the factor producing the greatest drop in volts. Armatures having a high self-induction will give a characteristic rapidly drooping, and in some cases reaching the horizontal axis, with a current only slightly in excess of the rated full-load current. The point at which the characteristic meets the horizontal axis is the point of zero voltage, and shows the current given by the machine when short-circuited. Early machines made for supplying a number of arc lamps in series at constant current were purposely designed with a steeply falling characteristic, so that the change of current was very slight when the voltage taken by the lamps varied considerably.

An instructive experiment in which a rapidly falling characteristic may be obtained without the use of a specially-designed alternator is to connect an inductive coil in series with the armature of an alternator of any type available. The alternator is then loaded, and the voltage measured is that of both armature and coil in series. The readings are plotted in the same way as in Fig. 100, and show practically the characteristic of an alternator whose armature resistance and self-induction have been increased by those of the coil. For purposes of calculation and illustration of the behaviour of an arc-lamp alternator such a curve is interesting.

Further Consideration of Characteristic of an Alternator.—Drawing a horizontal line through the highest part of the curve (see Fig. 100) the drop of voltage at any load is represented by the vertical distance between the curve and this horizontal line.

As already stated, the drop in voltage at full load is due to several causes, and it is important to separate them as far as possible.

(1) **Armature Resistance.**—The loss of voltage due to armature resistance may be separated easily, as follows :—

While the machine is still hot from the run, send a direct current through the armature and determine the resistance, either by accurately measuring the current with an ammeter

ALLIDIE LIBRARY OF THE UNIVERSITY

and the voltage drop with a milli-voltmeter, or by comparing the drop with that in a standard resistance in the same circuit by means of a galvanometer.

Calculate the ohmic drop (= armature current × armature resistance) at full load. On the ordinate corresponding to this load set up this voltage drop vertically and draw a straight line through this point and zero. This line then gives the armature ohmic drop at all loads.

In the case illustrated in Fig. 100 the armature resistance was found to be ·14 ohm. At 40 amps. this would correspond to a drop of $40 \times \cdot 14 = 5 \cdot 6$ volts.

The sloping line drawn at the bottom of the figure shows the ohmic drop for all loads.

The approximate method of determining the self-induction which is usually more convenient to employ, is described on page 204. The curve, Fig. 100, obtained in the present experiment may be employed in the same manner, and will then give the impedance and self-induction of the armature at various loads.

The curves, Figs. 94 and 100, apply to the same machine, so that they may be compared. From the two curves taken together, the armature impedance at various excitations and loads may be calculated. The exactness of the results, it must be remembered, depends on the possibility of assuming the effects of armature magnetic reactions and eddy current screening to be unimportant.

(2) **Armature Self-induction.**—The best method of determining the armature drop due to self-induction at various loads has been explained in Experiment VII., page 52.

From the readings taken in the manner then described, it is easy to separate the loss of voltage due to induction from the drop due to armature resistance, by the formula

$$e^2 = e_s^2 + e_r^2$$

Where e = total armature drop obtained in Experiment VII.

e_s = voltage drop due to self-induction.

e_r = voltage drop due to resistance.

It must be remembered that the loss of voltage due to armature resistance and armature self-induction cannot be simply added together on the curve to give the total voltage lost due to both causes, since these two voltages differ by 90° in phase.

The construction for showing graphically the resultant drop due to both causes has already been given in connection with Figs. 94 and 95, page 205.

On non-inductive loads, the ohmic drop is the most important factor, since it is in phase with the terminal volts. On inductive loads the reactance voltage has a greater effect in lowering the voltage.

It will be found that the loss of voltage in overcoming the impedance of the armature accounts for the greater part of the total loss shown in the characteristic curve of the machine. Practically the whole of the remaining loss of voltage is due to the third of the causes enumerated above, viz., weakening of main field.

(3) The weakening of the main field, due to magnetic reactions of the armature currents, is due to two causes: (*a*) The direct weakening effect of the armature field acting in opposition to the main field, and (*b*) the "screening" effect of eddy currents induced in the pole faces of the field magnets, and possibly also in the magnet windings themselves. The direct weakening effect is small when the armature is working on non-inductive load, as the action of the armature field is chiefly to distort and not to oppose the main field. There will be a slight weakening effect due to the fact that the lines will be more concentrated at one pole tip owing to the distortion, and that this will decrease the permeability of the pole in consequence. The direct weakening effect will be increased by a lagging current and diminished by a leading current. The screening action of the eddy currents is due to the induction of local currents in the pole faces and any other metal parts which are not laminated, caused by the pulsations of the field. The direction of these currents is always such that the lines of force due to them opposes the field inducing the currents, and the formation of the currents thus has the effect of a "screen" hindering the penetration of magnetic effects.

These eddy currents will be more readily produced in cases when the self-induction of the armature varies considerably in passing from pole to pole, *i.e.*, with armature winding concentrated in few open slots. In order to lessen the eddy currents, the poles or pole faces of large alternators are frequently made of laminated iron.

(4) Alteration of wave-form producing different virtual value of voltage. This cause would be too slight on a non-

inductive load under ordinary circumstances to be appreciable.

Effect of Armature Currents upon the Field.—In Experiment VII., page 52, was given the method of determining the impedance of the armature while stationary. It was further stated that currents might be induced in the pole faces, poles and field windings, by the alternating armature flux. It is now desirable to consider in greater detail the nature and measurement of these effects.

The winding of the armature acts like the primary winding of a transformer in producing an alternating magnetic flux. The path of this flux will vary with the position of the armature winding relative to the field. In certain positions the flux will pass across the air-gap from the armature core to one pole tip, then through the iron of the pole to the other pole tip of the *same* pole, and across the air-gap into the armature core. In the position occupied by the armature $\frac{1}{4}$ cycle later, the flux has to complete its path external to the armature core *between* a pair of poles, instead of through the iron of one pole. In this case the effect of the flux on the poles is comparatively slight. With an alternating current flowing in the stationary armature it will therefore depend on the position of the armature relative to the poles how much these will be affected by the alternating flux of the armature.

If the poles are not laminated, the effect of the alternating field will be to produce eddy currents in the iron of which they are composed. If they are surrounded by a winding, an alternating voltage will be induced in the winding which will produce an alternating current if the winding is closed.

The effect of such induced currents upon the armature winding will be similar to the effect of currents in the secondary of a transformer upon the primary winding. That is, the self-induction of the winding is reduced and some of the power of the primary circuit is lost in producing these secondary currents.

The actual extent to which these losses occur in actual working is not easily measured, but the following description gives some idea of the way in which they are formed and the extent to which they may exist.

By making connections exactly as described in Experiment VII. for the determination of amature impedance while stationary, but including a wattmeter in the supply circuit,

we can measure the power spent in sending the current through the armature. The power thus measured is the sum of the losses in the armature and in the fields, the latter being due chiefly to eddy currents, as the hysteresis losses will be small at the low induction produced.

Now, the losses in the fields will vary with the position of the armature, being greatest when the flux in the fields is greatest, and being small when the armature is so situated as to produce a minimum of flux through the poles. The hysteresis and eddy-current losses in the armature will, however, not vary very much with the position of the armature. By taking a series of readings of the power developed when supplying a constant current to the armature in a succession of different positions, it is thus possible to form an idea of the approximate amount of the losses which occur in the fields owing to eddy currents.

If the poles are surrounded by closed circuits having a high conductivity, the magnetic flux will produce currents in these round the pole face. The magnetic effect of these currents will be to oppose the varying flux, and to greatly

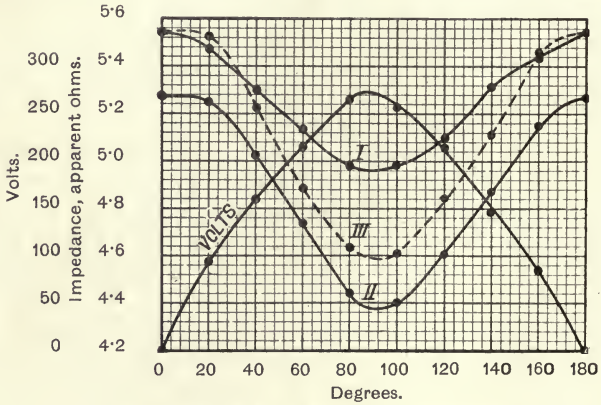


FIG. 101.—CURVES OF ARMATURE IMPEDANCE.

Curve I. Field winding open.
„ II. 1 ampere excitation.
„ III. Field winding short-circuited.

CURVE OF "VOLTS" AT TERMINALS WITH 10 AMPS.

diminish its amount. This is the effect of the amortisseurs often used on the poles of alternating-current machines. The ordinary magnet winding, if short-circuited, will have a somewhat similar effect in reducing the eddy currents in the poles as is illustrated in the curves to be described.

In order to illustrate what has been said, the curves of impedance and watts absorbed in Figs. 101 and 102 have been drawn. They were taken on a 1 kw. 4-pole alternator, with closed ring armature winding and solid poles. The armature while stationary was supplied with an alternating current of 10 amperes, the voltage at the terminals and the watts absorbed were observed for a series of positions of the armature. In Fig. 101 is shown the impedance of the armature (1) with the field circuit open, (2) with the field terminals short-circuited, (3) with the normal excitation of 1 ampere.

Fig. 102 shows the readings taken on the wattmeter in the same experiment, the numbers on the curve indicating the conditions as before. It will be noticed that the losses

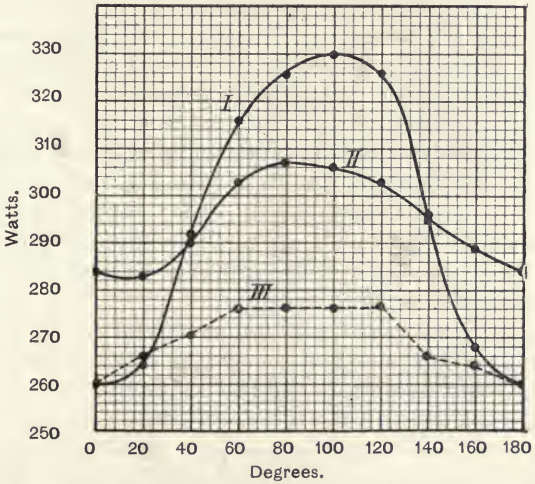


FIG. 102.—POWER TAKEN BY ARMATURE IN DIFFERENT POSITIONS WHEN SUPPLIED WITH CONSTANT CURRENT OF 10 AMPERES

Curve I. Field winding open.
,, II. 1 ampere, excitation.
,, III. Field winding short-circuited.

vary considerably with the position of the armature, reaching a maximum of 330 watts, when the coils are exactly opposite to the magnet poles. Further, this loss is very much less when the field windings are short-circuited. In the position where the armature coils lie between the poles the losses are the same, whether the field windings are short-circuited or not. This tends to show that the effect of the armature currents on the magnets is very small in this position. We

may, consequently, assume that the 260 watts supplied with the armature in this position is practically that due to the losses in the armature itself. The resistance of the armature was 2 ohms, so that 200 watts were lost in the armature winding and about 60 watts in armature hysteresis and eddy currents. It is interesting to notice that while the short-circuiting of the magnet windings diminishes the impedance of the armature winding on account of the transformer action which produces currents in the magnet winding, the power absorbed is less, owing to the more effectual magnetic screening of the currents in the winding as compared with the eddy currents in the iron of the poles to which the losses are due when the magnet winding is open.

The effect of excitation of the magnets is to diminish the armature impedance owing to increased saturation of the core added to the production of alternating currents in the magnet windings super-posed on the normal excitation.

The power curve (II.) in this case shows the influence of the field distortion resulting from both armature and exciting currents. There is thus no position when the fields are unaffected by the armature flux. In Fig. 102 the areas enclosed by curves I. and II., and the horizontal axis are almost identical, showing that the average effect of eddy currents for all positions of the armature is about the same with the field coils excited and open.

When the alternator works under normal conditions, the variations of the current will occur with the same frequency as the variation in position of the armature. Thus if the current is in phase with the induced electromotive force, the current would have its maximum value when the conductors are exactly under the centre of the poles, and the centre of the coil is midway between the poles, *i.e.*, the position corresponding to 90° on the curves Figs. 101 and 102. The maximum rate of change of current will consequently occur when the coil is at the point 0° on the curve, where its effect on the poles will be greatest. We should thus obtain the maximum loss due to eddy currents with armature current and induced volts exactly in phase. This condition is, however, scarcely ever found, since, even with a non-inductive load circuit the current lags behind the induced voltage. It is thus easy to see that the action of the armature currents in inducing currents in the field windings and pole faces depends on the power factor of the circuit.

Variation in Exciting Current.—Another method of showing the effect of the armature current on the field is to trace the variation of the exciting current by the method described in Experiment XV.

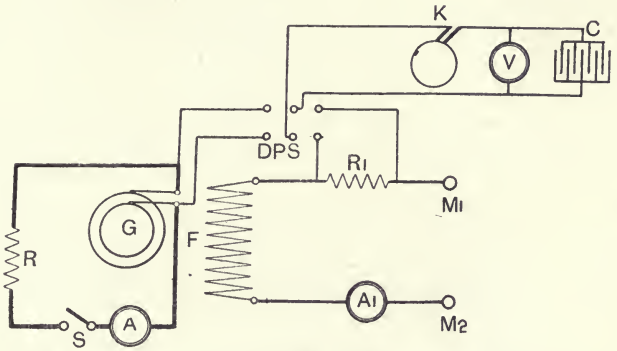


FIG. 103.—DIAGRAM OF CONNECTIONS FOR DETERMINING VARIATION OF FIELD CURRENT.

The connections required for this determination are shown in Fig. 103, where the symbols employed have the following meaning :—

G Alternator armature.
F Alternator field winding.
R Load resistance.
A, A_1 Ammeters.
M_1 M_2 Source of direct current for excitation.
K Revolving contact-maker on alternator shaft or driven synchronously with the alternator. (See page 84).
V Electrostatic voltmeter.
C Condenser.
D P S Two-pole throw-over switch for enabling readings of armature voltage and exciting current to be taken alternately.
R_1 Non-inductive resistance of known value, in field circuit enabling the excitation to be calculated from the observed voltage at its terminals.

The connections will be clear from the above diagram. The method of carrying out the experiment, is to run the alternator at constant speed and constant load. The fields are excited through an ammeter and non-inductive resistance from a source of constant continuous voltage. The electro-

static voltmeter is connected through the revolving contact-maker alternately to the terminals of the armature and the non-inductive resistance in the field circuit by means of the throw-over switch. The readings are repeated for a succession of positions of the contact maker. The value of the exciting current is calculated from the observed voltage E and known resistance R_1, since $C = \frac{E}{R_1}$.

Fig. 104 shows some results obtained in this manner with a small Pyke & Harris inductor alternator. The vertical

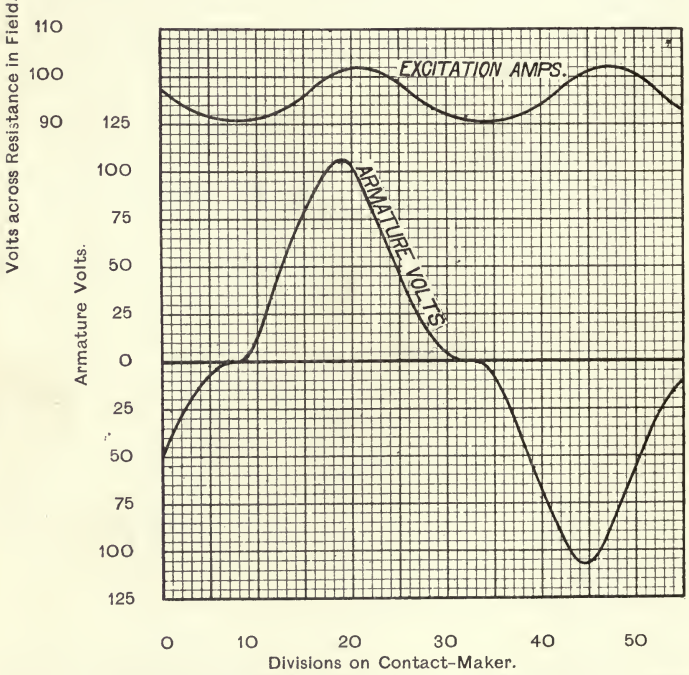


FIG. 104.—CURVES SHOWING EFFECT OF ARMATURE CURRENT ON FIELD CURRENT.
Excitation 1·6 R.M.S. amperes.
Armature current 1·5 R.M.S. amperes.

scale is plotted in terms of the volts registered by the electrostatic wattmeter, the horizontal scale indicating divisions on the contact-maker, which in this case were of an arbitrary value. From the known resistance of the armature circuit and that in the field circuit, the instantaneous values of armature and field current may be obtained. The values of these resistances were 19·3 ohms in the armature circuit, 59 ohms

in the field circuit. The currents as registered on the ammeters in the two circuits were 1·5 amperes in the armature, and 1·6 amperes excitation.

The most important result illustrated by the curves is that the excitation of an alternator, although supplied from a uniform source of electromotive force is subject to regular pulsations. These pulsations have the peculiarity that they occur with exactly twice the frequency of the fluctuations of the armature current. It is a general rule that if the wave form of the voltage generated by an alternator is composed of a number of waves of harmonic form, the frequency of these waves will be 1, 3, 5, &c., times the frequency of the fundamental wave. The frequency of the variations in the excitation will be 2, 4, &c., times the frequency of the fundamental wave. This may be stated briefly by saying that the alternator voltage contains only odd harmonies, while the field current contains only even harmonies.

The double frequency of the induced currents is due to the combined action of the rotation of the armature and the alternating character of the armature current. Thus if the armature were supplied with a direct current a flux would be induced in the field windings due to the armature rotation. This would have a frequency equal to the frequency of the alternating currents actually produced by the machine. Similarly if the armature were stationary and supplied with an alternating current there would be a flux at the poles equal in frequency to the alternation of the current. In the case of the loaded alternator both of these effects occur at once, and produce a variation in the field current which has double the frequency of the main armature current. The variations thus represent those due to rotation, and to the alternations of the current superposed. The double frequency of the field variations is well shown in Fig. 104, where thevariations of the armature electromotive force are plotted below for comparison.

Characteristic Curves with Leading and Lagging Current.—Since alternators have frequently to supply inductive circuits, and since the power-factor of the circuit affects the voltage, it is sometimes necessary to determine characteristic curves with the alternator working on inductive or capacity load.

The difficulties already referred to in connection with magnetisation curves on inductive load arise in the case of

the characteristic curve with even greater force. It would thus be extremely difficult in practice to take a series of readings of current with the power-factor adjusted in each case to a constant value.

If complete curves at different power-factors are required, and not an isolated point only on each curve (which is often enough for practical purposes), the same process of plotting

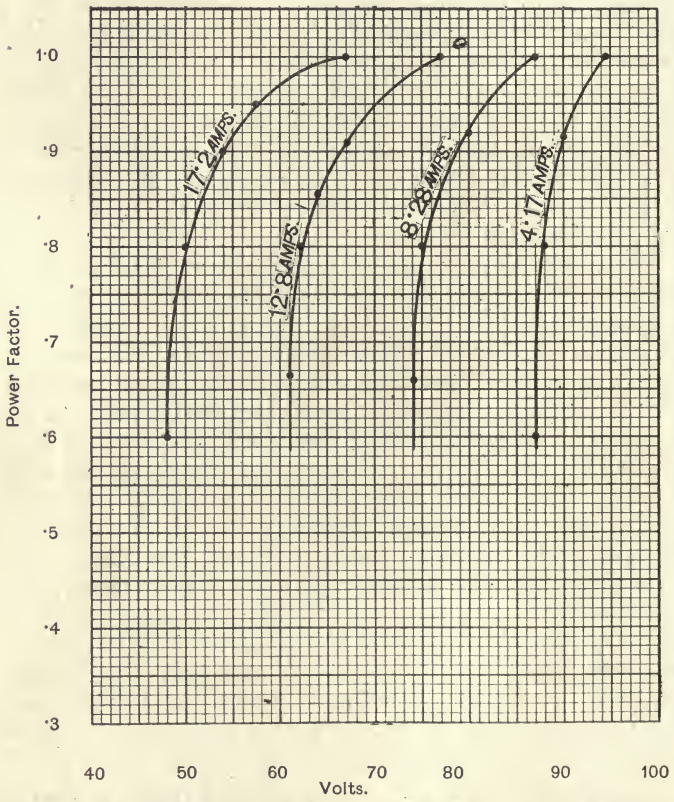


FIG. 105.—VARIATION OF VOLTAGE WITH POWER FACTOR. LAGGING CURRENT.

auxiliary curves comparing power-factor and voltage must be resorted to in this case also.

The following description of an actual test will indicate the method to be adopted in obtaining complete curves.

The machine experimented upon was a 1 kw. 4-pole Crompton alternator, with rotating armature.

(a) Determination of Characteristic Curve on Inductive Loads.—An inspection of the characteristic of the alternator on non-inductive load showed that four readings taken at about 4, 8, 12, and 16 amps. respectively, would be sufficient to enable the curves for other power factors to be plotted.

Accordingly, a series of readings, each corresponding to a fixed value of the current, but each being taken with a different proportion of inductive and non-inductive resistance in the circuit, was taken.

The four series of readings were then plotted with voltage horizontal and power-factor vertical, as shown in Fig. 105. The actual values of the currents, for which these curves were drawn, were respectively 4·17, 8·28, 12·8, and 17·2, as these currents corresponded to exact readings on the ammeter of 4, 8, 12, and 16 amps., the exact value being subsequently obtained from the calibration curve of the instrument. From the curves obtained in this manner, the lower characteristics shown in Fig. 107 for power-factors of ·9, ·8, and ·6, were plotted, each curve in Fig. 105 giving a single point on each characteristic. The general similarity between the curves in Fig. 104, and those drawn for a similar purpose in Fig. 96, comparing power-factor and excitation, will be noticed.

This was to be expected, since the variation in exciting current to maintain a given voltage must bear a fairly constant ratio to the drop of voltage, with a constant exciting current. The curves are, consequently, very similar in their significance.

(b) Determination of Characteristic Curve on Capacity Loads.—The characteristic curves obtained in Fig. 107, which lie above the characteristic for non-inductive load, were obtained with currents *leading* instead of lagging behind the voltage. Such curves may be obtained by choosing as load a circuit possessing considerable capacity, such as a long concentric cable or a number of condensers. The curves shown were determined in a manner which is usually more convenient than this, viz., by connecting the alternator to a synchronous motor, the fields of which were excited so that the motor voltage was higher than the alternator voltage. It will be explained later in the section devoted to synchronous motors that over-excited synchronous motors produce a leading current in the circuit supplying them, and thus have the same effect as condensers or cables with capacity.

For the purposes of the present experiment it was much more convenient to employ motors, since the amount of lead of the current could be readily adjusted by regulating the motor excitation, and the amount of current taken from the alternator could be easily varied by loading the motor by means of a brake upon its pulley.

In order to obtain complete curves for several values of the power-factor, the same series of readings were taken as in the case of the inductive characteristics. It was comparatively easy to adjust the motor load and excitation to give a number of readings at different power factors at a fixed current, whereas it would not have been easy to take a series of readings at a constant power-factor, without employing special instruments, which were not available.

A number of readings were taken with different conditions of load and excitation at each of the following values of the current, 10·65, 8·6, and 6·4 amps. From the readings the curves comparing power-factor and voltage shown in Fig. 106 were plotted. The general shape of these curves is similar to that of those shown in Fig. 105 ; but in this case they cross each other. This is because, with a current leading only very slightly, a high current produces nearly the same effect as the current in a non-inductive circuit, *i.e.*, it produces a drop in voltage. The same current, when leading by a large angle, however, produces a considerable rise in voltage, so that the highest current produces the greatest drop at high power-factor and the greatest rise at low power-factor, and consequently cuts across the curve of a smaller current which is more nearly vertical. The curve corresponding to zero current would, of course, be a vertical line showing neither increase nor decrease in voltage—in the present case it would correspond to the ordinate of 100 volts.

It is interesting in comparing the curves in Figs. 105 and 106, to observe that the curves in Fig. 105 appear to show that a further decrease in power-factor below about ·6 does not affect the voltage, while in the case of the leading current curves in Fig. 106, there does not appear to be a limit to the value of the power-factor which continues to change the alternator voltage.

One feature which will be noticed about the characteristics for lagging or leading currents is that the distance between the curves for $\cos \phi = 1{\cdot}0$ and $\cos \phi = {\cdot}9$, is much greater than that between the curves for $\cos \phi = {\cdot}9$ and $\cos \phi = {\cdot}8$.

In fact the greater the angle of lag or lead the less is the proportionate effect of decrease in the power-factor. This is owing to the fact which has been stated already, that the demagnetising action of the armature current is proportional to sin ϕ. The rate of increase of sin ϕ is most rapid when ϕ is small and decreases as sin ϕ approaches its maximum value of 1. Thus, a given increase in the value of ϕ produces a much greater effect when the actual angle of lag is small. Further, the angle whose cosine is ·9, is 26° and the angle whose cosine is ·8 is 37°, so that by taking successive equal increments or decrements of power-factor or cos ϕ, we are taking successively *decreasing* values of ϕ. There is thus a double reason for the apparently rapidly diminishing effect of decrease in power-factor upon the slope

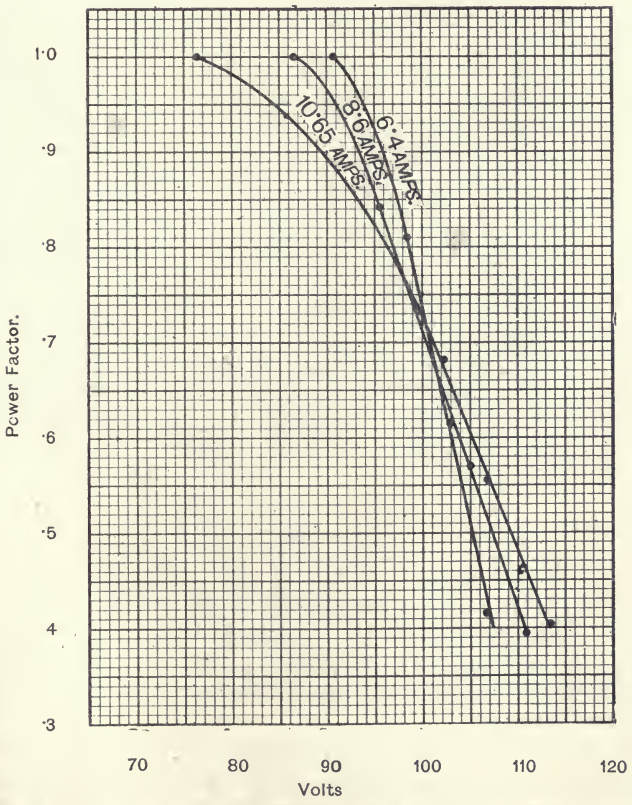


FIG. 106.—VARIATION OF VOLTAGE WITH POWER FACTOR. LEADING CURRENTS.

of the characteristic, viz., each successive change in power-factor corresponds to a smaller change of angle of phase difference and each successive change of angle corresponds to a diminished proportional armature re-action.

It will be seen from Fig. 107 that the characteristics for low power-factors appear as straight lines. They are not accurately so, but approximate to straight lines in the case of most machines. The characteristic on non-inductive load is considerably more curved than the other characteristics.

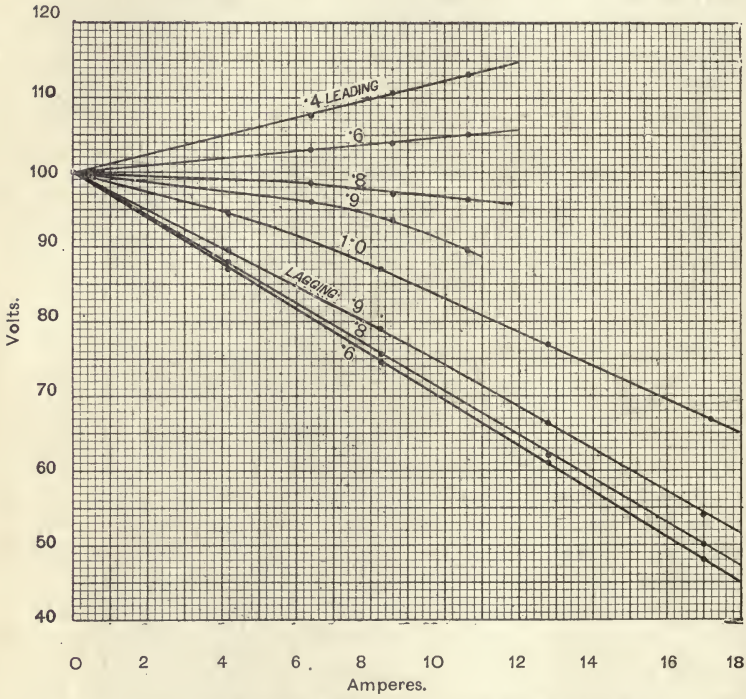


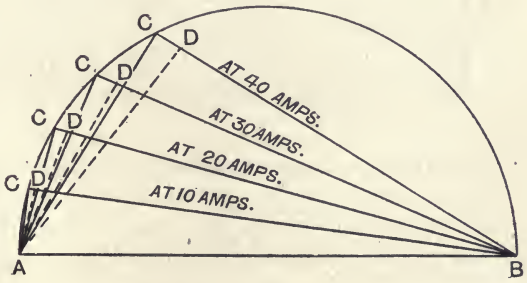
Fig. 107.—Characteristics with Various Power Factors.

The non-inductive characteristic can never be exactly a straight line, even when the armature core is unsaturated, and eddy current losses, &c., can be neglected. The reason for the curvature of the line under these circumstances may be easily explained.

Assuming the total volts generated to remain the same at all loads, they may be represented by the side $A\ B$ of the triangle shown in Fig. 95, re-drawn in Fig. 108. $A\ B$ is the

resultant of the voltage $A\ D$ spent in overcoming armature impedance, and the voltage $D\ B$ measured at the terminals of the machine. When the current is small, the line $A\ D$ is short, since the voltage overcoming armature impedance is small. Under these conditions the measured voltage $B\ D$ and the total voltage $A\ B$ are nearly equal, since $A\ D$ is nearly perpendicular to both of them. As the current increases the voltage $A\ B$ remains the same, but the voltage $D\ B$ decreases at first slowly, but afterwards more rapidly. The drop in armature impedance increases in proportion to the current, and consequently the sides of the triangle $A\ C\ D$ all increase in the same proportion. The angle $A\ C\ B$ is, however, always a right angle and consequently the point C will always lie on a semi-circle described on the line $A\ B$, and must move round this semi-circle towards B as the current (and consequently the length of $A\ C$) increases. It thus follows that the line $C\ B$ will continuously decrease as the current increases on account of the altered position of the point C. It will further decrease owing to the fact that the voltage represented by $C\ D$ (= armature ohmic drop) increases also in proportion to the current. It would then be easy to draw a characteristic curve on the assumption that the armature impedance, and total voltage generated, both remained constant.

Fig. 108 shows the form which the diagram in Fig. 95 would take when the current in the circuit has the values 10, 20 30, and 40 amps. In each case the length of $D\ B$ represents the terminal voltage. The external circuit in



C D = Armature and line drop.
D B = Terminal volts.
A B = Total volts generated.
A C = Reactance voltage.

FIG. 108.—CALCULATION OF EXTERNAL CHARACTERISTIC. NON-INDUCTIVE LOAD.

this case is assumed to be non-inductive, and the curve I. in Fig. 109 shows the values thus obtained plotted in the form of a characteristic. The resemblance to the curve obtained experimentally (with a different exciting current) and shown in Fig. 100, for the value $\cos\phi = 1{\cdot}0$ is apparent.

Fig. 110 shows a similar series of diagrams tot hose on Fig. 108 drawn for the same currents, but with an angle of lag of 37° in the external circuit, corresponding to a power-factor of ·8.

The lettering in diagrams 95, 108, and 110 has been made to correspond, so that the various quantities may be followed without difficulty.

Fig. 110 has been drawn under the same assumptions as

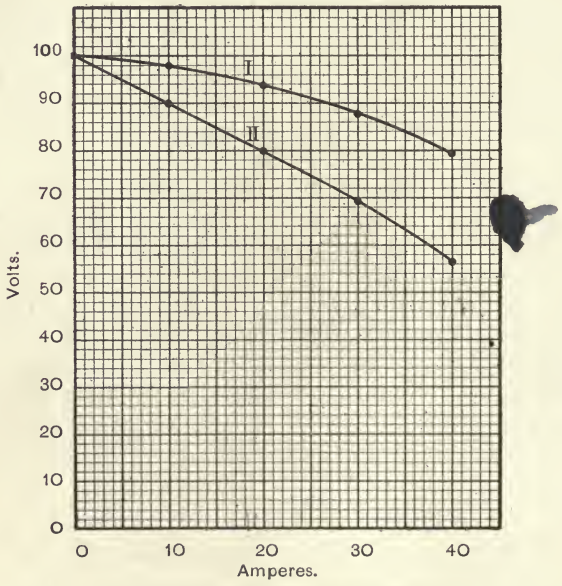


FIG. 109.—CALCULATED CHARACTERISTICS.

I.—Non-inductive load.
II.—Inductive load.
Power factor = ·8.
Excitation, 1·64 amperes.
Speed, 1,200 revolutions per minute.

the previous diagrams, viz., that the total voltage generated in the armature remains constant at all loads, and that the armature impedance also is constant. The first of these

assumptions has already been shown not to be accurate, since the lagging current decreases the main field and reduces this voltage. The curve of voltage obtained from the diagram will consequently be higher than its actual position. The curve II. in Fig. 109 is the curve obtained from the diagram and illustrates at once the fact that the characteristics obtained with inductive loads are nearly straight lines, owing to the voltage lost in armature impedance being much more nearly in phase with the terminal voltage, and thus producing almost a direct drop in voltage proportional to the load current of the alternator.

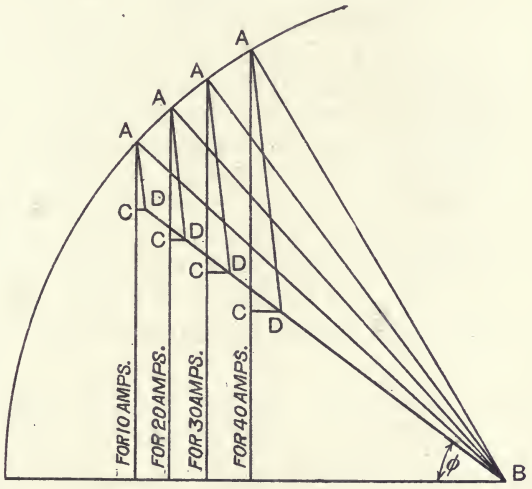


FIG. 110.—CALCULATION OF EXTERNAL CHARACTERISTIC INDUCTIVE LOAD.

Cos ϕ = ·8.

C D = Armature and line drop.
D B = Terminal volts.
A B = Total volts generated.
A C = Reactance voltage.

Phase Difference between Total Volts and Load Current with Non-inductive Load.—From the discussion of the two preceding experiments it is evident that, even when working on a non-inductive circuit, the phase difference between the current and the total voltage generated in the armature of the alternator will vary considerably with a varying load. This is perhaps brought out most clearly by the diagram Fig. 108, where the line *A B* represents the total armature

voltage and *D B* shows the phase of the current in the external circuit (since the external current and terminal voltage will be in phase with a non-inductive load circuit). Thus the angles *D B A* in Fig. 108 show the angle of phase difference between load current and total volts for various loads. The angles *B D A* show the angle of phase difference between current and voltage in the armature itself.

The reason for this varying phase difference may be repeated in the following general terms.

The voltage generated in the armature has to send current through a circuit composed partly of reactance (due to the armature self-induction) and partly of non-inductive resistance (in external circuit and conductors of the armature). The angle of lag of the current behind the total generated voltage will depend entirely on the relative magnitude of the reactance and resistance of the circuit. If the load current increases this must be due to a decrease of the non-inductive resistance in the external circuit. Since the armature self-induction remains nearly constant, this change must correspond to a smaller proportion of non-inductive resistance in the whole circuit and a greater angle of lag.

The general result of an increase of load must therefore be an increase in the angle of lag between current and total volts.*

From the characteristic curve of an alternator on load or from the short-circuit characteristic, the angle of lag between current and total volts can be approximately determined by the construction given on p. 205. This construction assumes, however, that the total voltage generated remains constant at all loads. This is by no means an accurate assumption, on account of the weakening of the field due to armature reactions.

A nearer approximation to the true voltage relations in the circuit and armature may be made by the following modification of the method already given.

Determine first the variation in excitation required to maintain a constant terminal voltage, when the alternator runs at normal speed, and supplies a non-inductive circuit.

* The student must note the distinction between "total volts generated in the armature" and "terminal volts." The terminal volts will, of course, always be in phase with the current in the case of a non-inductive circuit.

(See Experiment XXXVII.) Then connect the armature to a supply of alternating current, and pass a series of increasing currents through the stationary armature, at the same time exciting the magnets with the excitation previously determined to be that required to maintain the armature voltage constant when rotating and supplying each value of the current. In this way the excitation will always be so adjusted that its effect upon the armature impedance will be the same as when the alternator is working under load with constant terminal voltage.

After adjusting current and voltage as just described, take readings on a wattmeter of the watts supplied to the armature and of the current, and the volts applied to the armature terminals.

Calculate for each set of readings the value of cos ϕ by dividing the true watts by the apparent watts supplied to the armature. The value and also the phase of the armature drop is thus determined in relation to the terminal voltage for any load.

As an illustration, the following readings are given, taken on a small 4-pole ring-wound Crompton alternator having an output of 5 amperes at 100 volts.

TEST OF VOLTAGE DROP WITH STATIONARY ARMATURE.

Excitation to Maintain Constant Volts.	Armature Current.	Terminal Volts.	Watts.	Cos ϕ	ϕ
1·345	1	5·0	2·2	·438	64°
1·37	2	10·0	5·0	·5	60°
1·41	3	15·3	24·6	·537	57·5
1·465	4	21·5	47·5	·552	56·4
1·495	4·5	24·5	61	·554	56·3

From the values of ϕ given in this table the lower curve of ϕ in Fig. 111 has been plotted. It is seen that the angle of lag between current and volts in the armature varies between 56° and 65° for the range of loads taken.

From the readings thus taken the total voltage which would have to be generated in the armature when the machine works at a constant terminal voltage of 103 was calculated in the manner described below, and the results were plotted as shown in the top curve in Fig. 111 marked "Total volts."

To obtain the total volts generated, a horizontal line $B\ D$ (Fig. 112) was drawn to represent 103 terminals volts. At D the line $D\ A$ was drawn, making the angle $C\ D\ A$ equal to the angle of lag given in the table. Thus for the load of 3 amperes

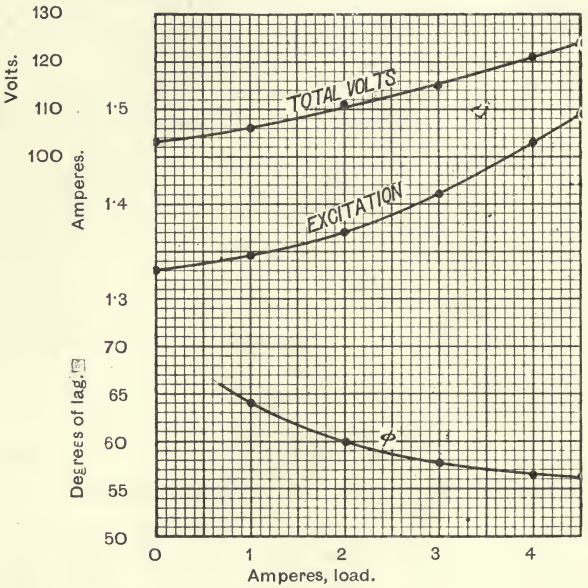


FIG. 111.—TOTAL VOLTAGE AND TOTAL ANGLE OF LAG IN ALTERNATOR ARMATURE.

the angle $C\ D\ A$ was made 57·5°. The length of $D\ A$ was made equal to the voltage spent in overcoming armature impedance at this load, in this case 15·3 volts. The line $B\ A$ then gives the total voltage which must be generated in order

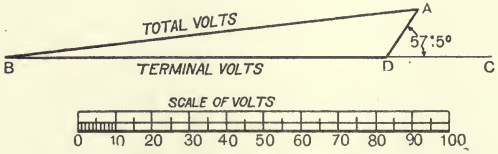


FIG. 112.—DIAGRAM OF VOLTS IN ALTERNATOR ARMATURE.

to overcome the armature impedance, and maintain the required terminal voltage. This being repeated for a number of values of the current gave the total voltage generated in the armature at each load.

The diagram of voltages constructed in this manner is a

more accurate representation of the relations which are indicated in Fig. 108. It is, however, not quite accurate, as the losses in the armature when stationary and when rotating are not identical. The method shown in Figs. 108 and 110 is the one usually adopted as sufficiently accurate in practice, although the more complete diagram just obtained is of considerable interest theoretically.

Experiment XXXVII.—Determination of Excitation Required to Maintain Constant Voltage.

Diagram of Connections.

As for Experiment XXXVI.

(See Fig. 93, page 201.)

Instructions. — Make connections as in Experiment XXXVI. Run the alternator at normal speed with the load circuit open, and adjust the excitation to give the full voltage. Close the main circuit switch and gradually increase the load to its full value. For each value of the load current adjust the field regulator so that the machine gives its normal voltage. Then read the load current and exciting current. The speed of the machine must be kept constant throughout the experiment.

If the machine is required to work on inductive loads, the readings should be repeated with inductive loads, having the minimum power-factor on which the machine will be required to work. The connections for determining the power-factor of the load may be made in any of the ways described in Experiments VIII., XIII., or XIV.

The results of the test should be entered in tabular form as indicated below, and a curve plotted with load current horizontal and exciting current measured vertically.

Determination of Excitation for Constant Voltage.

Alternator No..... Type.....

Normal output, ..amps., ..volts, at ..revs. per minute.

Load Current.	Voltage.	Exciting Current.	Speed.
0	95	1·33	1200
15	95	1·68	1200
25·7	95	2·05	1200
35·0	95	2·445	1200

The readings obtained in the case of a 6 kw. 3-phase generator with rotating field when loaded on one phase only are plotted in Fig. 113. The voltage was kept constant at 95, and the speed regulated to give a periodicity of 40 ~. The machine employed was the same as that for which the magnetisation curves, Figs. 92 and 94, and the characteristic, Fig. 100, were drawn. On referring to the magnetisation curves, it will be seen that the lowest excitation employed in the present experiment, viz., 1·33 amps., is well above the knee of the magnetisation curve, and that the ratio of increase in voltage to increase of excitation is fairly constant. Consequently, if the characteristic were straight, indicating a uniform drop of voltage as the load increased, the curve in Fig. 113 would also be practically a straight line. The characteristic in Fig. 100 begins, however, to bend downwards at about 20 amps., and consequently with currents above this value the excitation must increase more rapidly in order to keep up the voltage to its original value. This is indicated by the upward bend in the excitation curve, Fig. 100, occurring at 20 amps.

A curve of the kind just described shows the range which a field-regulating resistance must have in order to maintain constant voltage, and also shows what variation of resistance will be required for any given variation of load which may be contemplated.

In America alternators have frequently been arranged with a compound winding in the same way as direct-current dynamos In this case usually a portion of the current is rectified and passed through a series winding which adds its magnetising effects to those due to the ordinary separately-excited field windings. In such cases, the curve under consideration would give the required information for determining the number of series windings necessary to maintain an approximately uniform voltage. For such a purpose the number of amperes excitation obtained from the curve must be multiplied by the number of turns through which it flows in order to give the total number of ampere-turns required. The difference between this total number and the number supplied by the separately excited winding gives at once the ampere-turns to be provided by the series winding.

In making use of the curves such as that shown in Fig. 113, it must be remembered that a different curve showing a

greater increase in excitation would be obtained on an inductive load circuit, since the armature drop in this case would be greater.

Since the curve of variation of excitation may be approximately obtained from the results of the determination of the characteristic and magnetisation curves,

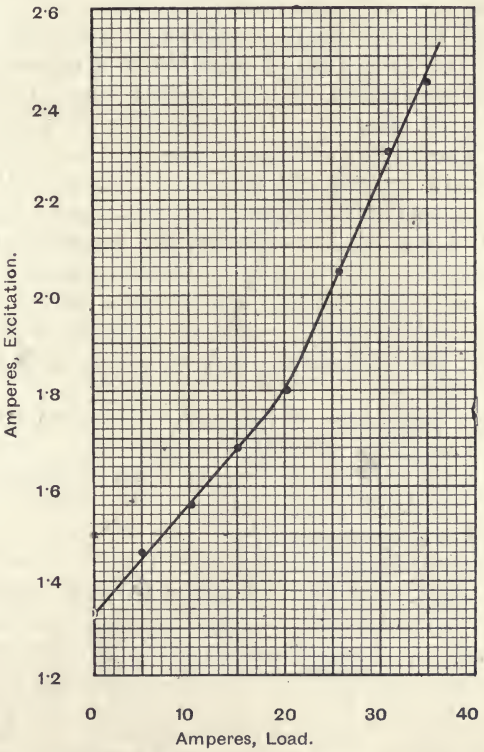


FIG. 113.—VARIATION OF EXCITATION TO MAINTAIN CONSTANT VOLTAGE.

it will in most cases be better to make use of these rather than to obtain a separate curve for inductive load by direct experiment.

The method of deriving the curve from those already taken would be approximately as follows, the method of procedure depending somewhat upon the information required, and the curves already obtained.

Suppose that the loss of voltage at a certain load, either inductive or non-inductive has been obtained by experiment

or calculation. On referring to the magnetisation curve, Fig. 92, p. 198, the point on the curve corresponding to the excitation under the assumed conditions, must be found. A line is then drawn vertically upwards through this point, its length being equal to the loss of voltage on the volt scale. A horizontal line drawn through the upper end of this vertical line to meet the magnetisation curve will represent by its length the increase in excitation required.

Strictly, the load magnetisation curve, Fig. 94, should be employed, but as this is not often taken experimentally and is practically parallel to the no-load curve, the latter is generally employed.

Short Circuit Magnetisation Curve.—If an alternator is only partially excited and its terminals are short-circuited through an ammeter, the whole of the electromotive force generated in the armature is spent in overcoming the armature impedance. By making measurements of the current under these conditions for various values of the excitation, useful information may be obtained. There are four factors which tend to diminish the current which the armature is able to produce :—

(*a*) The weakening of the main field produced by the armature magnetic reaction.

(*b*) The counter electromotive force produced by the armature self-induction.

(*c*) The armature resistance.

(*d*) Production of eddy currents.

The weakening of the main field is due to the setting up of lines of force by the armature current, and entails the loss of a certain electromotive force exactly in the same way as the self-induction of the circuit requires the expenditure of a certain amount of voltage in setting up a magnetic field.

The first two factors may consequently be treated together as being similar in their effects to a back electromotive force or idle voltage 90° out of phase with the current.

The last two sources of loss may similarly be taken as giving rise to a single energy electromotive force in phase with the current. We may, consequently, represent the conditions in the armature by a triangle of electromotive force, as in Fig. 116, p. 245, the values of the voltages being determined in the experiment to be described.

EXPERIMENT XXXVIII.—DETERMINATION OF SHORT-CIRCUIT MAGNETISATION CURVE, OR SHORT-CIRCUIT CHARACTERISTIC.

DIAGRAM OF CONNECTIONS.

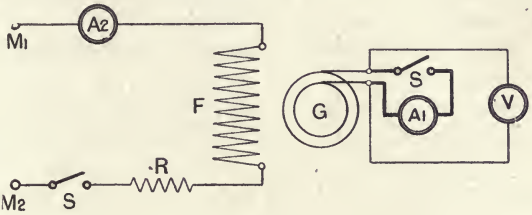


FIG. 114.

M_1, M_2	Source of direct current.
G	Alternator armature.
F	Alternator field windings.
R	Field regulating resistance.
A_1, A_2	Ammeters.
V	Voltmeter.
S, S	Switches.

Instructions.—Excite the alternator from a source of direct current through a regulating resistance, switch and ammeter. Connect the armature terminals together through an ammeter and switch. Connect a voltmeter to the alternator terminals. Run the alternator at normal speed, close the switch in the armature circuit, and excite the field at first with a small current. Read the ammeters, and then open the switch, and after adjusting the speed to its previous value, read the voltmeter. Maintaining the speed fairly constant,* gradually increase the excitation and observe the readings of the ammeters with the main circuit switch closed, and the voltmeter after opening the switch at each value. Take a similar set of readings for decreasing values of the excitation.

In the case of any but small machines, it will be inadvisable to suddenly open or close the switch, and a resistance which

* From the results of the next Experiment, shown in Fig. 120, it will be seen that when running at approximately normal speed, the short-circuit current remains constant for considerable speed variations. The importance of exact speed regulation does not therefore exist in this experiment, except that, of course, the corresponding ammeter and voltmeter readings at a given excitation must be taken at identical speeds.

can be varied and ultimately short-circuited should be put in series with the switch.

If a magnetisation curve at no load for the same speed has been previously taken, it will be unnecessary to take voltmeter readings in this experiment, since these readings form a no-load magnetisation curve.

In such a case, the voltage corresponding to each value of the excitation should be taken from the previously plotted magnetisation curve and entered in the column "Volts on Open Circuit" after correction for difference in speed, if necessary.

Results should be entered in tabular form as indicated below.

DETERMINATION OF SHORT-CIRCUIT CHARACTERISTIC.

Alternator No..... Type.....
Normal output ....amps. ....volts at ....revs. per min.
Frequency..... Normal excitation ....amps.

Exciting Current.	Speed.	Amperes on Short Circuit.	Volts on Open Circuit.
·12	1200	8·6	17
·23	„	15·45	32
·43	„	27·5	55
·55	„	35·0	67

The readings should be varied so that the maximum armature current is somewhat greater than the normal full-load current.

A curve comparing exciting current and short-circuit armature current should be plotted from the readings. For moderate values of the excitation this will be found to be practically a straight line, but bends upwards rather sharply if continued for higher excitations.

Fig. 115 shows the results of a test carried out in the manner just described on a 6-kw. 4-pole Schuckert alternator with rotating field, the speed being 1,200 revs. per minute, and the normal maximum current 35 amps. The curve is a straight line cutting the vertical axis at a point corresponding to 1·3 amps. This shows that the residual magnetism is sufficient to send a current of 1·3 amps. through the short-circuited armature.

The chief interest of the preceding experiment lies in the conclusions which may be drawn from it regarding the voltage drop of the alternator at various loads, and with various power factors. The usefulness of this form of measurement is increased by the fact that the machine requires only a small amount of power to drive it, although the currents taken from the armature may be large. The

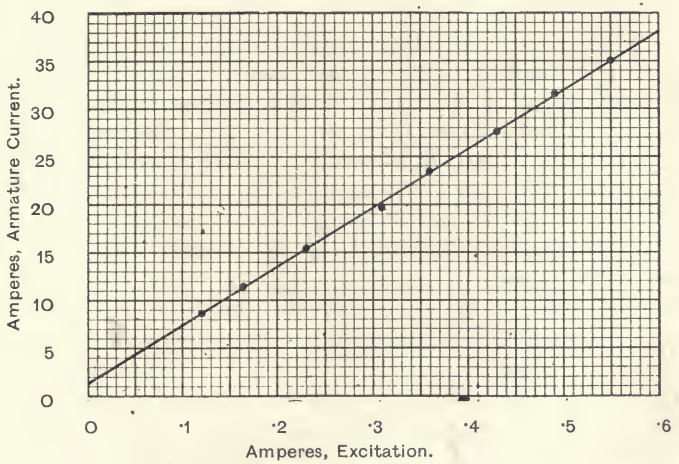


FIG. 115.—SHORT CIRCUIT CHARACTERISTIC.

experiment may thus be applied to a large machine without great expenditure of power.

As already pointed out, the voltage measured at the terminals of the alternator with the circuit open represents the voltage generated in the armature at the particular speed and excitation employed. On closing the switch in the armature circuit the voltage generated falls, owing to the weakening of the main field by the armature magnetic reactions. For the purposes of calculation it is simpler to assume that the total voltage generated remains the same on short circuit as on open circuit, but that the portion which is actually lost due to magnetic reaction disappears as part of the idle voltage lost in the armature. Making this assumption, therefore, we consider the whole of the no-load voltage as being spent in sending current through the armature and ammeter, and overcoming the self-induction and resistance of the circuit composed of them.

The resistance of the armature, ammeter, and connecting leads should be determined by measurement with direct current or other means. To the resistance thus measured should be added a percentage depending on the type and speed of the machine, in order to allow for the power spent in producing eddy currents in the pole faces and armature core. It will be shown later that errors in computing the eddy current losses have a very small effect upon the calculated value of the armature drop.

Since it is not possible to measure directly the amount of the losses due to eddy currents, they can only be estimated from the results of experience in machines of similar type, unless a special experiment can be made to determine them.*

It is thus possible to construct a right-angle triangle of electromotive force for the alternator on short circuit. The hypotenuse of the triangle will be the total electromotive force generated, *i.e.*, the open circuit voltage. The two other sides will represent respectively (1) the idle voltage, due to self-induction of the armature and loss of voltage by armature reaction, and (2) the energy voltage spent in overcoming armature resistance and eddy current losses. The voltage (1) will be $\frac{1}{4}$ period in advance of the current in phase, while the energy voltage (2) will be coincident in phase with the current. The angle between the energy voltage and total voltage will give the angle of lag for the circuit.

An example taken from the measurements plotted in Fig. 115 will illustrate what has just been stated.

The no-load magnetisation curve given in Fig. 92 was taken from the same machine while running at the same speed; consequently the voltages on Fig. 92 give the total voltages producing the currents plotted on Fig. 115 for the same excitation. Thus with ·4 amp. excitation, we see from Fig. 92 that the total voltage generated is 52·3. The armature resistance (warm, and including the short-circuiting leads) was ·17 ohm. Hence the energy voltage overcoming resistance was $25{\cdot}8 \times {\cdot}17 = 5{\cdot}79$ volts, the short-circuit current corresponding to this excitation being seen from Fig. 115 to be 25·8 amps. In the present case the eddy currents are

* In small machines with low linear velocity the eddy currents are not likely to affect the behaviour of the machine seriously. They may therefore be neglected in comparison with the loss in armature resistance, or taken into account by adding a percentage to the losses in the resistance. An approximate rule sometimes used for larger machines is to assume the eddy current losses to be equal to the armature resistance loss.

assumed to be too small to seriously affect the result on account of the low peripheral speed of the machine.

The idle voltage of the machine

$$= \sqrt{(\text{total voltage})^2 - (\text{energy voltage})^2}.$$

In this case idle volts $= \sqrt{52{\cdot}3^2 - 5{\cdot}79^2} = 52{\cdot}0$ volts.

The triangle of voltages for the figures just given is shown in Fig. 116, the line $a\ b$ showing the phase of the current in the circuit.

The total voltage shown by the line $b\ c$ in Fig. 116 is the voltage necessary to overcome the self-induction and resistance of the armature and leads when the alternator is giving a current of 25·8 amps. In a similar way the two curves given in Figs. 92 and 115 will give the voltage taken up in the armature for any value of the current shown in Fig. 115. It would not, however, be permissible to simply subtract this voltage from the total no-load voltage of the

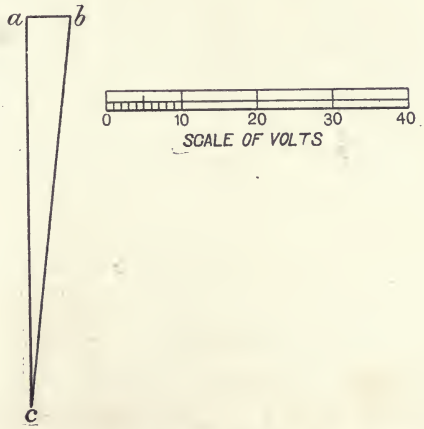


FIG. 116.—DIAGRAM OF ELECTROMOTIVE FORCE, EXPERIMENT XXXVIII.

alternator at normal excitation in order to get the terminal voltage available for sending current through the external circuit, for the reason that the voltage spent in overcoming armature impedance and the voltage applied to the external circuit are not generally in phase with each other. It depends upon the difference in phase between these two voltages, to what extent the volts spent in the armature affect the external voltage.

Thus, in Fig. 116 it is seen that the voltage $b\ c$ is nearly $\frac{1}{4}$ period in advance of the current in phase. If the external circuit is non-inductive, the terminal voltage of the alternator will be in phase with the current and nearly $\frac{1}{4}$ period behind the voltage $b\ c$. In this case the armature impedance will only diminish the terminal voltage to a slight extent. If the external circuit is so highly inductive that the terminal voltage makes the same angle with the current as the armature loss voltage $b\ c$, the full value of this voltage will be subtracted from the total voltage generated to produce the terminal voltage. This action has already been discussed at some length on page 233.

The following is a simple approximate construction for determining the loss of voltage for any value of the power

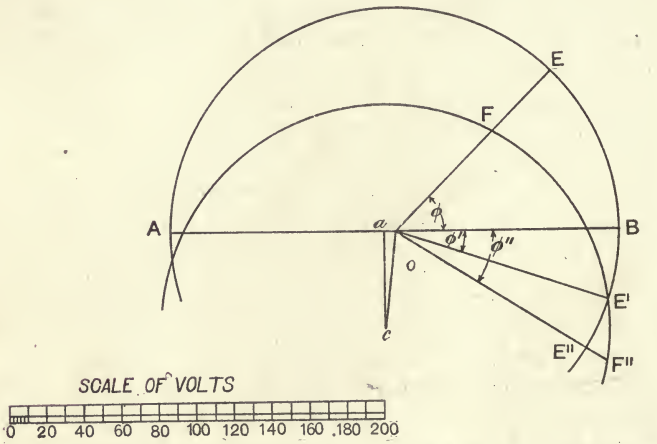


FIG. 117.—CONSTRUCTION FOR FINDING LOSS OF VOLTAGE AT ANY POWER-FACTOR.

factor of the circuit from the results of the experiment just given. The principles upon which it depends are exactly similar to those given for a similar purpose in connection with a transformer, p. 162 and seq.

Let it be desired to determine the voltage drop of an alternator when working with its full load on a circuit of varying power-factor.

Draw a semi-circle $A\ E\ B$ with centre O and radius equal to the open circuit voltage of the machine. (See Fig. 117.)

Construct the triangle $a\ b\ c$ obtained from the experiment in the manner just described, choosing the values correspond-

ing to the full-load current for which the construction is to be carried out. Draw the triangle $a\ b\ c$ so that the point b coincides with O, the centre of the circle. $c\ a$ drawn vertically is the short-circuit idle voltage, and $O\ c$ is the total voltage with the same excitation on open circuit; $O\ a$ is the armature ohmic drop and eddy current loss.

With c as centre, describe a circle with a radius equal to $O\ E$. Then for any value of the angle of lag in the circuit $\phi = B\ O\ E$ the terminal voltage of the machine will be given by $O\ F$ and the voltage drop by $F\ E$.

When ϕ is so great that the line $O\ E$ coincides with $O\ c$ produced, the maximum drop of voltage will occur. When ϕ is negative, *i.e.*, when there is a leading current in the circuit, the terminal voltage may be greater than on open circuit. Thus, when the current leads by the angle ϕ' in Fig. 117, the terminal voltage is just equal to the voltage on open circuit. When the lead is still greater, as shown at ϕ'', the voltage is greater than at open circuit by the amount represented by $E''\ F''$.

It must be remembered that the triangle $O\ a\ c$ is drawn in Fig. 117 for one particular value of the current, and that the diagram gives the loss or increase of voltage for this load. In order to determine the voltage corresponding to any other load, its approximate value may be obtained by altering the scale of the triangle $O\ a\ c$ in the same proportion as the load.

Relation Between Speed and Voltage of an Alternator.—When supplied with constant excitation and giving no armature current, the voltage of an alternator varies in direct proportion to the speed at which it is driven. This is evident from the formula already given for the voltage of an alternator (see page 194), in which it is seen that the voltage is directly proportional to the speed. A curve comparing terminal voltage and speed of an alternator on no load will consequently be a straight line passing through zero, since the voltage is zero when the speed is zero. A single observation at any speed would consequently be sufficient to enable the curve to be drawn, and the voltage at any other speed can be at once deduced.

Two other cases, in which the relation between speed and volts is not quite so simple, are considered in the following experiment, viz.: the case where the alternator supplies a constant current, and the case of a short-circuited alternator.

EXPERIMENT XXXIX.—DETERMINATION OF RELATION BETWEEN SPEED AND VOLTAGE OF AN ALTERNATOR.

(1) At no load,
(2) At full load,
(3) On short-circuit.

DIAGRAM OF CONNECTIONS.

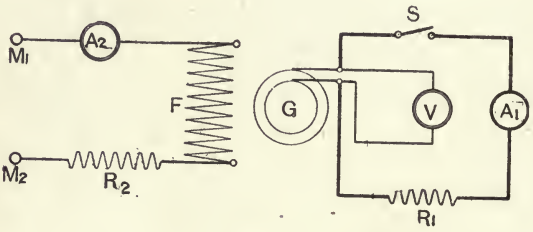


FIG. 118.

M_1, M_2 Source of continuous current.
G Alternator armature.
F Alternator field windings.
R_1 Non-inductive resistance in load circuit.
R_2 Resistance for regulating exciting current.
A_1 Ammeter for measuring load current.
A_2 Ammeter for measuring excitation.
V Voltmeter for measuring armature voltage.
S Switch for breaking main circuit.

Instructions.—Connect the field windings to a source of direct current through a regulating resistance and ammeter.

Connect the armature terminals to an ammeter, variable load resistance, and switch in series. Connect a voltmeter to the armature.

Maintain the excitation constant throughout each set of readings.

(1) Take a series of observations of speed and voltage with the switch in the main circuit open, beginning with the lowest speed for which the voltmeter will give readings and increasing up to full speed or slightly higher.

(2) Take a similar series of readings with the switch in the load circuit closed, and the resistance of this circuit so regulated that the current is the same (as nearly as possible) for each reading.

(3) Repeat the readings at similar speeds with the armature terminals short-circuited through the ammeter and

switch. In this case the current will vary, and should be recorded. Judgment must be exercised in the experiment as regards overloading the alternator; machines vary with regard to the magnitude of their current on short circuit. It is usually three or four times the normal full-load current. Probably it will be necessary to employ a lower value of the excitation in this case in order to prevent the current becoming excessive at full speed.

The results should be entered as below. Curves for the readings (1) and (2) should be plotted on the same sheet of squared paper, speed horizontally and voltage vertically. The curve for readings (3) should be plotted separately, if the excitation employed varies from that adopted for the first two sets of readings, and in this case the vertical scale is made to read amperes.

SPEED CURVES.

Alternator No..... Type.....
Normal output ....amps. ....volts, at ....revs. per min.
Normal excitation .....frequency.....

Speed. Revolutions per Minute.	Terminal Volts.	Excitation.	Armature Current.

The curve (1) comparing speed and voltage at no load should be a straight line, as already explained.

If the loss of voltage in the armature were the same for a given current independently of speed, the curve (2), in which the armature carries a constant current, would be a straight line making the same angle with the horizontal axis and lying at a constant distance above curve (1).

In the case of the curves given in Fig. 119, this is seen not to be the case. The variations in the armature drop caused by a change of speed are mainly due to two causes: (*a*) change in armature reactance ($=2\pi n L$) at different speeds; (*b*) eddy currents induced in the pole faces, and to a less extent in the armature core. These eddy currents themselves set up lines of force opposing the field producing them.

In consequence of these influences the lines shown in Fig. 119 diverge slightly as they rise, showing that the voltage of the alternator when loaded does not increase with speed at quite the same rate as when unloaded. The lower curve, like the upper one, closely approximates to a straight line. This may be explained by the fact that the chief cause which produces the difference between the no-load voltage and the voltage under load is due to the voltage spent in overcoming

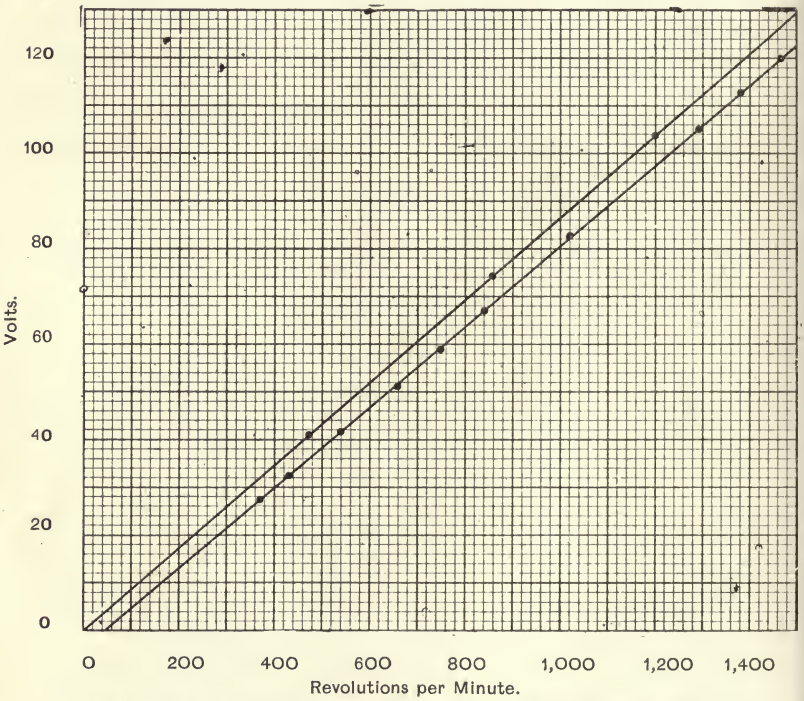


FIG. 119.—RELATION BETWEEN SPEED AND VOLTAGE OF AN ALTERNATOR
Upper curve no load.
Lower curve load = 20 amperes.
Excitation = 1·95 amperes.

armature impedance. The armature current remains the same, but the impedance ($= \sqrt{R^2 + (2\pi n L)^2}$) varies with the periodicity of the current. The value of R, the armature resistance, is small compared with the reactance, $2\pi n L$, and hence the impedance varies almost in direct proportion with the value of the periodicity n, and thus the loss of voltage due to this cause increases also in direct proportion

with the speed. The loss of voltage due to eddy currents is probably too small at moderate speeds to show much effect upon the curve, and this would increase in approximately direct proportion to the speed. The increase in the voltage drop thus varies directly as the speed, and the speed-volt curve for a constant load remains a straight line.

The curve shown in Fig. 120 shows the result of the test upon the same alternator when short-circuited. The excitation in this case was ·55 amp. This curve shows the

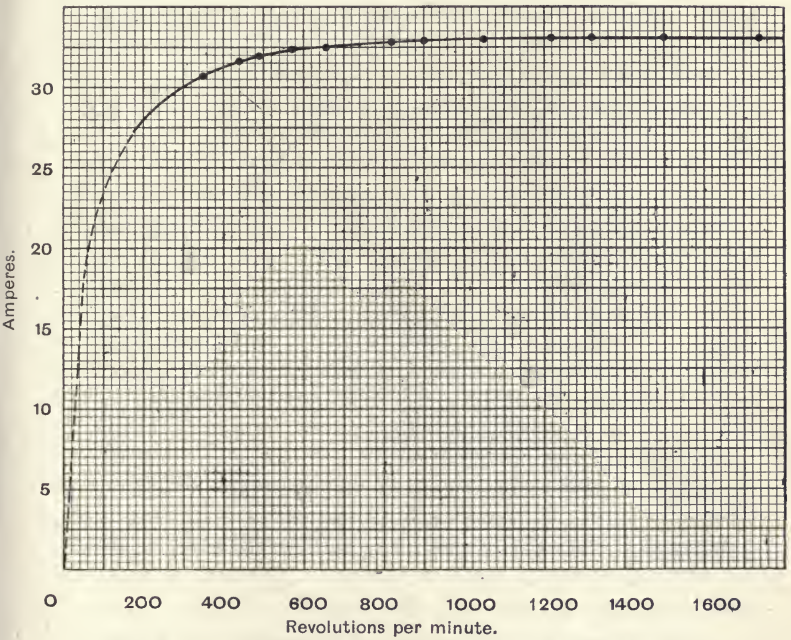


FIG. 120.—VARIATION OF SHORT CIRCUIT CURRENT WITH SPEEDS.
Excitation ·55 ampere.

interesting fact that after normal speed has been reached, the short-circuit current is practically independent of speed. This illustrates in another way the statement just made, that the armature impedance increases in almost direct proportion with the speed. The voltage generated also increases directly as the speed, and consequently the current remains constant, since it is the quotient of the volts divided by the impedance.

The curve rises very steeply from zero, since the impedance at low speeds is small. The dotted portion could not be obtained experimentally, as it was not possible to drive the machine steadily at such low speeds. The approximate position of this part of the curve was obtained by calculation from the value of the armature impedance previously calculated, and the calculated voltage at low speeds derived from the measured values at normal speed.

Efficiency Tests of Alternators.—The direct method of determining the efficiency of an alternator, by measuring the power supplied to drive it and the power given out in the form of electrical horse-power, is not generally adopted on account of the difficulty of measuring accurately the mechanical power supplied. The nearest approach to this method of measurement which is frequently adopted in testing small machines, is to drive the alternator by means of a motor whose efficiency is known at various loads. By measuring the power supplied to the motor, and multiplying this by the motor efficiency, the power transmitted to the alternator is obtained.

In the case of large alternators the power necessary to drive the machine at full load is costly, and the absorption of the load current becomes a difficult matter. Also the measurement of the output with a sufficient degree of accuracy to make the value obtained for the efficiency reliable is by no means an easy matter.

It is consequently usual to calculate the efficiency of alternators from measurements of the losses occurring in the various portions of the machine, and to divide the output by the calculated quantity (output + losses) at various loads to arrive at a curve of efficiency.

Losses in an Alternator.—The chief losses in an alternator may be summarised under four headings: (*a*) Armature resistance loss, (*b*) iron losses (hysteresis and eddy currents), (*c*) excitation losses, (*d*) friction losses (air and bearings friction).

The losses (*a*) can be calculated for any load by multiplying the armature resistance by the square of the current at that load, care being taken in the case of polyphase alternators to take the value of the current actually flowing in the windings, which is not always the same as that in the external circuit (*e.g.*, in a delta-connected 3-phase winding).

The losses (*b*) should be determined by a special experiment. Some of the most-used methods are given below.

An approximate calculation of the iron losses can be made in the same way as that described in the case of transformers, from the volume and induction of the iron subjected to the alternating flux.

The losses (*c*) can be directly calculated from the resistance of the field windings and voltage of the excitation.

The losses (*d*) can usually be determined at the same time as the iron losses.

Determination of Losses.—For the copper losses (*a* and *c*) the only determination to be made is that of the resistance of the windings, which is usually performed by sending a measured direct current through them and measuring the fall of potential in the windings. In each case watts lost = (current)^2 × resistance.

In order to determine the frictional and iron losses, the simplest method is to run the alternator as a synchronous motor. After running the machine up to normal speed and switching on the supply, the field is adjusted until the current taken is at its least value, and the power-factor of the driving circuit is consequently high. The power taken by the armature under these conditions is read on a wattmeter, and is the sum of the losses due to friction, hysteresis, and eddy currents. The copper losses in the armature due to the driving current will probably be too small in most cases to be worth subtracting. Allowance can easily be made for them from the known resistance of the armature and the value of the current, which must in any case be observed in order to ascertain whether the excitation is correct. Assuming (which is nearly though not accurately true) that the friction and iron losses remain the same at all loads, the total loss at any load is obtained for any load current by adding together the watts thus measured and the calculated copper losses in the armature and field.

Another method which is easily carried out, if a direct-current motor of known efficiency is available, is the following:

The motor is coupled to the alternator, and made to drive it at the full speed of the alternator. If the alternator fields are unexcited, there will be practically no iron losses, and the whole of the power supplied by the motor will be due to mechanical friction. If the normal excitation of the

magnets is applied, the increase of power given to the motor in order to maintain the same speed will be equal to the power taken up by the iron losses of the alternator.

If the alternator is provided with a direct-coupled exciter, it is convenient to employ this exciter as the motor in the test, after having made a careful determination of its efficiency. The quantities to be measured are small and liable to variation; great precautions must therefore be taken to run the alternator as nearly as possible under working conditions as regards temperature, &c., and only to take readings after the machine has been rotating for some time, so that the bearings may be thoroughly lubricated. The first readings should be repeated several times between the other readings, so as to insure that the conditions have remained constant.

This test is often carried out in the following way:—

The motor is made to drive the alternator unexcited. The power supplied to the motor W_1 is noted. The alternator fields are then excited, and the power taken by the motor to maintain the same speed is noted ($=W_2$). The motor is disconnected from the alternator, and the power required to drive it when running light is noted ($=w$). Then the friction losses of the alternator are taken to be $(W_1 - w)$, and the iron losses $(W_2 - W_1)$. The excitation of the motor should be maintained practically constant during the three measurements. It is assumed that the losses in the motor are the same at no-load and when driving the alternator. The motor should be large enough to make the alternator only a light load for it, in order that these assumptions may be justified.

The Retardation Method of Measuring Iron Losses.—This is a method which is especially applicable to alternators of the flywheel type. The machine is brought up to speed by means of a motor and belt, or by its own exciter. The belt is then slipped off, or current shut off from the exciter, as the case may be, and observations are taken of the rate at which the machine slows down, by reading a tachometer at equal intervals during the retardation. Curves plotted from these readings give the information required to enable the iron losses to be determined. If the machine is not of the flywheel type, it can sometimes be conveniently tested by the same method after mounting on the shaft a flywheel of sufficient weight to render the time of retardation sufficiently

long for readings to be taken. A curve of retardation (plotted with time horizontally and the tachometer readings observed vertically) is first obtained from the machine when allowed to slow down from full speed to rest with the fields unexcited, *i.e.*, under the action of air and bearing friction only. A second curve is then taken in a similar manner, but with the fields excited. The more rapid retardation which will occur in this case is due to the effect of hysteresis and eddy currents.

The principle of this measurement is as follows :—

The energy of rotation possessed by a rotating body is

$$W = \tfrac{1}{2} I \left(\frac{2 \pi n}{60}\right)^2$$

where I is the moment of inertia of the body,

n is its speed in revolutions per minute.

If the moment of inertia is measured in units of the absolute C.G.S. system, *i.e.*, in gramme-cm.^{2}, the energy is expressed in the same system as dyne-centimeters or ergs. In order to bring the ergs to kg. metres we must divide by $9 \cdot 81 \times 10^7$, or if to watts-seconds by 10^7, or if to foot-pounds by $1 \cdot 356 \times 10^7$. Thus the energy of a rotating body in watt-seconds or joules is

$$W = \tfrac{1}{2} I \left(\frac{2 \pi n}{60}\right)^2 \times 10^{-7} = \cdot 545 I n^2 10^{-9}$$

If the speed of the alternator when slowing down under the influence of the frictional and other losses is observed at successive intervals of time, the difference in the energy possessed by the machine at two periods is a measure of the work done in overcoming the losses. The amount of this work divided by the time between the observations is the mean power thus absorbed. The power absorbed will, of course, grow less as the speed diminishes. By plotting a curve of mean power for a number of speeds obtained as the machine gradually slows down, the losses at full speed may be obtained by continuing backwards the curve of losses to cut the ordinate corresponding to full speed.

Thus, for example, readings were taken every half-minute in obtaining the upper curve in Fig. 121 ; the reading at full speed was 172 revs. per minute. The next reading after half-minute was 164. The stored energy at full speed was $\cdot 545 I \ 172^2 \times 10^{-9}$ watt-seconds. The energy lost during the first half-minute was $\cdot 545 \ I . (172^2 - 164^2) \times 10^{-9}$. The

average power absorbed by the losses at a speed whose average value was 168 revs. per minute, was therefore

$$\frac{\cdot 545\ I\ (172^2 - 164^2)}{30 \times 10^9} \text{ watts.}$$

From a series of such values a curve was plotted with watts vertical and speeds horizontal, showing the watts lost at various speeds, the watts lost at full speed being then

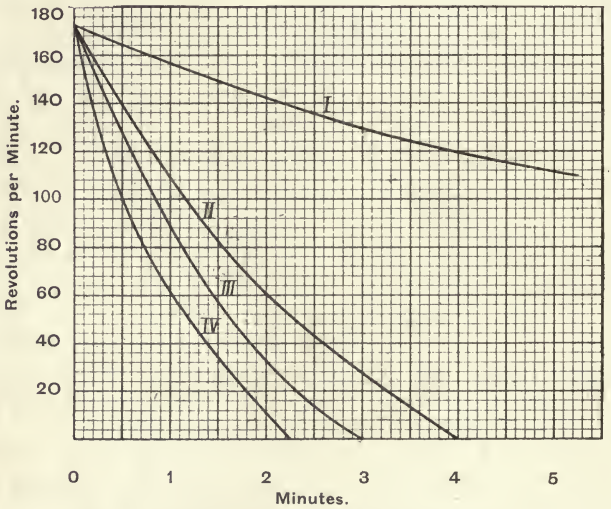


FIG. 121.—RETARDATION CURVES OF AN ALTERNATOR.

I. - Unexcited.
II.—Excitation 27 amperes.
III.— „ 34 „
IV.— „ 45 „

taken from the point where the curve would cut the ordinate corresponding to full speed. It is to be noted that such a set of readings gives the watts in terms of I, the moment of inertia of the rotating parts. In order to eliminate this unknown factor a separate observation is made to determine the total friction and iron losses at full speed. This is usually done in the manner already alluded to, by running the alternator as a synchronous motor and measuring the power absorbed by means of a wattmeter. This observation then gives the value of the total losses at full speed, and enables the actual value of all the other proportional readings to be assigned. The losses due to iron losses and to friction can thus be obtained at any speed. By taking readings with

several values of the excitation, the iron losses at varying induction are obtained.

Two Similar Alternators Connected in Parallel.—The Hopkinson method of testing which is so much used in the case of direct-current machines is also applicable to alternators. The alternators are connected in parallel, but are not mechanically coupled, and supplied with current of suitable voltage and frequency to drive them both as synchronous motors. If the machines are similar and equally excited, they will take equal small currents, and run in phase with each other. The power supplied, when measured on a wattmeter, will give the no-load losses of both machines.

By changing the excitation of one machine, it may be made to lag behind or lead the other in phase, and in doing so it will supply current to the other machine or receive current from it. By suitable adjustment of the excitation it is thus possible to fully load the machines, the power given out by one being spent in driving the other. For any value of the load the power supplied from the mains is that spent in the losses occurring in both. The efficiency of each is got by taking the square root of the joint efficiency of both.

Running Alternators in Parallel.—Alternators can be run in parallel if brought to the same speed and voltage, and if switched together on to a single supply circuit when exactly coincident in phase. This is owing to the fact that if one machine tends to lag behind the other, it will receive current from the other which tends to drive it as a motor, and it is thus prevented from falling further out of step. If the machines are not driven with exactly the same regularity, they will vary slightly in phase, and in doing so the leading machine will always supply current to the lagging one, which will have the double effect of slightly retarding the leading machine and accelerating the lagging machine. The difference in phase between the two machines must never exceed $\frac{1}{4}$ period, or they will fall out of step, since in that case the direction of the current supplied by the leading machine will be reversed, and tend to still further retard the lagging alternator.

The condition for parallel running is, therefore, that with a small angular displacement between the machines the

current sent by the leading machine must be sufficient to prevent the displacement becoming greater.

Synchroniser.—In order to ascertain when two machines are running at the same speed and are in phase, so that they may be switched into parallel, a synchroniser is employed. The principle of its action is now to be discussed.

Fig. 122 represents two alternators G_1 and G_2 connected in parallel to an external circuit A B, of which A represents one main conductor and B the other.

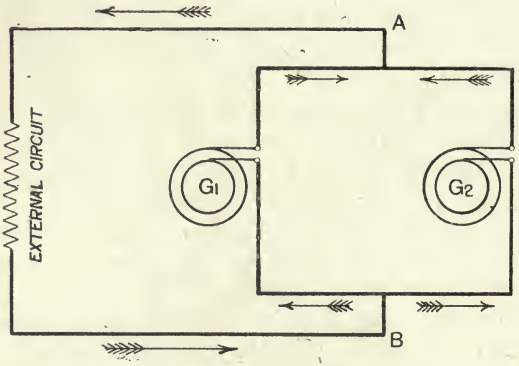


FIG. 122.—TWO ALTERNATORS COUPLED IN PARALLEL.

Obviously, in order that the machines may both supply current to the circuit A B, the brushes, which are simultaneously of the same sign, must be joined to the same conductor. This is shown in the diagram, where, at the instant represented, the upper brushes are both +, and are connected to A.

The conductors joining the alternators form a second closed circuit distinct from the main external circuit. In this circuit the two machines generate electromotive forces tending to send currents in opposite directions round this smaller circuit. At the instant represented on the diagram the directions of these electromotive forces will be those represented by the arrows. Consequently, when the machines are in correct phase for working in parallel, there will be equal and opposite electromotive forces acting in this local circuit, having a resultant electromotive force equal to zero. If the machines were wrongly connected together (*i.e.*, + to − instead of + to + and − to −) the resultant electromotive

force given to the external circuit A B would be zero, while the electromotive force of the local circuit joining the machines would be twice the voltage of either machine.

The condition for switching in parallel is that of *no resultant voltage* in the local circuit joining the machines—*i.e.*, the machines must have :—

(1) Equal voltage.
(2) Equal periodicity.
(3) Equal phase.

The simplest method of determining the condition is shown in Fig. 123, which shows an incandescent lamp S L connected in parallel with the switch S employed for connecting the machines together.

This lamp serves to show whether there is any resultant voltage in the circuit joining the alternators, since it glows

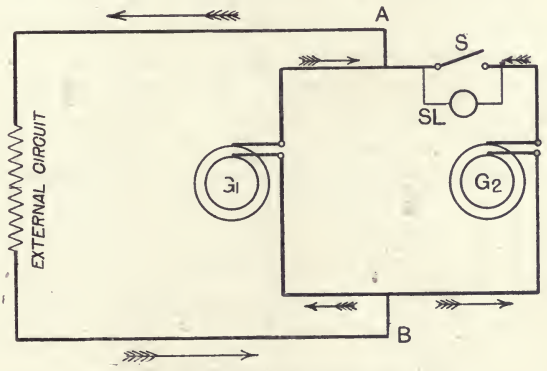


FIG. 123.—USE OF LAMP AS SYNCHRONISER.

when the machines are in series (*i.e.*, joined + to −) and ceases to glow when they are truly in parallel, and the resultant voltage is consequently zero.

If the two machines run at different speeds, the lamp will appear alternately bright and dull, the changes in the lamp occurring less frequently as the difference in speed becomes less, and as the time taken for the faster-running machine to catch up the slower machine becomes greater.

A voltmeter with a range equal to double the normal voltage of the alternators may be used instead of the lamp, if it is provided with a damping device to prevent the needle

swinging too much, and if its moving parts are light and free enough to follow rapidly the variation of voltage.

It is important to notice that in the direct form of synchroniser just described, parallelism is indicated by zero reading on the voltmeter or darkness of the lamp. A single-pole switch and single lamp are shown in Fig. 123. Usually a double-pole switch with a lamp across each break in the circuit would be employed. The lamps would, in this case, be in series with one another, and would, consequently, only receive half as much voltage, *i.e.*, the full voltage of one alternator as a maximum.

Synchronisers Employing a Transformer.—If the pressure of the alternator is too high to make possible direct connection to a lamp, a transformer may be used, the primary winding taking the place of the lamp shown in Fig. 123, and the lamp being connected in series with the secondary, in

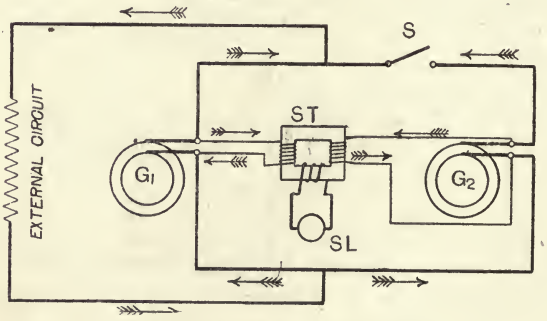


FIG. 124.—DIAGRAM OF SYNCHRONISER.

order to reduce the pressure applied to the lamp. The action is exactly the same as that just described, since the current, passing round the transformer, will be nil when the alternators are in phase, and the lamp will not glow under these conditions.

The above methods both have the peculiarity that a lamp is required for each pole of the switch, and that the lamp ceases to glow when synchronism is attained, unless the lamps are connected from one switch to the other, instead of in parallel with the switch.

The following modification (see Fig. 124) is of more general use, as a 2-pole switch may be used with a single lamp or voltmeter, and the lamp can be made to glow at coincidence or at opposition in phase as desired.

The generators are connected to separate equal windings on a small transformer. The lamp or voltmeter is connected to a third winding having fewer turns than the other windings, and consequently giving a lower voltage.

The two high-tension windings will produce a magnetic flux in the core of the transformer. This flux will be the sum or the difference between the fluxes which would be produced by the windings acting singly, according to the relative direction of the currents in the windings. It depends how the alternators are connected to these primary windings, whether the lamp in the secondary will be bright or dull when the machines are in parallel. In this country it is usual to choose the connection so that the voltmeter reads its maximum, or the lamp burns most brightly, when the machines are in phase. In America the opposite practice has generally been adopted, and the machines are switched " in " when the lamps are out.

Thus, on referring to Fig. 124, with the connections as shown, the current in the left-hand winding of the synchronising transformer under the conditions indicated will produce a magnetic flux in the transformer core such that a N-pole would be urged in an anti-clockwise direction in the interior of the core. The current in the right-hand winding will at the same time produce a similar flux, on account of the direction of winding and direction of current assumed. If the applied voltages in both windings are equal, the resultant flux will be the sum of that due to either winding acting alone and the centre winding connected to the lamp has a maximum voltage induced to it. Under these conditions we may suppose the machines are in phase for correct parallel working. If now the connections between alternator G_2 and the right-hand winding of the transformer are reversed, so that the current produced by the machine flows in the opposite direction through this winding, the flux produced by this winding will be clockwise, and will consequently be subtracted from that formed due to the current in the other winding. Hence the same relative phases of the machines would produce zero flux, instead of a maximum flux, through the third winding.

When more than two alternators have to be connected so as to be capable of being run in parallel, as is usually the case at a large generating station, the connections have to

be modified somewhat from those given above. Since it may be necessary to run any machine with any other, or with several other machines, a simple means must be provided whereby the low-pressure winding of the synchronising transformer of any machine may be readily connected to that of any other. The usual method adopted is to have synchronising bus bars extending across the back of all the generator switch-board panels, as shown in Fig. 125 where

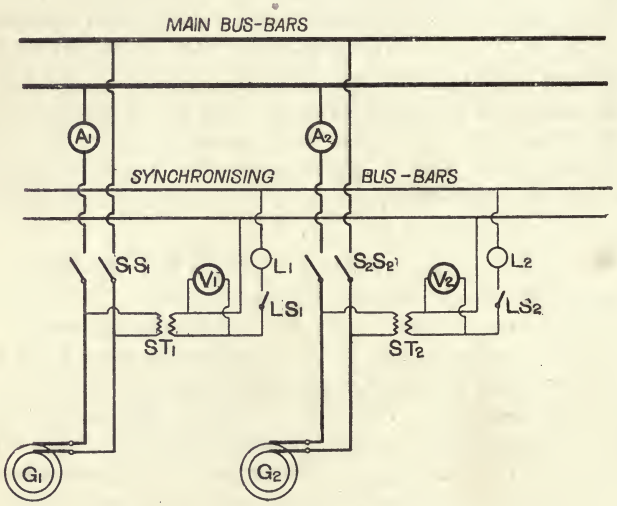


FIG. 125.—CONNECTIONS FOR SYNCHRONISING ALTERNATORS.

G_1, G_2 represent the armatures of two of the machines, of which there may be any number connected to the switch-board in a similar manner.

The letters given on the diagram, Fig. 125, have the following meaning :—

G Alternator armature.
S S Main 2-pole switch.
A Alternator ammeter.
V Alternator voltmeter.
S T Synchronising transformer.
L Synchronising lamp.
L S Synchronising lamp switch.

It will be noticed that the generator voltmeter is shown connected to the secondary of the synchroniser transformer.

This arrangement has the advantage that an ordinary low-pressure voltmeter may be used, while its dial may be graduated so as to indicate the full pressure of the alternator, since the actual voltage at the voltmeter terminals is always a definite fraction of that generated by the alternator.

Special forms of synchronisers, indicating whether the alternator runs too fast, or too slow, are also employed, but their description hardly comes within the scope of this book.

EXPERIMENT XL.—TO SYNCHRONISE AN ALTERNATOR WITH ANOTHER ALREADY RUNNING.

DIAGRAM OF CONNECTIONS.

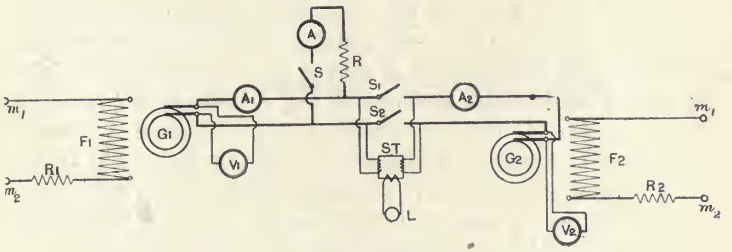


FIG. 126.

m_1, m_2	Source of direct current.
G_1, G_2	Alternator armatures.
F_1, F_2	Alternator field windings.
R_1, R_2	Field regulators.
R	Variable resistance in load circuit.
A	Ammeter in load circuit.
A_1, A_2	Ammeters reading output of each generator.
V_1, V_2	Voltmeters reading voltage of each generator.
$S\,T$	Synchroniser transformer.
L	Synchroniser lamp.
S_1 S_2	Paralleling switches.
S	Switch in load circuit.

Connections.—These will necessarily depend upon the nature of the machines and upon the nature of the permanent connections, if the machines are connected to a switchboard.

Supposing the machines to be without permanent connections to a board, the method of running to be adopted may be that indicated in Fig. 123, for low-tension machines, or that shown in Fig. 124, if the pressure is higher.

The second method of connection is indicated above as being of more general application. The modifications required in order to make the instructions applicable to the first method will be readily seen on referring to the diagram Fig. 123.

Instructions.—Make connections between the alternators to be synchronised through a 2-pole switch. Make connection to one high-tension winding of the synchroniser from the terminals on one side of the switch, connecting the other winding similarly to the other pair of switch terminals. Connect the low-tension synchroniser winding to a lamp or voltmeter*, or to both in parallel.

In order to measure the circulating current between the machines, insert an ammeter in series with one of the switches. In the figure an ammeter is shown for each generator. This enables simultaneous readings of the output of each machine to be made. If the machines are to be loaded, connect a resistance to the circuit in series with an ammeter and switch.

Connect a voltmeter to each alternator. Excite the machines from a source of continuous current with an independent regulating resistance for each generator.

If one alternator is already running at normal speed and voltage, run up the second to about the correct speed, and adjust its field current until the voltages of the machines are equal.

The lamp will probably blink rapidly. The speed must now be adjusted in the direction giving less rapid flickering. The lamp should then go completely out and light up brightly more and more slowly. When the action is sufficiently slow and marked, so that in the case of a small machine two or three seconds elapse between each period of brightness, close the double-pole switch as the lamps are brightest (if this is the condition when in synchronism). Then note the sudden throw of the ammeter in the circuit between the machines, and after this has assumed its steady reading, regulate the field of the alternator until the ammeter reading is as low as can be obtained. For large machines it is essential that the speeds should be practically identical before switching in.

After this, the switch in the load circuit may be closed, and the machines tested under load.

* If the machines are of large size a voltmeter is a necessity, in order to judge the synchronism sufficiently accurately.

The machines should divide the load between them in proportion to their rated output for considerable variations in the load without subsequent regulation of the excitation.

It may be desirable to test this by increasing the load gradually, and noting the current given by each machine without alteration of the field regulators.

Chapter VIII.

Synchronous Motors.

A Synchronous Motor is in construction similar to an alternator. It requires consequently a supply of direct current for exciting the magnets, and a supply of alternating current to the armature.

Mode of Operation.—Let Fig. 127 represent the armature of a 2 pole alternator or synchronous motor. If current is supplied at the brushes so as to flow in the direction indicated, the action will be like that of a direct-current motor. The armature will exhibit the polarity indicated in the Figure by the arrow *n s*, and rotation will take place in the direction shown. If current flowed in the opposite direction through the armature, rotation would take

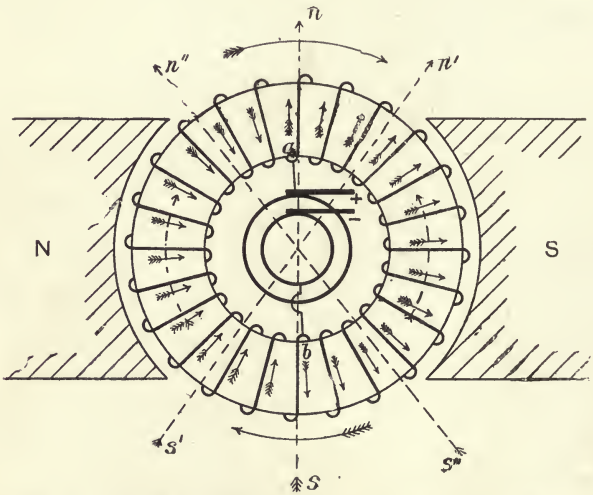


Fig. 127.—Diagram of Armature Current in Synchronous Motor.

place in the opposite direction. A rapidly alternating current supplied to the armature while at rest would produce a series of impulses tending to drive the armature first in one direction and then in the other. The result would be a rapid vibration of the armature, but no rotation, the inertia of the moving part being too great

to allow it to move appreciably from the neutral position between each alternation of the current. Suppose, however, the 2-pole armature shown in Fig. 127 were made to rotate at exactly such a speed that it made half a revolution in the time taken by the current to change from its maximum value in one direction to its maximum value in the opposite direction. In that case the direction of the current would be reversed each time the position of the conductors *a*, *b* connected to the rings was reversed. Consequently, the polarity of the armature would be the same as before, after half a revolution.

When followed out in detail, the action of the currents would be as follows: Beginning with the armature in the position shown, suppose that the current is at its maximum value, and flows, as shown by the small arrows. The poles induced in the armature core will be in the positions indicated by *n*, *s*. As the armature rotates in clockwise direction, in virtue of the attraction between the poles of the armature and those of the field, the current dies down, because it is an alternating current. After a quarter of a revolution the current will have become zero, and the armature poles will at the same time have gradually diminished in strength to zero while approaching the centre of the field poles. After the conductors have passed the centre of the field poles, the armature current gradually increases, but flows in the opposite direction, so that the points in the armature marked *n* and *s* are now respectively of south and north polarity. With increasing armature current and the simultaneous movement of the conductors connected to the slip rings towards the mid-position between the poles, the turning moment increases, until the newly-formed south pole reaches the position *s* on the diagram, formerly occupied by the opposite point of the armature. The cycle now repeats itself.

The revolution of the machine would under these conditions be maintained by the current. The continuance of the rotation depends upon coincidence of the reversal of the direction of the current and the reversal of the position of the armature. If the current reversed too soon or too late, the polarity induced by the current would oppose the rotation of the armature during part of the revolution. If the *rate of alternation* of the current were changed relatively to the speed of the armature, the current would sometimes

assist and sometimes oppose the rotation, and the motor would probably cease to rotate.

The essential point about a synchronous motor is that it can only operate when rotating *synchronously*, or in step with the current supplied to it.

Effect of Increase in Number of Poles.—As explained above, a 2-pole synchronous motor will make exactly one revolution for each cycle of the alternating current supplied to it. Thus a 2-pole machine supplied with current having a periodicity of 25 would make $25 \times 60 = 1{,}500$ revs. per. minute.

If the motor has a greater number of poles, the speed will be such that an armature conductor passes from one pole to the next of the same character in the time of each period of the current. Hence, if the machine has 12 pairs of poles, it will take the time of 12 periods to make one revolution. Thus,

$$\text{Speed of motor} = \frac{n \times 60}{p} \text{ revs. per minute.}$$

When n = periodicity of current supplied.
p = number of pairs of poles.

Method of Starting a Synchronous Motor.—As just explained, the action of a synchronous motor depends on its running at the speed of synchronism, and the machine will not rotate if switched on to an alternating supply while stationary. It must, therefore, be started by some external means from rest, and, when it has been brought up to the correct speed, it may be switched on to the alternating circuit, and will then continue to rotate.

Synchronous motors are often started by means of direct-current motors either intended for that special purpose, or intended as direct-current generators for supplying the exciting current required by the synchronous motor. The use of a direct-current motor necessitates a supply of direct current which is sometimes not available before the machine is started, *e.g.*, in cases where the motor drives its own exciter.

Sometimes a synchronous motor can be started from rest by the action of the eddy currents formed in the pole faces of the magnets. This can only be done in the case of 2 or 3 phase machines, and will be referred to in discussing them.

In cases where only a single motor is to be driven by an alternator, as is frequently the case in testing, the motor

may be made to start by running the alternator very slowly, and then giving the motor a few sharp turns by hand to bring it to the reduced speed of the alternator. In starting the motor in this way it will be found easiest to turn the motor by hand without excitation, and to switch on the field when sufficient speed is attained for it to fall into step with the alternator. If the alternator speed is then gradually increased, the motor will speed up and keep in step.

If a synchronous alternating-current motor is to be connected to an alternating circuit, it must first be run up to the *speed* of synchronism, its *field* must be adjusted to the correct amount, and the switch connecting it to the supply must be closed when the *phase of the motor* corresponds to that of the circuit.

The process is consequently similar to that adopted in synchronising two alternators (see page 259).

The following experiment is suggested specially as being suitable for giving practice in the starting of synchronous motors.

EXPERIMENT XLI.—TO START A SYNCHRONOUS MOTOR AND ASCERTAIN THE LIMITS OF EXCITATION FOR WHICH IT WILL START.

DIAGRAM OF CONNECTIONS.

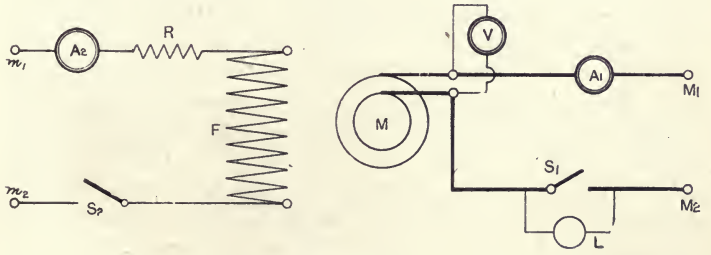


FIG. 128.

M_1, M_2	Source of alternating current.
m_1, m_2	Source of direct current.
M	Motor armature.
F	Motor field windings.
A_1	Ammeter reading armature current.
A_2	Ammeter reading exciting current.
V	Voltmeter.

R	Field regulating resistance.
S_1	Main switch.*
S_2	Field switch.
L	Synchronising lamp.

General.—Although it is possible in some cases to switch small synchronous motors directly on to the supply after running them approximately up to speed without synchronising them, it is a practice which cannot be adopted with all motors, and which is liable to produce considerable fluctuations in the supply voltage. It is very dangerous to attempt to do this with any but very small motors. The form of synchroniser to be employed may be either the simple lamp shown in the diagram (Fig. 128) above, or it may be the form of synchroniser shown in the diagram (Fig. 125). The instructions given below can be readily modified to suit the latter case by referring to those given on page 264 for paralleling alternators.

Instructions.—Connect the armature of the motor to the source of alternating supply through a switch in series with an ammeter. Connect a voltmeter to the machine terminals. Excite the magnets with direct current through a field regulator and ammeter. For synchronising, connect a lamp, or several lamps in series, across the terminals of the main switch. Remember that these lamps must be able to stand a voltage equal to *twice* that of the supply, if a single pole switch is employed.

Before closing the main switch, run the motor up to full speed by an auxiliary motor or other means. Adjust the excitation until the motor voltage is equal to that of the supply. Vary the speed until the lamp fluctuates slowly, and becomes quite extinguished between each period of brightness. Close the switch when the lamp is out. Then adjust the field until the current taken by the armature is a minimum. The machine may now be loaded without falling out of step.

Although it is generally advisable to excite the motor so that its armature voltage before switching in is as nearly as possible equal to the voltage of the source of supply, the motor will start with either more or less excitation than this.

* A double-pole switch with a lamp suitable for the line voltage across each pole may be employed, and is preferable in the case of permanently connected motors.

The experiment should therefore be repeated first with gradually decreased excitation for each operation of switching on to the circuit, and then with increased excitation, until the lower and upper limits for which the motor can be started are reached. The variation in the armature-starting current produced by under and over excitation should be noticed.

The results of the experiment should then be tabulated in the following manner.

EXPERIMENT IN STARTING SYNCHRONOUS MOTOR.

Synchronous motor No..... Type.....

Output.....h.p. at ....volts and ....periods per second.

Voltage of supply....

Motor Armature.			Excitation.	Notes as to Manner of Starting.
Voltage Before Switching In.	Current.			
	At Starting.	When Steady.		

The next experiment shows more fully the effect of varying the excitation upon the behaviour of a synchronous motor—a matter of the greatest importance for the satisfactory working of motors of this type.

EXPERIMENT XLII.—DETERMINATION OF THE EFFECT OF VARIATION OF THE EXCITATION OF A SYNCHRONOUS MOTOR.

DIAGRAM OF CONNECTIONS.

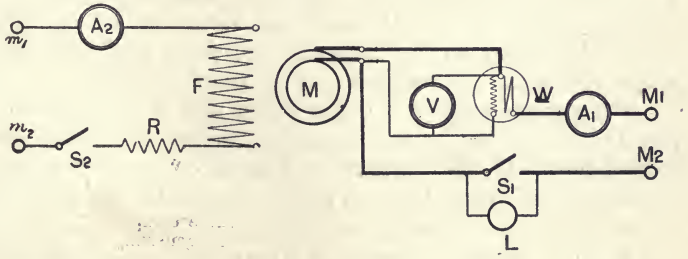


FIG. 129.

M_1, M_2	Source of alternating current.
m_1, m_2	Source of direct current.
M	Motor armature.
F	Motor field windings.
A_1	Ammeter reading armature current.
A_2	Ammeter reading exciting current.
V	Voltmeter reading armature voltage.
W	Wattmeter reading power supplied to armature.
R	Field regulating resistance.
S_1, S_2	Switches.
L	Synchronising lamp.*

Instructions.—Excite the motor field from a source of direct current through a regulating resistance, ammeter, and switch. Connect the armature to the source of alternating current through a switch, ammeter, and series coil of a wattmeter. Connect a synchroniser in parallel with the switch for synchronising the motor before switching the armature on to the supply. Connect a voltmeter and the volt coil of the wattmeter across the terminals of the armature.

Run the motor, close the switch in the exciting circuit, and synchronise as described in the previous experiment, closing the switch in the armature circuit when synchronism is obtained.

Next determine the most favourable excitation by varying the exciting current until the armature current indicated by A_1 reaches its lowest value.

The experiment should now be continued as follows: With the motor running on no load, vary the excitation, first decreasing and then increasing the exciting current, and for each value of the excitation note the current and power taken by the motor armature. If this can be done without too great fluctuations in the source of supply and excessive heating of the motor armature, the variations of excitation should be taken so far in both directions that the motor stops, the point of breakdown being carefully noted.

The voltage applied to the motor and the periodicity of the current must be maintained constant throughout the experiment. If the current is derived from an alternator which is not very large in comparison with the motor, it

* The synchroniser will frequently be of the more complete type indicated in Fig. 126. A double-pole switch will in this case generally be employed.

will be necessary to adjust both the speed and excitation of this generator as the current taken by the motor varies.

After taking a complete set of readings as described with the motor unloaded, several similar sets of observations should be taken with the motor loaded, the load being kept constant for each complete series of readings. The load may be applied either by a band or other form of brake, on the motor pulley, or the motor may be made to drive a dynamo, the current of the dynamo being kept constant during each set of readings, and varied when a change of load is desired. If the efficiency of the dynamo is known, the actual load on the motor can at once be determined by measuring the output of the dynamo. If the efficiency is not known, it is sufficient for the purpose of the present experiment to assume an approximate value of the efficiency in order to show the difference between the curves obtained. The direct-current motor employed to start the synchronous motor in the first place may often be conveniently employed as the dynamo for this purpose.

If the excitation of the direct-current machine is maintained constant its armature current will be directly proportional to the torque exerted by the motor.

The results should be entered in tabular form as indicated below.

EXCITATION CHARACTERISTIC OF SYNCHRONOUS MOTOR.

Synchronous Motor No..... Type.....
Output ....h.p. at ....volts and ....revs. per minute.
Periodicity....cycles per second.

Excitation Amperes.	Terminal Volts.	Armature Current.	Armature Watts.	Load.	Cos ϕ

The results of each set of readings should be plotted on squared paper, excitation being plotted horizontally and volts and watts vertically, giving one curve of current and a curve of watts for each load applied to the motor (see Fig. 130, page 278).

The nature of the curves obtained in this experiment will be discussed after a general account of the behaviour of a synchronous motor under variable excitation has been given.

Behaviour of Synchronous Motors with Variable Excitation.—The experiment just described illustrates the most important point in the behaviour of a synchronous motor. The following general reasoning is given as an introduction to the graphical representation of the conditions, which is given later.

In the direct-current motor the turning effort on the shaft is due to the action of the armature current on the magnetic field, and the magnitude of the turning moment is proportional to the product (armature current × field strength).* Consequently, with a constant load, any change in the strength of the field necessitates an inverse change in the current. This is automatically brought about by a change of speed, which varies the armature back voltage until the current attains the right value.

Thus in the case of a direct-current motor

$$\text{turning effort} \propto (\text{field} \times \text{armature current}),$$

when the position of the brushes, and consequently the magnetic axis of the armature, remains unchanged, and

$$\text{armature current} = \frac{E - e}{R},$$

where E = applied electromotive force.
e = back electromotive force induced by rotation.
R = resistance of armature.

For constant load e varies automatically due to change of speed when the field is changed, so as to maintain the product (field × current) the same as before.

In the case of an alternating-current synchronous motor the turning effort on the shaft is due to the same causes, and is proportional to the mean force existing between the armature (due to the current in it) and the field. We may, therefore, again in this case write

$$\text{turning effort} \propto (\text{field} \times \text{armature current}),$$

but must make the same limitation as before, viz., that the magnetic axis of the armature field produced

* See Chapter IX. in the author's "Practical Dynamo and Motor Testing."

by the current remains constant in direction relative to the field magnets. This condition means rather more in the case of an alternating motor than it does with a direct-current machine. Its meaning in this case is that the armature current must attain its maximum value when the armature is in the same relative position as regards the fields, *i.e.*, the difference in phase between armature current and back electromotive force (the phase of which depends only on the position of the armature) must be constant. Consequently, if the phase of the armature current changes, the turning effort produced by a given armature current and field will change also.

This may be put in another way by saying that it is only that component of the armature current which is in phase with the back electromotive force of the motor which can be looked upon as producing the torque, and the component which is out of phase with this is inoperative.

Thus, if ϕ be the angle of phase-difference between the armature current and back electromotive force of the armature, and if C be the total armature current, then the component which may be looked upon as producing the turning moment on the shaft is $C \cos \phi$.

Thus, with a constant load on the motor shaft, the product (field $\times$ $C \cos \phi$) must have a definite value and remain constant for this load, and if the field is varied the factor $C \cos \phi$ will change inversely in order that the load may continue to be overcome. With a constant value of ϕ the load will be proportional to the product (field $\times$ current) exactly as in the case of the direct-current motor with fixed brushes.

Further, in the case of the alternating-current motor the armature current

$$= C = \frac{\text{resultant of applied and back voltage}}{\text{armature impedance}}$$

in which the resultant voltage is the resultant, obtained by the parallelogram law, of the applied and back volts. The armature current will be this voltage divided by the impedance of the armature.

Rotary Converter

If the excitation be changed, a synchronous motor cannot change its speed like the direct-current motor,

because its speed depends only on the periodicity and not the voltage of the supply. It cannot consequently adjust the back electromotive force generated in the armature, since this depends only on the product of speed and field strength. The motor can, however, bring the current to the value required to overcome the load by altering the phase angle between the voltage applied and the back volts of the armature. This changes the resultant voltage and consequently also the armature current. At the same time the phase difference between armature current and back voltage changes, and consequently the amount of their mutual action changes also. In general terms the effect of alteration in field strength will be as follows :—

With excitation so adjusted that the back voltage and applied voltage are equal, the armature of the motor will assume a position such that the back voltage is not exactly opposite in phase to the applied voltage, since in this case the armature current would be zero, the two voltages being equal and opposite. On the contrary, the armature will fall behind, so that the back electromotive force lags in phase until the product of the back voltage and the component of the current in phase with the back voltage produces the requisite torque.

If the excitation is now decreased, the pull of the field on the armature is lessened, and the armature lags still further, thereby increasing the resultant electromotive force in the armature and also the armature current, until again the product of back electromotive force and energy current produce the torque required to overcome the load.

If the excitation of the motor be increased instead of decreased, the required armature current will flow with a smaller angle of phase difference between applied and back voltage, and the motor will consequently lag by a smaller angle, and $C \cos \phi$ will approach more nearly to C. When the excitation has been increased just up to the point where the armature current is exactly in phase with the back electromotive force and $\cos \phi$ is consequently equal to unity, the current required to drive the motor will have its minimum value, since the whole of the current acts usefully on the fields. On increasing the

excitation still further, the angle ϕ will change sign—*i.e.*, the current supplied will begin to get greater again for the same load, but the current will lead the voltage in phase instead of lagging behind it.

An undue increase of excitation will increase the current required to maintain the rotation of the armature, on account of the great difference of phase between the armature current and voltage. This can only be clearly seen from a diagram. The current will always lag behind the resultant electromotive force of the circuit by a definite amount on account of the self-induction of the armature. The difference of phase between the terminal voltage of the motor and the current will, however, vary, and consequently the amount of the current must vary also, since it is only the component in phase with the motor volts which is effective in driving. For every load there will be a particular value of the excitation which will give the lowest value of the armature current, *i.e.*, the highest power factor and smallest idle component of the current. This is obviously the most favourable condition. The excitation applied to a motor under working conditions should be such as to give the lowest armature current at the normal average load.

We may now observe how the effects of varying the excitation just discussed are illustrated by the curves obtained from the experiment.

Figs. 130 and 131 show curves of current and watts obtained by varying the excitation of a small 4-pole 1 h.p. motor having a ring-wound rotating armature, when running at a speed of 1,500 revs. per minute. The three curves correspond to three sets of readings taken each at a constant output. The loads were : No load, ·38 h.p., and ·81 h.p. respectively.

It will be seen that the curves of current consist of two inclined portions, which in the case of no load are nearly straight lines, meeting at the lowest portion of the curve corresponding to 1·47 amps. excitation.

It therefore appears that with 1·47 amps. excitation the armature current is most nearly in phase with the back voltage, and the current supplied to the motor is consequently a minimum. With a lower excitation the current lags behind the back electromotive force, and the amount

of current necessary to maintain the rotation of the shaft increases for a double reason: (1) It has to increase in order to make up for the weakening of the field by the lessened excitation; (2) the total current supplied must increase in a greater ratio than the weakening of the field, because only the portion $C \cos \phi$ is active, and ϕ increases as the current lags behind the back electromotive force.

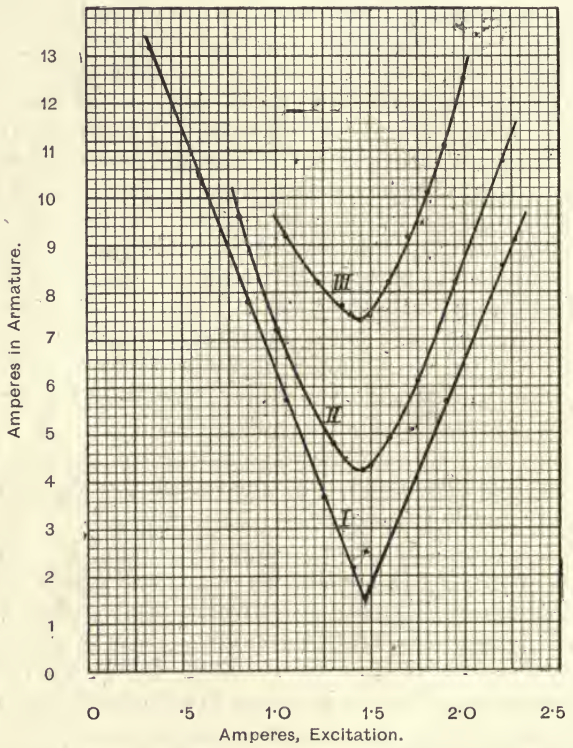


FIG. 130.—VARIATION OF ARMATURE CURRENT WITH EXCITATION IN SYNCHRONOUS MOTOR.

I. = Motor running light.
II. = Output ·38 h.p.
III. = ,, ·81 h.p.

The curve consequently rises to the left of the minimum point due to these causes.

The curve also rises, although rather less steeply, for values of the excitation higher than 1·47 amps. This is due to the fact that the armature current in this case leads

the back electromotive force in phase, so that a smaller and smaller component of the current supplied to the armature reacts on the fields so as to produce the rotation. Thus, in spite of the fact that the field is strengthened, the current supplied must be increased in order to keep the component of the current which is in phase with the back electromotive force at a sufficiently high value to produce the required constant torque.

It will be seen that the right-hand part of the current curve is somewhat less steeply inclined than the left-hand branch, since, as just explained, in this case the field is strengthened by the change of excitation, and makes it unnecessary for the value of $C \cos \phi$ to increase so rapidly as is the case for points on the left-hand portion.

From the slight difference between the inclination of the two branches of the curve, it is evident that the variation in strength of the field produces only a small direct change in the armature current, and that nearly the whole of the variation is produced indirectly because of the alteration in the lag of the motor armature which follows the change of field. The lack of symmetry in the two limbs of the curves will depend also on the magnetisation curve of the machines, since excitation, and not flux, is plotted horizontally. In the present instance the magnetisation curves were practically straight lines within the range here employed. The other curves follow generally the form of the no-load curve, but are more curved and show a more gradual bend at the lowest point, indicating a less sudden transition from lagging to leading current, and consequently more stable running.

The curves of watts in Fig. 131 will be seen to follow generally the shape of the curves of current, but show a less proportional variation. At first it might be thought that the power supplied to the motor would only vary very slightly, since the motor runs at a constant speed and exerts the same turning effort. From the curve it is evident that this is not the case, at anyrate when the motor is lightly loaded, the power supplied varying from a minimum of 165 watts to a maximum of 625 watts.

This variation is specially high in the present case, since the motor was small and had a high armature resistance and low efficiency. A variation in power necessary

to drive the motor would, however, be found with any motor, which would be similar in its nature although smaller in extent in the case of a more efficient machine. An increase in armature current necessarily gives rise to increased losses in the conductors, armature core, and pole faces of the machine.

The curve of watts shows very forcibly the importance of choosing the most favourable excitation for the motor,

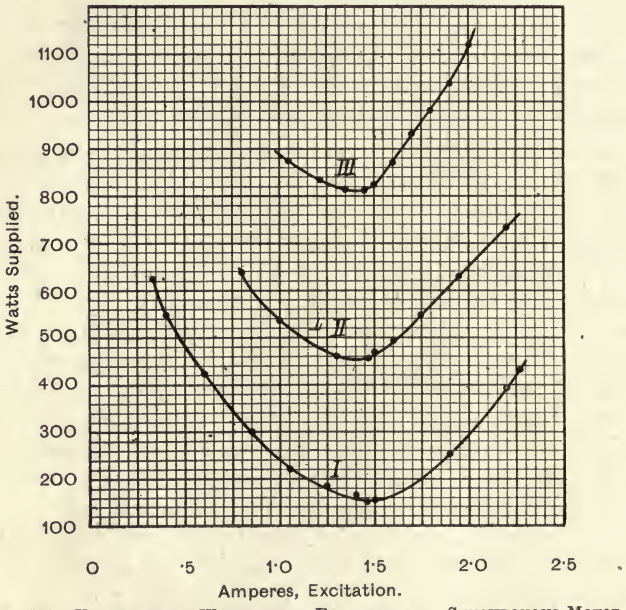


FIG. 131.—VARIATION OF WATTS WITH EXCITATION OF SYNCHRONOUS MOTOR.

I. = Motor running light.
II. = Output ·38 h.p.
III. = „ ·81 h.p.

since the *useful* work is the same for all points on the curve, although the power supplied to the motor varies so greatly.

The shape of the curves obtained in Experiment XLII. depends upon the self-induction of the armature of the motor. If the armature were without self-induction the armature current would be in phase with the resultant voltage and would increase in proportion to it. There would then be only one possible value of the exciting

current for a given load, and the two limbs of the curve would coincide, giving a single line.

The greater the self-induction of the armature the more divergent will the two limbs of the curve be, and the greater will be the difference between the two values of the excitation at which it will operate with a given load and current. This will be explained more fully in connection with the graphic representation of the conditions (see page 286).

Fig. 132 shows the curves of power factor obtained from the same readings as the curves of current and watts

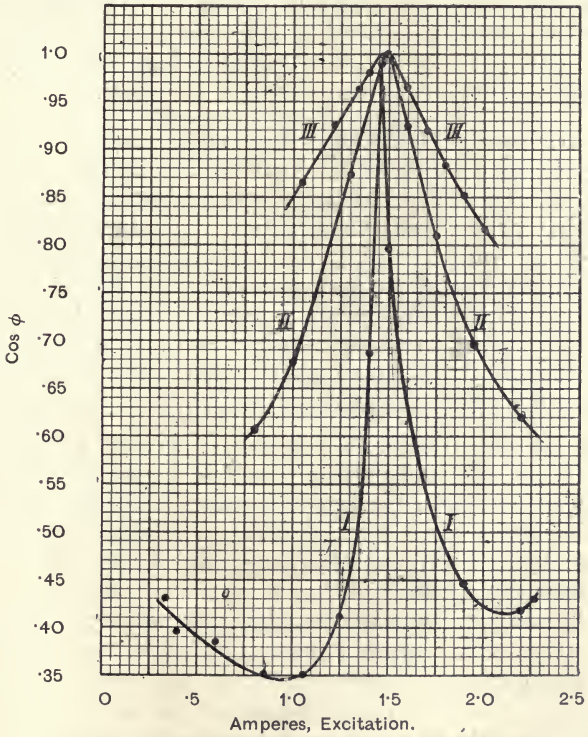


FIG. 132.—CURVES OF POWER FACTOR AND EXCITATION.

I. = No load.
II. = ·38 h.p.
III. = ·81 h.p.

given in Figs. 130 and 131. The no-load curve shows how rapidly the power factor changes with the excitation in the case of an unloaded motor. This curve shows the curious feature of a power factor rapidly falling and then

slightly increasing, with further divergence from the most favourable excitation shown by the upward bend of the lower branches of the curve. This must be explained by the fact that the rapid increase of current which flows through the armature when the excitation is increased above or decreased below the correct value produces a considerable loss of energy in the armature, due to its resistance and the iron losses in it. These losses ultimately become so great that the power spent in the motor armature increases more rapidly than the current. Although, therefore, the current increases and only does the same work as the smaller current in *driving* the armature, it does so much work in heating the armature that after a certain point the power factor begins to increase, instead of decreasing further, as more current is supplied.

It is important to observe how the curves just described are affected by an increase in load applied to the shaft of the motor. In each case the figures show three curves obtained from the same motor when unloaded and when loaded by a brake, and giving out ·38 and ·81 h.p. respectively at the same speed as before. It was found impossible to obtain as great a range of excitation with the motor loaded as when running light. In each case the extreme readings shown are the extreme values of the excitation for which the motor would run.

The curves of current and power factor are much flatter at the apex when the motor is loaded. This is especially noticeable in the case of the curves of cos ϕ. From this it is evident that, in the case of a loaded motor, a small variation in exciting current has only a very small effect when this variation is near the point of most favourable excitation. This has a most important practical result in the steady running of a synchronous motor under load. The extremely sharp point in the curves of the unloaded motor at the point of best excitation shows that very small variations in the exciting current on either side of the best value will produce considerable changes in the armature current and in the phase angle at which the motor will run. This illustrates one of the chief difficulties to be met with in the running of rotary converters from the alternating current side. A rotary convertor is really driven as an unloaded synchronous motor by an

alternating current, and the "hunting" of these machines is mainly due to the want of stability of the conditions under which such machines work, as shown in the curves in Figs. 130 to 132.

Effect of Change of Load at Constant Excitation.—Having shown the effect of an alteration in exciting current at various constant loads, it is of interest to ascertain how a change of load will affect the performance of a motor excited with a constant current. This forms the subject of the next experiment.

EXPERIMENT XLIII.—DETERMINATION OF EFFECT OF VARIATION OF LOAD UPON A SYNCHRONOUS MOTOR HAVING CONSTANT EXCITATION.

DIAGRAM OF CONNECTIONS.

Same as Fig. 129 for Experiment XLII.

Instructions.—Make the same connections as described for Experiment XLII., page 271. After exciting the field and synchronising the motor, close the main switch in the armature circuit, and disconnect the starting motor. Vary the excitation until the value is found corresponding to minimum current when unloaded. Keep this value of the excitation constant, and gradually load the motor by a brake, or by making it drive a generator and increasing the output of this machine. For each value of the load take readings of the current and watts supplied to the motor. Read also the terminal voltage, which should be kept as constant as possible.

A series of similar readings should then be taken for several different values of the excitation both above and below the most favourable value, the excitation being kept constant through each series.

The results should be entered in tabular form. The table given for the preceding experiment gives suitable headings. Three sets of curves should be plotted from the results, load being measured horizontally in each case The curves should show armature current, watts input. and values of cos ϕ respectively, plotted vertically.

The curves in Figs. 133 to 135 show the results of a test, made on the same motor as that from which the curves in Figs. 130 to 132 were obtained. The curves were taken for

each of four values of the excitation, viz., ·9, 1·1, 1·36, and 1·8 amps.

The most favourable excitation at the voltage employed in this experiment was about 1·36 amps., hence the curves show the relation between the load and the watts, current and power-factor for two values of the excitation below the normal, and for one value above the

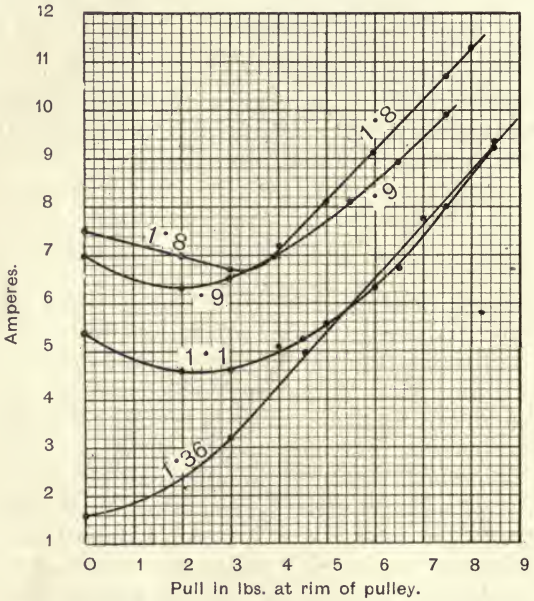


FIG. 133.—VARIATION OF ARMATURE CURRENT WITH LOAD IN A SYNCHRONOUS MOTOR AT VARIOUS EXCITATIONS.
Pulley, 8in. diam.

normal, as well as the relation for the condition of normal excitation.

The current curves (Fig. 133) show that with normal excitation the current increases almost in direct proportion to the load, in much the same way as it does with a direct-current shunt-wound motor. With a lower excitation of 1·1 amps. the current is much greater at light loads, on account of the low power-factor, but approximates to the same value as for normal excitation at higher loads. With the excitation still further diminished to ·9 amps. the initial current is still higher, and although the current

curves tend towards that obtained with normal excitation, it does not reach it for the range of loads shown.

With excitation 1·8 amps., considerably above the normal, values of the current are obtained which are greater than for any of the other curves, and the curve indicates little tendency to approach that taken at normal excitation.

The curves of watts (Fig. 134) show that the difference between the power given to the motor when normally and when under-excited is comparatively slight; indeed, the

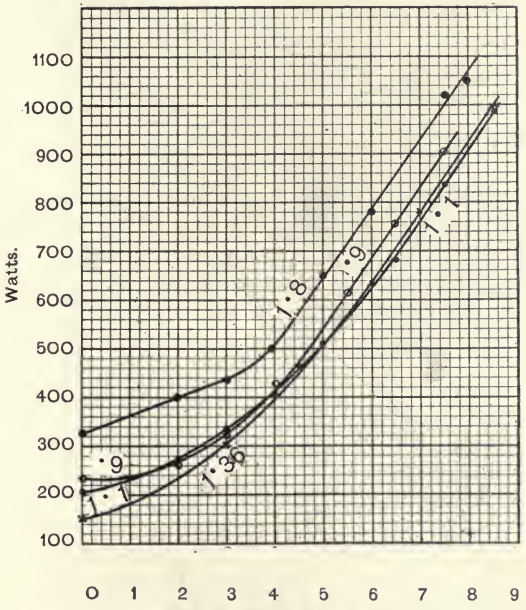


FIG. 134.—VARIATION OF POWER TAKEN BY SYNCHRONOUS MOTOR WITH LOAD AT VARIOUS EXCITATIONS.
Pulley 8in. diameter.

excitation of 1·1 amps. shows rather better results than the normal 1·36 at higher loads. The curve for over-excitation indicates a considerable amount of waste power as being supplied.

The curves in Fig. 135, showing the variation in power-factor, illustrate in a very decided manner the advantage at light load of correct excitation. The power-factor in the curve for 1·36 amps. excitation is high throughout. With the

other excitations the power-factor only becomes fairly high at the higher loads. The explanation of the curve for 1·8 amps. being higher than the ·9 amp. curve is to be found in the greater armature waste power in the motor when over-excited.

Graphic Representation of Conditions in a Circuit Containing Two Synchronous Machines.—In Experiment I. it was shown how to obtain the resultant voltage when two alternators

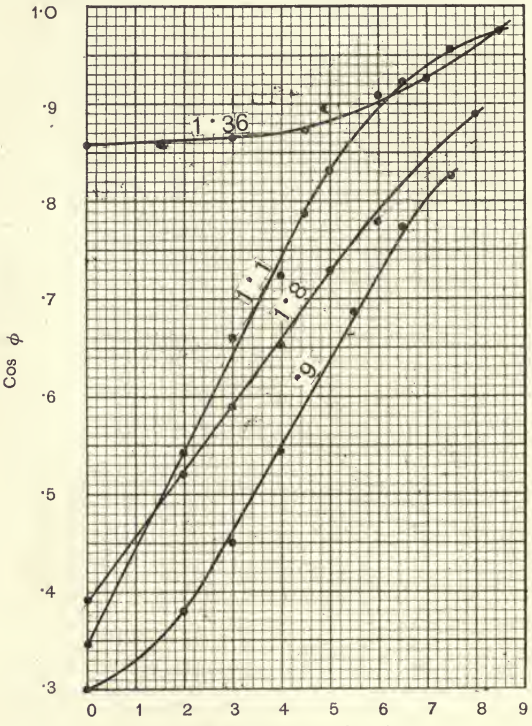


FIG. 135.—VARIATION OF POWER FACTOR WITH LOAD IN A SYNCHRONOUS MOTOR.
Pulley 8in. diameter.

differing in phase were connected to the same circuit. The running of two alternators in parallel, or the connection of a synchronous motor to an alternator, are special cases of the problem then discussed, the only additional feature introduced being that of a current in the circuit. In following discussion the voltages of the two machines will

be assumed to be equal unless a statement to the contrary is made.

Case 1.—Two Alternators in Parallel without Load.—When running in parallel connected in the manner shown in Fig. 122, each alternator will tend to send current in opposition to the other machine round the circuit which connects the two machines together. There will thus be two opposite electromotive forces acting on the circuit, and so long as the machines remain truly in parallel (*i.e.*, in exact opposition of phase as regards the local circuit joining them) and give equal voltages, there will be zero resultant electromotive force, and no current.

When represented graphically, the voltages of the alternators would form two lines OE_1 and OE exactly equal, and opposite to each other. The resultant of these lines would be zero.

If one alternator begins to lag behind the other, the two electromotive forces no longer remain opposite in phase, and the conditions in the circuit joining the machines then become similar to that shown in Fig. 136, where the alternator voltage OE_2 has assumed the position shown instead of its original position indicated by the dotted line.

The resultant voltage acting on the circuit is now seen to be OE_r, which is the diagonal of the parallelogram drawn with OE_1 and OE_2 as sides.

In this case there will be a current in the circuit, the value of which is $\frac{\text{resultant voltage}}{\text{impedance of circuit.}}$ Since the resistance of the conductors between the alternators will be very small, the value of the current will be practically $\frac{\text{half of resultant electromotive force}}{\text{impedance of armature of one alternator.}}$ The phase of the current can be determined by resolving this resultant voltage into its components. In order to do this, we must know either the energy or idle component of the current.

Usually the energy component is not easy of determination, since, in addition to the component spent in overcoming the resistance of the circuit and armatures, there may be an unknown energy component spent in iron

losses and in doing mechanical work owing to the interactions between the two machines.

The idle component of the resultant voltage is numerically equal to the product of current and the reactance of the circuit. Several methods of determining the approximate value of the armature reactance have been given; it is therefore usually possible to determine the amount of reactance voltage or idle voltage ($= 2\pi n L C$).

Describe a semi-circle on $O E_r$, and from E_r describe an arc with radius equal to $2\pi n L C$, the reactance voltage, to cut the semi-circle in P. Then $O P$ is the energy component of the resultant electromotive force in the circuit, and is in phase with the current.

The power given to the circuit by either machine = current × voltage of machine × cos ϕ, where ϕ is the angle enclosed between the line OP and the voltage of the machine. For the lagging machine, the value of cos ϕ will be negative, hence the power given by this machine to the circuit is negative. In other words, this machine absorbs power and is driven by the other.*

The difference between the positive and negative power is seen at once by observing whether the projection of the current vector on the voltage vector would have its direction in the same or the opposite direction as the voltage. Thus in Fig. 136 the projection of the current on the line $O E_2$ is $C\epsilon_2$, which is directed in the opposite direction from O to the voltage $O E_2$ itself.

The power given to the circuit by the first machine is the product of $O C\epsilon_1$ amperes and $O E_1$ volts. The power taken by the second machine is the product of $O C\epsilon_2$ amperes and $O E_2$ volts. In each case this is the product of the machine voltage and that component of the current which is in phase with it.

The fact that, when one machine lags behind the other, it receives a driving current, is the reason why it is possible to run alternators in parallel. Whichever machine tends to fall behind is at once accelerated by power supplied by the other. Thus machines will run in parallel and resist any tendency to fall out of step.

* If there is any doubt as to the side of OE_r upon which the semi-circle should be drawn, this can always be decided by remembering that the circuit is inductive, and consequently the current must lag behind the resultant voltage E_r in phase. If the semi-circle in Fig. 136 had been drawn below E_r instead of above it, the current would apparently *lead*.

If the machines are connected in *series, i.e.*, terminals of unlike sign together, the conditions will be unstable, since a lagging machine would receive currents tending to retard instead of to accelerate it. The machines would thus not continue to work in series (unless mechanically coupled together), but would rapidly alter their phase relations until they run in *parallel*. If two machines connected as in Fig. 122 are connected in series they will, of course, produce a short circuit in the connections between the machines. They could only work in series with each other when also in series with the load circuit.

Case 2. Synchronous Motor Driven by Alternator.—When the motor is excited so as to give a voltage equal to that of the alternator, the case is exactly similar to that just described. The motor will always lag slightly behind exact opposition in phase to the alternator; but since it runs synchronously, its back electromotive force will be constant and equal to that of the alternator. The phase relations and value of the current flowing between the machines will be the same as those discussed in Case 1. The phase and amount of the current will vary with the load on the motor, which will lag behind the generator by exactly the amount necessary to give the current its value necessary to overcome the load.

It will frequently occur, however, that owing to armature reactions and other causes the back voltage of the motor becomes less than that of the alternator. In such a case the electromotive forces of the circuit become those represented in Fig. 137, where the motor voltage represented by OE_2 is smaller than OE_1. The resultant voltage is found as before, and from this the phase of the current is determined as already described. Again, the power given out by the alternator is the product (current $\times$ OE_1 $\times$ cos POE_1), while the watts absorbed by the motor = current $\times$ OE_2 $\times$ cos POE_2.

Fig. 137 is drawn with the voltages of the two machines in the same phase relations as Fig. 136, the only difference between the diagrams being the decreased voltage of OE_2. The effect of this change is seen to be the rotation backwards of the current vector OC, so that the current in Fig. 137 is almost in phase with the voltage OE_1. The power given out by the alternator is seen to have increased, since the component current $OC\epsilon_1$ is greater. The power given

to the motor is also considerably increased, since the product $O E_2 \times O C\epsilon_2$ is considerably greater in Fig. 137 than in Fig. 136. It is thus seen that decreasing the excitation of a lagging machine has the effect of increasing the power supplied by the other to drive it, if the machines remain at the same relative angle.

If the conditions indicated by Fig. 136 represent an alternator and motor with equal armature voltage, and if

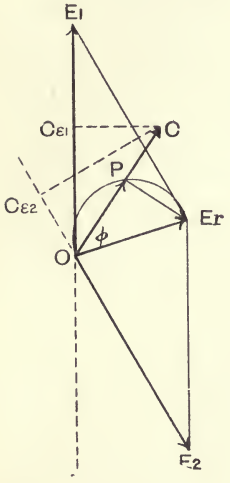


FIG. 136—DIAGRAM OF CIRCUIT CONNECTING TWO ALTERNATORS IN PARALLEL.

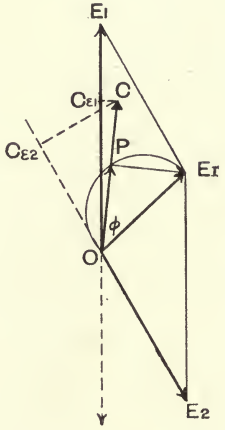


FIG. 137.

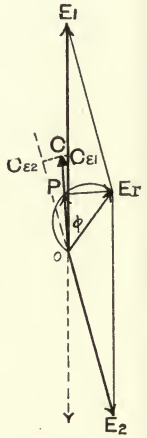
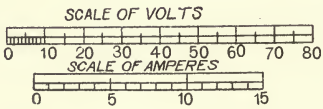


FIG. 138.

DIAGRAMS OF CIRCUIT CONNECTING ALTERNATOR AND SYNCHRONOUS MOTOR.



the excitation of the motor is then diminished to the value $O E_2$ in Fig. 137, the load on the motor remaining the same as before, the motor will receive an increased amount of power, and will be consequently accelerated The phase angle between the two machines will consequently increase, until the power given to the motor becomes the same as originally sufficed to drive it, as shown in Fig. 136. Under these circumstances, the conditions will become those indicated in Fig. 138, where the two voltages have become more nearly opposite, and the resultant voltage and current have both decreased in value. The power-factor in the case of both motor and

generator is seen to be far higher than in the case of equal excitation of the machines, but the product of OE_2, and $OC\epsilon_2$ is the same as in Fig. 136.

It is to be noted that the angle of lag of the current in the circuit joining the machines is COE_r in each case, and the cosine of this angle would be the power-factor of the circuit. The power transmitted from one machine to the other would be measured by a wattmeter with its current coil inserted in this circuit and its volt coil connected to the terminals of either machine. The angle of phase difference between the armature back voltage of either machine and the circulating current would be $E_1 O C$ and $E_2 O C$ respectively, but this could not be obtained by any method of direct measurement.

The voltages measured at the terminals of the generator and motor will be equal, except for the small loss in the line, which may be neglected for the present. This does not, however, mean that the electromotive force generated in the armature of the alternator is equal to the back electromotive force generated in the motor armature. As in the case of corresponding direct-current machines the voltage at the generator terminals = total voltage generated − volts lost in armature; while in the case of the motor the terminal voltage = back electromotive force + volts lost in the armature.

It must be borne in mind that the various voltages are not in phase with each other, and the negative or positive sign must be taken to apply to vector summation and not algebraic summation.

Case 3. Two Alternators in Parallel Supplying Current to an Outside Circuit.—This condition is similar to that considered in Case 1, with the addition of an added current flowing through the armatures of the machines to the external circuit. This current does not affect the interaction of the two machines in any material way so long as the output of both remains equal. It will reduce the external voltage of the machines slightly, and this will produce a small modification in the voltage diagram already given. Even when the machinese are not quite equally excited, or when one tends to fall behind the other in step, the diagrams already given apply. The diagrams should in each case represent the

alternator voltages and the local current circulating in the circuit joining them. The current supplied to the external circuit will consequently not be shown on the diagram, and will only affect it by introducing an increased voltage drop in the armature voltage of the machines.

Effect of Variation of Excitation of Synchronous Motor.— We are now in a position to consider somewhat more in detail the causes of the special form of the curves shown in Fig. 130.

If we neglect the losses in the machine, the curves in Fig. 130 show that for a given load there are two values of the excitation, for which the load will be overcome with a certain current in the armature. The two points corresponding to a fixed armature current and load indicate that the resultant voltage in the circuit is the same for both points, because the resultant voltage determines the value of the current. Further, the value of the power supplied to the motor must be equal in both cases, and consequently the angle of phase difference between current and applied voltage is the same for both points. The difference between the points is due to the fact that in one case the current is in advance of the applied voltage, while in the other case it lags behind it by an equal angle.

Fig. 139 is drawn to show that these conditions can be produced by variation of the motor excitation. Thus for a certain excitation the phase relations of the circuit are shown

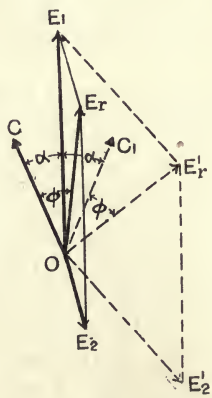


FIG. 139.

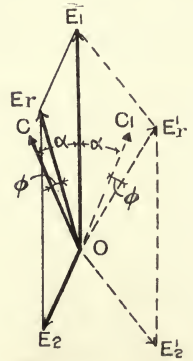


FIG. 140.

EFFECT OF VARIATION OF EXCITATION IN CASE OF TWO SYNCHRONOUS MOTORS WITH DIFFERENT ARMATURE SELF-INDUCTION.

by the full lines, the lettering corresponding to the figures already given. In this case, the current lags behind the applied voltage by the angle α . By increasing the excitation of the motor the motor voltage increases from $O\ E_2$ to the dotted value $O\ E_2^1$, when the angle between current and applied voltage becomes again the same as before. The power supplied to the motor, the resultant voltage and the current in the circuit are the same as in the first case. The two conditions shown in the diagram by the full and the dotted lines respectively would correspond to points at the same height, but on different limbs of one of the curves in Fig. 130.

In order to illustrate the statement previously made, that the distance apart between the limbs of the curves depends upon the self-induction of the armature, the conditions shown in Fig. 139, have been repeated in Fig. 140, with the only difference that the armature self-induction in the latter case is assumed to be smaller, and consequently the angle of lag ϕ is smaller. It is at once obvious from an inspection of the figures that the difference in length between $O\ E_2$ and $O\ E_2^1$ is much less in the second figure, showing that the difference in excitation for the two points giving equal armature currents is much less in the second case than in the first. In other words, the distance between the limbs of the curve will be less the lower is the armature self-induction. With a non-inductive armature the two limbs would coincide, and the curve showing the relation between excitation and armature current would become a straight line, since the value of E_2 would be the same whether the current was earlier or later than E_1 in phase.

It would prove a most useful exercise for the student to construct several of the " Vee " curves, like those in Fig. 130, from diagrams representing several assumed conditions as to armature self-induction, &c.

Chapter IX.

The Polyphase Circuit.

Generation of 2 and 3 phase Currents.—If the armature of an alternating-current generator has conductors situated at equal distances apart round its circumference, the phase of the electromotive forces induced in them by the field will differ. Only those conductors which are situated in similar positions with regard to the poles at any instant will be identical in phase, while conductors situated midway between these will be intermediate in phase. As already explained, it is usual to connect in series coils which are formed of conductors lying in several slots, and which are, therefore, not identical in phase, although differing but slightly. In order to produce an alternating current, generally only those conductors can be advantageously joined together which are approximately in phase with each other, being similarly situated under similar poles, or which are 180° out of phase with each other, the latter being situated in corresponding positions under opposite poles. There are thus portions of the armature between the conductors which are joined in this way which cannot be advantageously wound with conductors connected to the same circuit.

Two-phase Current.—If an exactly similar set of conductors is wound in the intermediate positions between the first set, and these are independently connected together, they will give an alternating electromotive force similar in voltage and periodicity to that induced in the first winding, and only differing from it in phase.

An alternator having two armature windings with the conductors of one winding situated in advance of the other by half the pitch of the magnet poles, *i.e.*, the coils of one exactly half-way between the coils of the other, will supply a 2-phase current at the four terminals with which the armature is then provided.

Thus, a 2-phase current consists of two independent alternating currents, each carried by a single pair of conductors. The two circuits have the same voltage and periodicity, and a fixed relative difference of phase of a

quarter period, so that one current passes through its maximum value as the other is at zero, and vice versâ.

The relation between the currents or voltages of the two circuits may be represented by two equal rotating lines at right angles to each other, as in Fig. 141. The lines are not shown joined together, since the quantities represented by them are in two separate circuits. The power supplied by the generator will be twice the power given to either circuit.

Three-phase Current.—If three windings are applied to the armature with equal spacing between the conductors of each winding, the phases of the electromotive forces induced in the windings will differ by one-third period, so that there will be a point of maximum electromotive force in each direction induced in each winding in rotation.

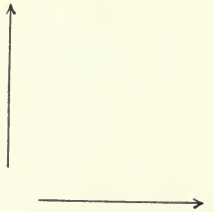


FIG. 141.—RELATION BETWEEN VOLTAGES IN A 2-PHASE CIRCUIT.

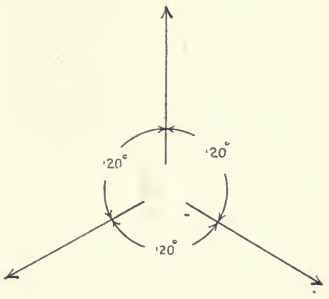


FIG. 142.—RELATION BETWEEN VOLTAGES IN A 3-PHASE CIRCUIT.

If the three windings form completely separate circuits, the armature will have six terminals, and the current and voltage relations may be represented by three rotating lines, each making an angle of 120° with the other two (See Fig. 142.) The power developed by the machine will be three times the power of each winding. A system of this kind is termed a *3-phase* system, and consists of three alternating circuits, having equal voltage and periodicity, and a fixed phase relation of such a kind that there is one-third period phase difference between the voltages of any pair of circuits.

From what has just been said, it appears that a 2-phase current requires for its transmission four wires, and a 3-phase current similarly requires six wires, and an *n*-phase

current would require $2n$ wires. This is true in general; but for a 3-phase circuit the special relations existing between the currents in the three circuits make it possible to employ in this case three conductors instead of six. The special conditions which make this possible must now be explained.

Transmission by Three Wires.—Let Fig. 143 represent a generator supplying direct current to a distant point P,

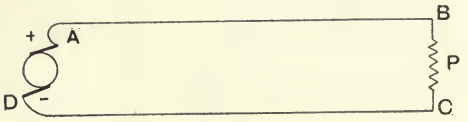


FIG. 143.

and let us consider the conditions governing the flow of current in the conductor $A\ B$.

Current cannot flow along $A\ B$ unless this conductor forms part of a closed circuit connected to the generator Hence the circuit must be closed at the end P, and there must be a return conductor from P capable of transmitting at any moment the same current from C to D as is flowing from A to B. The return current need not flow by a single conductor, but may flow partly through the earth, as in the case of a tramway installation, or may be unequally divided between a number of conductors. The only absolutely essential condition for the flow of a current

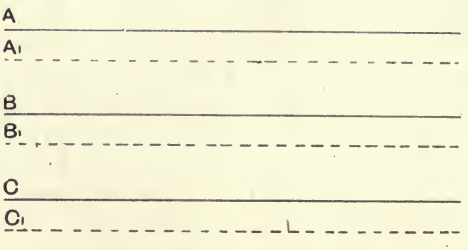


FIG. 144.

from A to B is that the same amount of current shall flow at the same moment from C to D. This condition applies to any kind of current, whether direct or alternating.

Let $A\ A^1$, $B\ B^1$, $C\ C^1$ (Fig. 144) represent the six conductors carrying a 3-phase current, each conductor shown

dotted forming the return wire for the current carried by the conductor just above it.

Then the current in A^1 is equal and oppositely directed to the current in A *at every instant*, and similarly for the remaining pairs.

It will be shown later that the special characteristic of a 3-phase system is that the sum of the currents in any two circuits at any instant is equal and oppositely directed to the current in the third circuit. Thus, the sum of the currents in B and C is always equal and opposite to the current in A. Consequently, if a suitable connection were made between the conductors A, B, and C, the conductors B and C might, without any change in the current flowing in them, provide the return path for the current in A. That is, the current supplied by the generator to the circuits B and C may be looked upon as being the return current of the circuit A. Thus the conductor A^1 might be dispensed with, without the current in A being affected.

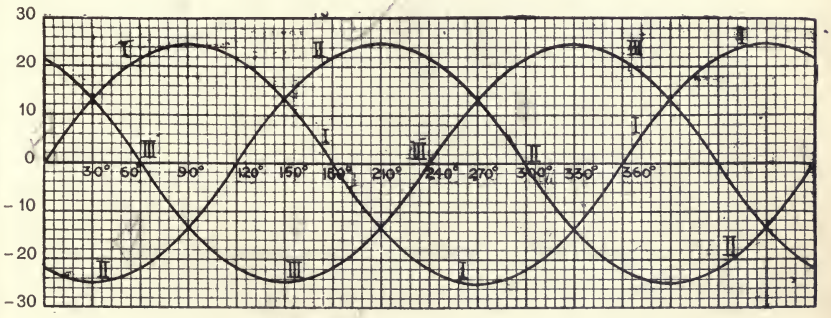


FIG. 145.—CURVES OF CURRENT IN A 3-PHASE CIRCUIT.

Similarly, the conductors A and B carry an equal and opposite current to that in the conductor C, and the conductor C^1 may be dispensed with without altering the current in C, if A and B are so connected to C that they can form the return path for the current in it.

In this way no circuit requires a separate return conductor, and the 3-phase currents can be transmitted along three conductors instead of six.

The simplest way of showing that the sum of the currents in two circuits is equal and opposite to that in

the third is to draw curves representing the simultaneous values of the currents. This has been done in Fig. 145. Taking at random any instant represented on the curve it will be seen that the algebraic sum of the ordinates of two curves is equal to the height of the ordinate of the third, and is opposite to it in sign.

The mathematical proof of the proposition is simple when the quantities are represented by vectors, as in Fig. 146, where $O\,C_1$, $O\,C_2$, $O\,C_3$ show the three currents.

The angles referred to are plainly marked. C is the common maximum value of the currents.

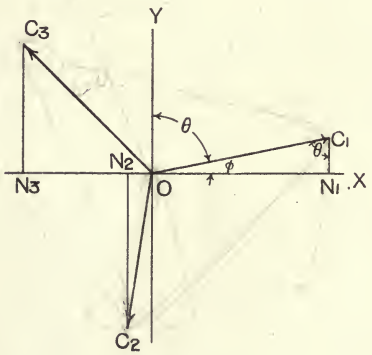


FIG. 146.

The instantaneous values of the currents are obtained by horizontal projection. It should be remembered that the cosine of an angle is equal to the sine of its *complement*.

(1) $O\,N_1 = C \cos \phi = C \sin \theta$.

(2) $O\,N_2 = C \cos (120 - \phi)$.
$= C (\cos 120 \cos \phi + \sin 120 \sin \phi)$.
$= C \cos 120 \sin \theta + C \sin 120 \cos \theta$ .. (A)

(3) $O\,N_3 = C \cos (120 + \phi)$.
$= C (\cos 120 \cos \phi - \sin 120 \sin \phi)$.
$= C \cos 120 \sin \theta - C \sin 120 \cos \theta$ .. (B)

Therefore, from equations (A) and (B) above we get :—

$O\,N_2 + O\,N_3 = 2\,C \cos 120 \sin \theta$.
$= -\,2\,C\,\frac{1}{2} \sin \theta = -\,C \sin \theta = -\,O\,N_1$.

Evidently, also, this equality is independent of the value of θ and is true throughout the cycle.

Methods of Connection for three-phase System.—In the preceding paragraph it was assumed that each circuit was connected to the other two, so that they should form the return path for its current. The method of connection between the circuits must evidently fulfil two conditions. (1) It must be symmetrical; (2) the current must flow through the resistances or machines forming the three load circuits.

There are two methods of connecting the load circuits fulfilling these conditions, both in general use. They are indicated in the diagrams, Figs. 147 and 148, and are called respectively the *star* and the *delta* or *mesh* connections.

In the star connection, Fig. 147, the three load circuits are connected with one end to a common or neutral point, the free ends being joined directly to the supply lines.

In a mesh-connected system, Fig. 148, the load circuits are all joined end to end in series, so as to form a closed circuit, and the supply conductors are joined to each junction point.

Connection must be made in the alternator itself between the three windings or phases with which it is wound. This can either be done in the machine itself, so that it has three terminals for connection to the three transmission lines, or the alternator may have six terminals, and these may be inter-connected, so as to supply current of the right character to the line. In either case there

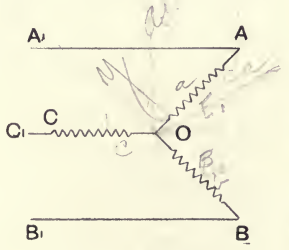


FIG. 147.—STAR CONNECTION.

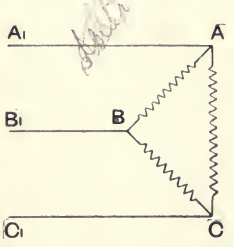


FIG. 148.—MESH CONNECTION.

are available only the two alternative methods just described, and the three windings of the armature must be connected in one of the ways shown in the diagrams, Figs. 147 and 148, in star or mesh connection. The effect of this interconnection of the phase-windings upon the voltage of transmission must now be discussed.

Star and Mesh Connection of an Alternator.—(1) Let the three spirals, Fig. 147, represent the three-phase windings of an alternator, and first suppose them to be star connected as shown. Then the voltage between a pair of conductors $A^1 A$ and $B^1 B$ forming the transmission line is due to the two windings $O A$, $O B$. These windings act in exactly the same way as two alternators giving equal voltages and coupled together at such an angle that they differ in phase by $\frac{1}{3}$ period or 120°. The resultant voltage due to these alternators might be found as in Experiment I., and would be represented in magnitude and phase by the diagonal of the parallelogram of which the separate electromotive forces are drawn to form the sides.

Thus the electromotive force due to the two coils $O B$, $O A$ would be represented by two lines $O E_1$, $O E_2$ in a similar manner to the currents in Fig. 146. It must, however, be remembered that when thus represented (see Fig. 149) the lines represent the voltages as + or − *with respect* to O, and for the conditions shown, the instantaneous values of the voltage of both coils are equal, and +, and would be represented by projections on $O E_R$, which lies to the right of O, and is therefore to be taken as positive. Hence the voltages E and E_2 in $O A$ and $O B$ will both be directed away from O, the common point of the windings, or both towards this point, according to the convention adopted regarding the positive and negative direction. The value of this resultant voltage is seen from Fig. 149 to be E_R, equal to the maximum value of each of the component voltages. The voltage which we wish to determine is, however, the resultant voltage between the lines $A A^1$ and $B B^1$, Fig. 147. Thus, if directions away from O are considered +, a positive electromotive force in $O B$ will be the reverse of a positive electromotive force in $O A$ as regards the resultant voltage from B to A. We must therefore reverse the sign of one voltage, shown in Fig. 149, in order to obtain the resultant voltage from B to A. The parallelogram consequently assumes the form given in Fig. 150, where the voltage $O E_2$ is shown with the arrow pointing in the reverse direction. The parallelogram is now $O E_2^1 E_R E_1$, the diagonal giving the resultant voltage $O E_R$ is therefore longer than in the case represented by Fig. 149. The resultant voltage is, in fact, seen to be

$$\frac{1}{\cos 30^0} \times \frac{3}{2} E = \frac{3}{2} \times \frac{2}{\sqrt{3}} E = \sqrt{3}\, E$$

i.e., the voltage measured from A to B in Fig. 147 is equal to $\sqrt{3}$, or 1·73 times the voltage generated in a single coil, such as $O\,A$, of the alternator.

Stated generally, in a star-connected alternator, the terminal voltage = voltage of one phase × $\sqrt{3}$. Evidently from the diagram the current supplied to any

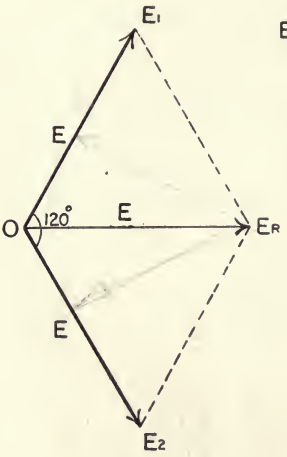


FIG. 149.

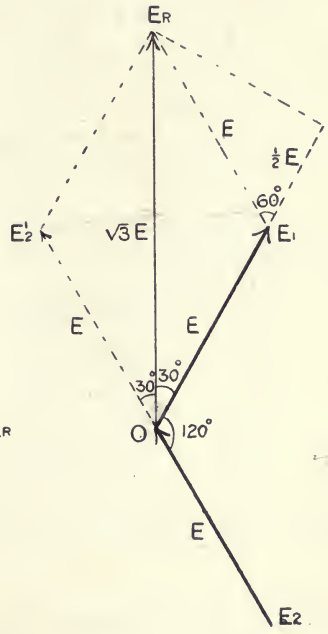


FIG. 150.

RESULTANT VOLTAGE OF TWO COILS OF STAR-CONNECTED GENERATOR IN 3-PHASE TRANSMISSION.

circuit is the same as the current in each winding of the armature.

(2) Suppose the same windings to be connected in mesh, as shown in Fig. 148.

The voltage between any pair of conductors $A^1\,A$, $B^1\,B$, &c., is in this case the same as that produced in the phase winding between them, since they are connected directly to the ends of the winding.

Hence, for a mesh-connected alternator the terminal voltage = voltage of one phase.

The relation between the currents in the transmission line and in the armature windings is in this case not so simple as in the case of the star-connected generator, since each line is connected to two phase windings, and will receive current from both. The relation between the two currents supplied to each conductor may be represented by an exactly similar diagram to Fig. 150, used to show the voltages in the case of the star-connected armature. By similar calculation the resultant current is seen to be $\sqrt{3}$ × current in either phase winding, since the same relations hold in the case of currents in mesh connection as previously were found to exist for the voltage of a star-connected system.

The results of the preceding paragraph may be summarised as follows :—

Let C, E be respectively the current in the line and the voltage between each pair of conductors, and

c, e the current and voltage of each phase winding in the generator armature.

For star-connected armatures

$$C = c,$$
$$E = \sqrt{3}\, e,$$

For mesh-connected armatures

$$C = \sqrt{3}\, c,$$
$$E = e.$$

The formulæ just given also apply to the case of motor armatures supplied from the line, c and e representing the current and pressure in each phase winding of the motor. Similarly, for any resistance or other form of load circuit supplied from the line, the above rule applies if c and e are taken as the current and voltage of each branch, when the branches are connected either in star or mesh connection, and C and E are the current and voltage in the main circuit.

Power of 3-phase Circuit.—Still employing the symbols just given, it is easy to see what the power transmitted by the 3-phase circuit is.

First, take the case of non-inductive load.

The power given to each branch load circuit is $c\,e$ watts. Since there are three such branch circuits, the total power transmitted by the line to the branch circuits is $3\,c\,e$ watts.

In the star connection $c = C$ and $e = \frac{E}{\sqrt{3}}$, hence in this case, total power $= 3\,c\,e = 3\,C\,\frac{E}{\sqrt{3}} = \sqrt{3}\,C\,E$.

In the mesh-connected system $c = \frac{C}{\sqrt{3}}$ and $e = E$, consequently power of system $= 3\,c\,e = 3\,\frac{C}{\sqrt{3}}\,E = \sqrt{3}\,C\,E$.

Thus in either case the power is the same and is equal to $\sqrt{3}\,C\,E$ watts.

The relations just given should be verified by means of the following Experiment:—

EXPERIMENT XLIV.—DETERMINATION OF THE POWER TRANSMITTED BY A 3-PHASE CIRCUIT (LOAD NON-INDUCTIVE).

DIAGRAMS OF CONNECTIONS.

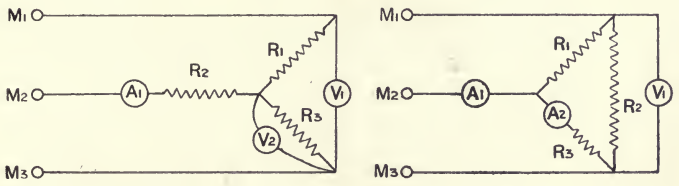


FIG. 151.

M_1, M_2, M_3 Terminals supplied with 3-phase alternating current.
R_1, R_2, R_3 Equal non-inductive resistances.*
A_1, A_2 Ammeters.
V_1, V_2 Voltmeters.

Instructions.—In order to verify the formulæ for both star and mesh connected systems, the connections should be made in turn according to both systems, as indicated in the diagrams shown above.

* Banks composed of equal number of similar incandescent lamps form a suitable non-inductive load. A still more convenient form of 3-phase resistance is a liquid form with three symmetrical blades capable of being dipped into the solution simultaneously to any desired extent.

The diagrams sufficiently indicate the connections to be made. It is to be noted that in the case of the star connection a single ammeter measures both the current in the line and in the branch circuit. With the mesh connection a single voltmeter measures both line and branch voltage.

For each of the connections indicated in the diagram, take readings of the line current and voltage, and the current and voltage of one of the branch circuits. This should, if possible, be repeated for two or three values of the resistances. In each case the values of all these resistances should be equal. Enter the results in the form shown below, and note that the two values of the power entered in the last two columns should be the same.

COMPARISON OF POWER IN MAIN AND BRANCH CIRCUITS.

Line.		Branch.		Total Watts.	
Current $= C.$	Voltage $= E.$	Current $c.$	Voltage $e.$	$\sqrt{3}\,CE.$	$3\,ce.$

Power in Inductive Circuits.—If the load is partly inductive, the current and voltage will not be in phase with each other. In this case the power in each branch circuit will be $c\,e\cos\phi$ instead of $c\,e$ watts. If the angle of lag in all the branch circuits is the same $(=\phi)$ the expression for the power in the line becomes

$$\text{power} = 3\,ce\cos\phi \text{ or } \sqrt{3}\,CE\cos\phi,$$

which is the general expression for the power transmitted by a three phase line with equal loading on all phases.

In general, the measurement of the power of a 3-phase system necessitates the employment of a wattmeter when the load is partly inductive, as the methods in which only ammeters and voltmeters are employed become very complicated.

There are several methods of connecting the wattmeter to the circuit, the most usual of which are now to be described.

Wattmeter Measurements in Three-phase Circuit.—Method I.—One obvious method of measuring the power given to or taken from a three-phase circuit is to connect the wattmeter in one of the three-branch circuits, as indicated in Fig. 152. If the

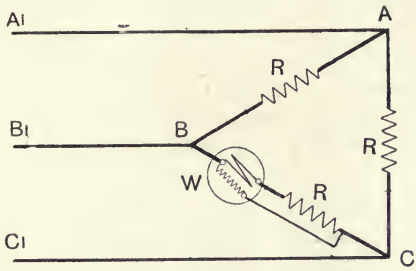


FIG. 152.—CONNECTION OF WATTMETER IN BRANCH CIRCUIT.

power given to the circuit is to be measured, the wattmeter would be so connected as to indicate the output of one phase of the generator. If the power measured is that given to a motor, transformer, or other set of branch circuits, the wattmeter would measure the power in one branch circuit. As explained in the preceding section, the total power of the system = $3\,c\,e$ where c and e are the current and voltage in one branch of the star or mesh. Thus the wattmeter will read one-third of the total power, and the reading must be multiplied by three, unless this use of it has been already foreseen and allowed for in the calibration of the instrument.

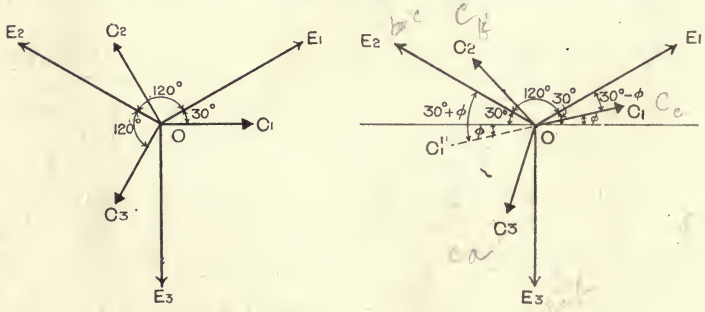
It is frequently more convenient to insert the wattmeter in the line circuit, as the windings of the generator or motors may not be easy to disconnect. This can be done by employing the next method of measurement.

Method II.—What would at first sight appear an equally simple method is to connect the wattmeter so as to read the product of current and voltage of the line, in the manner shown in Fig. 155. In this case it must, however, be remembered that the current C and voltage E are not in phase with each other, even in a non-inductive circuit. Thus in the line supplied by a star-connected generator, the line current C is the same as the current c

of one phase, and consequently C and c are in phase. The line voltage E is the resultant of the voltages e and e in two phases which differ from one another by 120°, and is 30° out of phase with either of them, as will be seen from the diagram Fig. 150. Consequently, in a non-inductive circuit, where c and e are in phase, E and C will differ in phase by 30°. Hence the reading of the wattmeter connected as shown in Fig. 155 will read the product $C\,E \cos 30° = \frac{\sqrt{3}}{2} C\,E$ when there is no lag or lead in the circuit, *i.e.*, when the power-factor is unity.

Thus for a non-inductive circuit a single reading of the wattmeter will give the power of the circuit when multiplied by two—since, as will be remembered, the total power is $\sqrt{3}\,C\,E$. Similar reasoning will lead to the same result with a mesh-connected generator, as the current and voltage are in this case also 30° out of phase.

If the load is inductive, so that the branch current c is not in phase with the voltage e, as assumed above, but has an angle of phase difference ϕ, then the phase difference



Load non-inductive. Load inductive.

FIG. 153.—PHASE RELATIONS IN THREE-PHASE LINE (STAR CONNECTED GENERATOR.)

between the current and voltage in the line will not be 30° but 30° ± ϕ according as ϕ is an angle of lag or of lead, in the case of a mesh-connected generator, or 30° ∓ ϕ under like conditions for a star-connected generator. This case is illustrated by Fig. 153.

Here the vectors $O\,E_1$, $O\,E_2$, $O\,E_3$, represent the phase of the three voltages measured from line to line as derived from a star-connected generator, and the lines $O\,C_1$, $O\,C_2$, $O\,C_3$, show the phase of the currents flowing in

the lines. The case of a non-inductive load and also of an inductive load are shown. Imagine the series coil of a wattmeter to be inserted in one line, carrying the current C_1, for instance, in the right-hand diagram. Suppose also that the volt coil of the wattmeter is connected so as to read the voltage between the same line and one of the others, so as to read the voltage E_1, for instance. The wattmeter reading will then be $C_1 E_1 \cos E_1 O C_1$, that is $C E \cos (30 - \phi)$. Now imagine the volt coil thrown over to read the voltage between the line in which the current flows and the other line, and suppose this to be E_2. The watts read will now be $C_1 E_2 \cos E_2 O C_1$, or $C E \cos (30° + \phi)$. Thus the two readings of the wattmeter will be respectively $C E \cos (30° - \phi)$ and $C E \cos (30° + \phi)$,

$$i.e.\ C E (\cos 30° \cos \phi + \sin 30° \sin \phi)$$
$$\text{and } C E (\cos 30° \cos \phi - \sin 30° \sin \phi)$$

The sum of these is $2 C E \cos \phi \cos 30° = \sqrt{3} C E \cos \phi$, which is the value in watts of the power in the circuit.

We have, therefore, the following rule for determining the power in an equally loaded inductive three-phase circuit. Connect the wattmeter with its current coil in one of the line wires, and join the volt terminal alternately to the other two line wires. The sum of the two readings thus obtained gives the total power on the circuit. The load and power factor on the three lines are of course assumed equal.

Incidentally it is worth noticing that the difference between the two wattmeter readings when divided by the line voltage gives the idle current directly.

The difference of the readings is $2 C E \sin 30° \sin \phi$.

$$= 2 C E \tfrac{1}{2} \sin \phi = E.\ C \sin \phi,$$

and $C \sin \phi$ is the wattless current in the line. (See page 74).

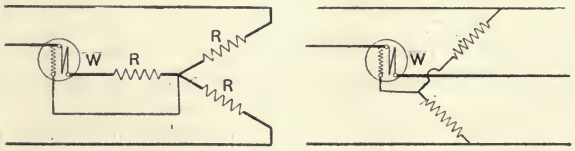
The method just given, although simple in use for occasional measurements, or when arranged with a throw-over switch, has the disadvantage that it requires two readings to be taken and to be added together. It is therefore not a direct reading method suitable for switch-board use.

The next method to be given is suitable for permanent connection in the circuit for obtaining direct readings.

Method III.—The method consists in connecting together three non-inductive resistances for the purpose of measurement, so as to form a star system, the neutral point of which will be at the same potential as the neutral point of the generator winding, and which will consequently answer the same purpose for measurement. The wattmeter is then connected so as to read the current in one line, and the voltage from this star point to the same line. (See Fig. 154b.) The voltage applied to the volt coil of the wattmeter is consequently the same as would be generated in one phase of a star-wound generator ($= e$). Also the current coil carries the same current as would be carried by the winding of the generator in the same case ($= c$). Hence (employing the previous notation) the reading of the wattmeter will be ce, or $\frac{1}{3}$ the total power of the circuit.

Usually a wattmeter intended for permanent connection to a circuit in this manner would have its scale so calibrated that the readings would be three times the power actually producing the deflection. In this case the power of the circuit would be registered directly.

If the neutral point of the generator winding, or the neutral point of the motors or branch circuits supplied, is



(a) Circuit with neutral point available. (b) Wattmeter connected to artificial neutral point.

FIG. 154.—WATTMETER CONNECTIONS IN THREE-PHASE CIRCUIT.

available, the resistances shown in Fig. 154b are not necessary, and connections are then made as in Fig. 154a. The resistances are only required in order to create a neutral point when this is not otherwise available.

In cases where the load on the three wires is not balanced, the usual way to determine the power of the system is to insert a wattmeter in each branch circuit,

The readings of the three wattmeters must then be added together to give the total power of the circuit.

The following experiment illustrates Method II. given above.

Experiment XLV.—Determination of Power and Power-factor in a Balanced Three-phase Line.

Diagram of Connections.

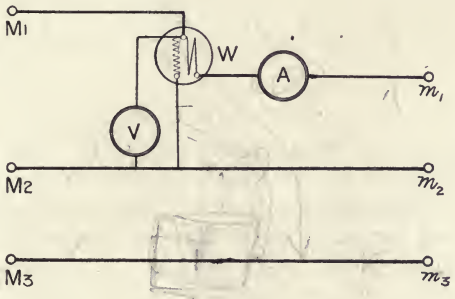


Fig. 155.

M_1, M_2, M_3	Source of three-phase alternating current.
m_1, m_2, m_3	Points connected to a balanced partly-inductive load.
W	Wattmeter.
A	Ammeter for measuring line current.
V	Voltmeter for measuring line voltage.

Instructions.—Connect three conductors from the source of three-phase alternating current to a partially loaded transformer, induction motor, or other inductive resistance.

Insert in one line wire the current coil of the wattmeter and an ammeter. Connect a voltmeter between one pair of wires, and connect the free end of the volt coil of the wattmeter alternately to the two wires not containing its current coil, *i.e.*, alternately to M_2 m_2 and M_3 m_3 in the diagram above.

Make sure that the voltage between each pair of line wires is the same, and then take readings on the ammeter, voltmeter, and two readings with alternative connections of the wattmeter for several values of the load on the circuit.

Enter the results of the readings thus :—

DETERMINATION OF POWER-FACTOR IN 3-PHASE CIRCUIT.

Line.			Wattmeter Readings.				
Current C	Volts V.	Volt-amperes $C \times V$.	First W_1	Second W_2.	Total Watts. $W_1 + W_2$.	Power Factor $\frac{W_1 + W_2}{\sqrt{3}\, C \times V}$	Idle Current. $\frac{W_1 - W_2}{V}$

Measurement in Unbalanced Circuit with Two Wattmeters.—Two wattmeters connected as shown in Fig. 156 suffice to measure the power in an unbalanced 3-phase system. The sum of the readings of the two instruments is equal to the total power transmitted. Sometimes the two volt coils are mounted on a single spindle, and the two current coils act on their respective volt coils in a single instrument. In this way the deflection of the needle is the sum of the deflections due to each pair of coils, and the instrument then reads the total watts of the circuit directly.

The proof that the sum of the readings of wattmeters W_1 and W_2 in Fig. 156 gives the true watts of the circuit may be stated as follows.

Referring to Fig. 156, it is desired to measure the power delivered to the three branch circuits, a, b, and c.

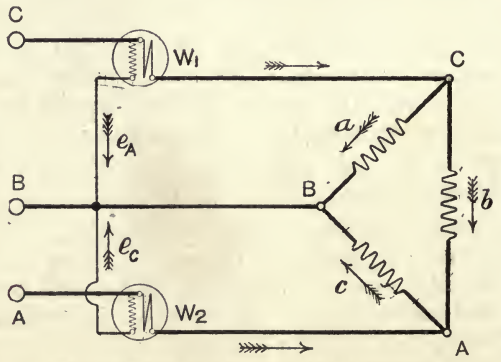


FIG. 156.—MEASUREMENT OF POWER IN AN UNBALANCED CIRCUIT.

Let e_a, e_b, e_c = the voltages of the branch circuits respectively.
c_a, c_b, c_c = the currents in the same circuits.
e_A, e_B, e_C = the voltages between the lines.
c_A, c_B, c_C = the currents in the line conductors.

Also consider the directions indicated by the arrows in the figure to be positive directions, in order to give definite meanings to the signs to be employed.

The power to be measured

$$= W = e_a c_a + e_b c_b + e_c c_c \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At the given instant it is evident from an inspection of the figure that

$$e_a = e_b + e_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{also } c_c = c_a + c_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } c_A = c_c - c_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The reading of wattmeter W_1 at any moment

$$= W_1 = e_A c_C = e_a (c_a + c_b) = e_a c_a + e_a c_b$$

Substituting from equation (2)

$$W_1 = e_a c_a + c_b (e_b + e_c) = e_a c_a + e_b c_b + e_c c_b \quad . \quad (5)$$

Similarly the reading of wattmeter W_2

$$= W_2 = e_c c_A = e_c (c_c - c_b) = e_c c_c - e_c c_b \quad . \quad . \quad . \quad (6)$$

Adding equations (5) and (6)

$$W_1 + W_2 = e_a c_a + e_b c_b + e_c c_c = W \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since the wattmeters in each case read the mean of the products of current and volts, the sum of the readings will be the mean value of the watts W as given in equation (7).

The proof just given is based on the distribution of currents and voltages in a mesh-connected circuit. A similar proof may be obtained for a star-connected circuit. This is, however, hardly necessary, since from the method of connecting the wattmeters, it is evident that they will read the power transmitted by the line quite independently of the manner in which the branch circuits may be connected. Thus the circuits *a*, *b*, *c*, in Fig. 156, might equally well be connected in star, instead of mesh, and the sum of the readings of the two wattmeters would still give the total power passing in the lines.

In the present case, as in the Method II. of measurement by one wattmeter given above, it is to be noticed

that the common terminal of the wattmeters is connected to the source of supply. Under these conditions wattmeters of the usual construction will read in the correct direction. If it is found that the reading of the instrument is reversed, the shunt coil connections should be interchanged and the readings of the wattmeter considered negative and must be subtracted from the value recorded by the positively reading wattmeter.

Armature Reactions in a Polyphase Alternator.—In a single-phase alternator the main field is only acted upon by the armature currents periodically, since these currents vary in value from zero to a maximum. The main field will consequently be alternately distorted and remain undisturbed according to the periodic variations of the armature current. In a polyphase alternator the conductors of the second phase come under the poles as the conductors of the first phase leave them. Also the maximum value of the currents in successive phases follow each other, so that the armature is never without current. In a three-phase armature the total current is always the same. The result is that the armature reactions of a polyphase alternator are much more uniform, and also on the average stronger than in a single-phase machine.

Chapter X.

The Rotary Converter.

The Rotary Converter.—The currents induced in the conductors of a direct-current generator are actually alternating currents which have given to them a single direction in the external circuit, owing to the action of the commutator. Thus, an alternator with rotating armature is essentially the same machine as a direct-current generator, except for the substitution of simple collecting rings directly connected to the armature winding, in the place of a commutator of which the segments are connected to a great number of separate conductors.

A closed circuit armature may be provided with both commutator and slip rings, and may then be used alternatively as a direct or alternating current generator. It may also be used to give simultaneously direct and alternating currents. A more frequent application of a machine constructed in this way is to drive it as a motor by means of current of one kind, and to take current of the opposite kind from the armature winding. In this way the machine acts as a motor-generator, with the important difference from the true motor generator, that the same armature winding receives the driving current and gives out the generated current.

As in the case of direct-current motor-generators, the fact that both generator and motor currents circulate round a single armature core renders the resultant armature magnetic reaction very small. This is due to the motor and generator currents always being in opposite directions, and consequently exerting magnetising forces which approximately neutralise each other.

In the rotary converter a further advantage is secured by the fact that the individual conductors carry only the difference between the two currents which they receive as motor conductors and give out as generating conductors.

The value of the current carried by any conductor consequently varies with the position of the armature, and goes through a complete cycle of changes as the conductor passes from one pole to the next similar pole.

The *average* amount of current flowing in a given conductor will further depend upon its position relative to the connections to the slip rings, since the whole of the alternating current must pass to or from the external circuit through the conductors situated next to these connections.

Thus, the heating of the conductors of a rotary converter is not uniform, but is less than for either a motor or generator rated for the same current.* For the reasons above stated it will be greater in those parts near to the connections to the slip rings.

The rotary converter may also be looked upon as a sort of rotating commutating machine through which the current passes, being changed from alternating to continuous, or vice versâ, in the process. This point of view is in some ways not so satisfactory as the one first suggested, since during certain periods the current given out by the armature is greater than the current supplied to it, and in this case it is obviously difficult to consider the input and output current as being identical.

Losses in the Rotary Converter.—As in the case of all other types of transformers, there is a certain loss which is independent of the output of the machine. This is, in the case of the rotary converter, the power required to excite and drive the converter, and may be called the no-load loss. The losses which depend on the load are almost entirely those due to the resistance of the armature conductors. These losses, except in the case of the single-phase converter, are smaller than in the case of any other type of generator or direct-alternating transformer for a given load current.

Ratio of Transformation.—The armature voltage of a direct-current machine is given by the following formula :—

$$E = \frac{N\,n\,Z}{60 \times 10^8}$$

E = Armature voltage.
N = Number of armature conductors.
n = Revolutions per minute.
Z = Number of magnetic lines passing from N poles to S poles.

* This is the case for a given a.c. current. With a single-phase converter the a.c. current is almost 1½ times the d.c. current, and the heating is greater than for a d.c. generator of the same d.c. output.

The virtual voltage of a closed circuit armature giving alternating current is (see page 194.)

$$E = \frac{N\,n\,Z}{60 \times 10^8} \times \frac{1}{\sqrt{2}}$$

assuming the wave form to be a sine curve, as is usually approximately the case with a distributed winding.

Hence, if we neglect the losses in the armature conductors, the ratio of direct to virtual alternating single-phase voltage is $1 : \frac{1}{\sqrt{2}}$, *i.e.*, 1 : ·707.

Thus, if the rotary converter is supplied with 100 volts direct current, it will give out 70·7 volts alternating. If supplied with 100 volts alternating, it will give out 141·4 volts continuous current. In either case, the *maximum value* of the alternating voltage is the same as the voltage of the direct supply, and the virtual value is the maximum value $\div \sqrt{2}$, if the wave form of the alternating current is a sine curve.

The ratio of direct to alternating current is the inverse of the ratio of the voltages. This follows from the fact that the input and output watts must be equal (except for the incidental losses in the machine), and hence, if we postpone the consideration of the effect of lagging or leading currents,

Volts (direct) × amperes (direct) = volts (alternating) × amperes (alternating).

But it has just been shown that

Volts (direct) = volts (alternating) × 1·414.

Hence,

Volts (alternating) × 1·414 × amperes (direct) = volts (alternating) × amperes (alternating);

Or,

Amperes (alternating) = 1·414 × amperes direct,

i.e., the (virtual) alternating current (with sine wave form) is nearly 1½ times the value of the direct current supplied to, or taken from the commutator.

Polyphase Rotary Converters.—A single-phase two-pole converter will have two slip rings connected to conductors situated at opposite points on the armature. If the machine has a number of poles, the slip rings will each have as many connections to the conductors as there are pairs of poles. The angular pitch between two successive

conductors to one ring will be equal to the angle between one N pole and the next N pole. The total number of connections from the armature will thus be the same as the number of the poles.

Assuming the case of a two-pole converter fed from the direct-current side to give a single-phase alternating current, the armature will have two connections to the slip rings which will in two positions in every revolution receive the full voltage given to the commutator. The value of the alternating voltage produced will consequently have a maximum value equal to the direct current voltage, and a virtual value of $\frac{1}{\sqrt{2}}$ or ·707 times this voltage.

In the case of a two-pole three-phase converter the conductors connected to slip rings will be situated 120° apart.

The maximum voltage between two rings will occur when the conductors joined to the rings are in such a position as the conductors a, b shown in Fig. 157, since the conductors between a and b will then be moving in the strongest part of the field. In this position the flux being cut by the conductors will be less than the maximum flux, which would

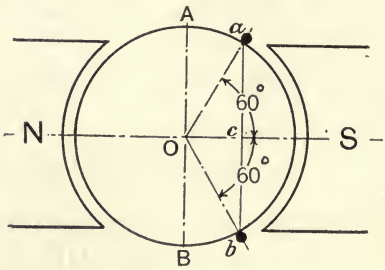


FIG. 157.—VOLTAGE OF THREE-PHASE CONVERTER.

be cut by the conductors between A and B in the ratio of ab to AB, if we assume the armature to rotate in a uniform field.

Since the angle $aOb = 120°$, the length $a\,c = O\,a \sin 60° = O\,A \sin 60°$. Thus the line $a\,b = A\,B \sin 60°$

Hence maximum voltage $= V_D \sin 60°$, where $V_D =$ voltage induced in the armature when employed for generating direct current, *i.e.*, direct-current voltage of the converter.

The virtual value of the alternating voltage $= \frac{1}{\sqrt{2}}$ maximum value $= \frac{1}{\sqrt{2}} \sin 60^\circ V_D = \frac{\sqrt{3}}{2\sqrt{2}} V_D =$ ·6123 V_D volts.

It may be stated generally that in a rotary converter with n rings, if the electromotive force between the direct-current brushes is V_D, and v_A the virtual alternating pressure between two adjacent rings

$$v_A = \frac{1}{\sqrt{2}} V_D \sin \frac{\pi}{n} \quad . \quad . \quad . \quad . \quad (1)$$

The relation between the direct and alternating currents is best obtained by equating $D\ C$ and $A\ C$ powers.

Thus, if C_D and c_A are the direct and virtual alternating currents of a converter having n rings, *i.e.*, arranged for an n-phase alternating current,

Direct current power $= V_D\ C_D$

The alternating current power per phase* $= v_A\ c_A$, and the total $A\ C$ power $= n v_A c_A$.

$\therefore\ V_D C_D = n\ v_A\ c_A$, or substituting the value already obtained for v_A,

$$V_D\ C_D = \frac{n\ c_A\ V_D}{\sqrt{2}} \sin \frac{\pi}{n}$$

$$\therefore\ c_A = \frac{\sqrt{2}\ C_D}{n \sin \frac{\pi}{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In a three-phase circuit we know the line current

$C_A = \sqrt{3} \times$ current per phase $= \sqrt{3}\ c_A$,

Also $n = 3$.

Thus we obtain the value of the line current by multiplying equation (2) by $\sqrt{3}$, and substituting for n :—

$$\sqrt{3}\ c_A = \frac{C_D \sqrt{2} \sqrt{3}}{3 \sin 60^\circ}$$

$$i.e.,\ C_A = \frac{C_D \sqrt{2} \sqrt{3}}{3 \frac{\sqrt{3}}{2}} = C\ \frac{2\sqrt{2}}{3}$$

Hence the ratio of the currents for a 3-phase circuit is

$$\frac{C_A}{C_D} = \cdot 943 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* c_A is taken to be virtual current in the armature conductors, and C_A the virtual current at the slip ring.

The ratios of direct and alternating currents and voltages may be conveniently tabulated in the following manner :—

	Direct Current.	Single Phase.	2-phase.	3-phase.	6-phase	n-phase.
Volts between slip rings	1	·707	·707	·612	·354	$\frac{\sin\frac{\pi}{n}}{\sqrt{2}}$
Amperes at slip rings	1	1·414	·707	·943	·472	$\frac{2\sqrt{2}}{n}$

Magnetisation Curve or Open-Circuit Characteristic of a Rotary Converter.—In order to ascertain the magnetic properties of the converter, and the most suitable excitation, this curve should be obtained, as in the case of a direct or alternating-current generator.

If it is possible to drive the machine by external means, the test should be carried out exactly as described in Experiment XXXIV. for an alternator. The machine is driven by a belt or coupling at a constant speed, and the excitation, which should be supplied from an external

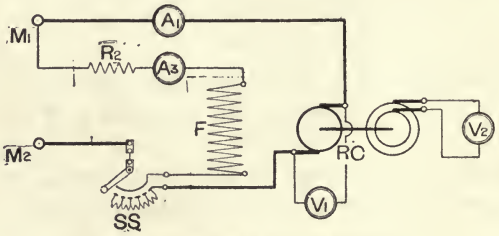


FIG. 158.—DIAGRAM OF CONNECTIONS.

source of direct current, is gradually increased from zero to a value considerably above the normal. Voltmeters should be connected to both alternating and direct sides of the armature, and must be read for each value of the excitation. After a similar series of readings have been taken with decreasing values of the exciting current, the mean curve should be plotted with amperes or ampere-turns horizontal, and direct and alternating voltages vertical.

If it is not possible to drive the converter mechanically, it must be driven electrically from the direct-current side. The experiment is then carried out as follows :—

Connect the direct-current side of the machine to a source of direct current in series with a variable resistance. Supply the field windings with direct current, also through a variable resistance, and through an ammeter. Connect a voltmeter to the alternating side of the armature.

The connections will then be as shown in Fig. 158.

Begin by giving the magnets their full excitation, and start the machine, gradually cutting out the armature resistance until full speed is attained. Read the excitation and alternating and direct-current voltage. Slightly decrease the field, and increase the resistance in the armature circuit, so as to obtain the same speed as before. Repeat the readings and take a succession of such observations with decreasing values of the excitation.

When plotted to give the magnetisation curve, the terminal voltage on the direct-current side will be higher than the true voltage, which should be considered to be the voltage transformed into alternating volts on the A.C. side, by the amount lost in armature drop and armature reactions. If the armature drop (= current × armature resistance) be subtracted from each D.C. ordinate of the curve, a curve which is practically the true magnetisation curve of the machine will be obtained.

Fig. 159 shows the magnetisation curve or no-load characteristic of a $2\frac{1}{2}$ kw. rotary converter obtained by driving the machine by a separate motor and varying the excitation. Curve I. shows the readings of a direct-current voltmeter connected to the direct-current side, and Curve II. shows similar readings obtained on the alternating side.

Curve III. shows the relation between the direct and alternating voltages. This ratio is seen to be constant over the whole range of readings taken, and to be $\frac{\text{alternating volts}}{\text{direct volts}} = \cdot 675$. This value is less than the theoretical value of ·707, which would have been found if the wave form of the alternating voltage generated had been truly sinusoidal. The normal wave form in a rotary converter is affected only by the shape of the poles, since the

armature winding is a distributed one, and consequently does not influence the form of the wave. The width of the poles is consequently the chief factor in modifying the wave form produced, and its effect is usually stated in terms of the ratio of the width of pole to the pitch of the poles (*i.e.*, the distance from centre to centre of the poles).

This ratio consequently largely determines the *virtual* value of the alternating voltage (its *maximum* value is

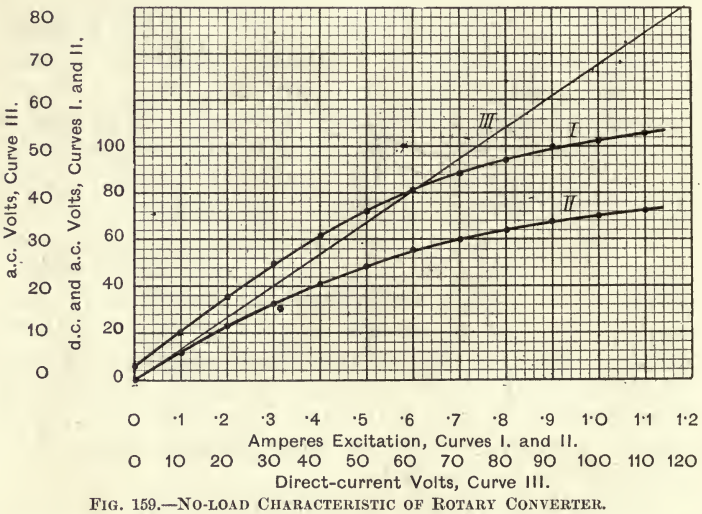


FIG. 159.—NO-LOAD CHARACTERISTIC OF ROTARY CONVERTER.

determined by the direct-current voltage) and the ratio of transformation of a rotary converter. The following table shows the influence of the pole width upon the ratio of transformation of a single-phase rotary converter. In a polyphase converter the ratio of transformation is altered in the same proportion by change in polar length.

$\frac{\text{Polar arc}}{\text{Pole pitch}}$	0·8	0·75	0·7	Sinusoidal field	0·65	0·6	0·55
$\frac{\text{Alternating volts}}{\text{Direct volts}}$	0·67	0·69	0·71	0·71	0·73	0·75	0·77

In the case for which the curve, Fig. 159, was drawn, the polar arc was rather more than ·75 of the pole pitch,

and the normal ratio of transformation, without armature drop, &c., was therefore ·675 instead of $\frac{1}{\sqrt{2}}$ or ·707.

EXPERIMENT XLVI.—DETERMINATION OF CHARACTERISTIC AND EFFICIENCY OF A ROTARY CONVERTER. (1) WHEN RUN FROM THE DIRECT-CURRENT SIDE.

DIAGRAM OF CONNECTIONS.

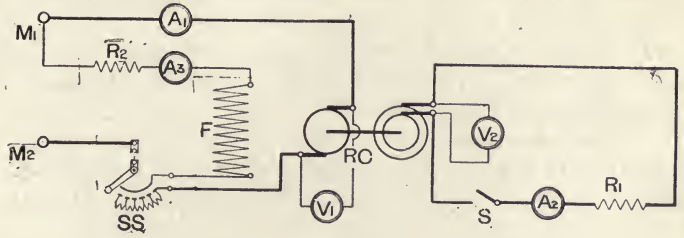


FIG. 160.

M_1M_2 Source of direct current.
$R\,C$ Armature of rotary converter.
F Field windings.
A_1 Ammeter reading direct current supplied to armature.
A_2 Ammeter reading alternating current given out by armature.
A_3 Ammeter reading exciting current.
V_1 Voltmeter reading direct voltage applied.
V_2 Voltmeter reading alternating voltage given out.
$S\,S$ Starting switch.
R_1 Variable non-inductive resistance in load circuit.
R_2 Shunt regulating resistance.
S Switch for breaking load circuit.

Instructions.—Connect the brushes of the direct-current side of the converter to a source of direct current. Connect also the field windings to the same supply in series with an ammeter and regulating resistance. Connect the terminals of the alternating side to a variable non-inductive resistance in series with an ammeter and switch. Connect voltmeters to the terminals of both the direct and alternating sides.

Start the machine in the same manner as an ordinary direct-current motor, with the switch in the alternating

circuit open. When running at normal speed and with normal excitation, read the voltage at both direct and alternating terminals, and also the direct current supplied to the armature. Keeping the direct-current voltage at the armature terminals and also the excitation constant, close the switch in the load circuit. For a series of increasing values of the load current take readings on both voltmeters and ammeters. The power in each circuit should be calculated and the results entered in tabular form as indicated below.

From the results the following curves should be plotted on a load base: (1) direct current supplied, (2) ratio of transformation, (3) efficiency.

It must be remembered that in all rotary converter experiments the position of the brushes affects the voltage ratio very much. The brushes should be kept fixed at the neutral position.

LOAD CHARACTERISTIC OF ROTARY CONVERTER.

Rotary converter No..... Type.....
Alternating voltage.....Current.....
Excitation.....Speed.....revs. per min.

Direct Current.			Alternating Current.			Ratio Alternating to Direct.		Efficiency.
Volts.	Amps.	Watts.	Volts.	Amps.	Watts.	Volts.	Current	

The efficiency is calculated by dividing the D.C. watts supplied by the A.C. watts given out. It must be remembered that the excitation losses should be added to the input watts to give the total efficiency. Since the load circuit is non-inductive, the output may be taken as the product of current and voltage on the A.C. side.

Figs. 161 and 162 show the results of such a test, which was carried out on the same 2-pole machine for which the magnetisation curves in Fig. 159 were drawn. The full-line curve in Fig. 161 shows the ratio of voltage transformation, the voltage on the direct-current side being constant at 100. The curve is seen to drop fairly rapidly at first, but afterwards to become practically a straight line. The loss of voltage indicated by the curve

is due to the same causes as those producing the drop in a separately excited generator, although the armature reactions and loss due to armature resistance will both be smaller in the present case in proportion to the current.

Fig. 162, which shows the relation between the primary direct current and the output alternating current, is also a straight line, but does not pass through zero. The ratio of current transformation appears consequently not to be a constant.

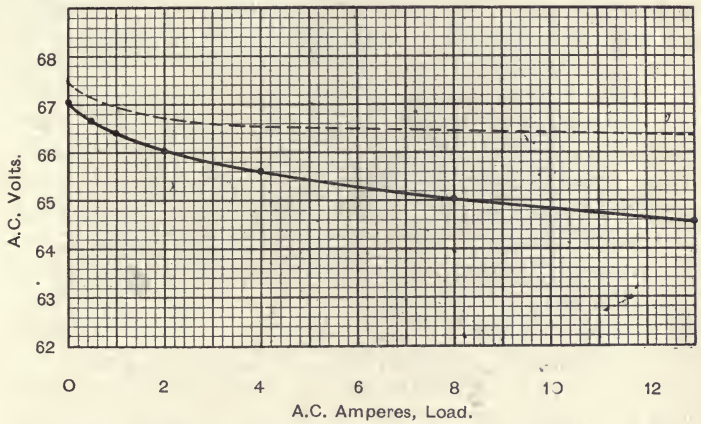


FIG. 161.—EXTERNAL CHARACTERISTIC OF ROTARY CONVERTER DRIVEN FROM D.C. SIDE.
D.C. VOLTS = 100.

The explanation of this apparent want of proportionality in the ratio of transformation is that the machine required 2·5 amps. to drive it when the alternating output as zero, as indicated by the point at which the curved cuts the vertical axis. As the power required to drive the converter when giving out current on the alternating side will be at least equal to the current taken to drive it at no load, there will always be at least 2·5 amps. of the direct current supplied which are not converted into alternating current, but which pass through the armature from brush to brush on the commutator, and are spent in overcoming the frictional losses, so as to maintain the revolution of the machine. Thus, if the power spent in driving the machine were a constant, the true ratio of current transformation would be obtained

by subtracting the constant no-load current from each reading of the direct current supplied. This can most simply be done on the diagram by drawing a horizontal line through the point where the curve cuts the vertical axis, and taking this as the horizontal axis from which to

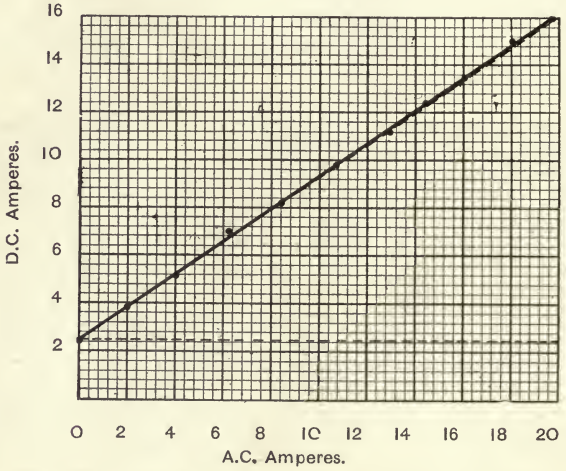


FIG. 162.—CURRENT CHARACTERISTIC OF ROTARY CONVERTER DRIVEN FROM D.C. SIDE.

measure that part of the direct current which is converted into alternating current. Proceeding in this way we find the ratio of transformation to be

$$\frac{\text{alternating current}}{\text{direct current}} = \frac{18}{12{\cdot}4} = 1{\cdot}45.$$

We can calculate from the true voltage ratios obtained from the open-circuit characteristic (Fig. 159) what the ratio of current transformation should be, since

D.C. watts = A.C. watts, and $\frac{\text{A.C. volts}}{\text{D.C. volts}} = {\cdot}675$ as already measured, hence

$$\frac{\text{A.C. amps.}}{\text{D.C. amps.}} \text{ should be } \frac{1}{{\cdot}675} = 1{\cdot}48.$$

The difference between the ratio 1·45 actually observed and the true ratio of 1·48 is due to the slight increase in driving current required to overcome the increased iron losses when the machine is loaded. These losses are probably chiefly due to eddy currents in the pole faces,

and to increased hysteresis losses in the armature core. In Fig. 162 the dotted line drawn just below the curve indicates the theoretical ratio of current transformation. The vertical distance between this line and the curve actually obtained is therefore a measure of the increased current taken in driving the machine when loaded on the alternating-current side.

Referring again to Fig. 161, showing the ratio of voltages, we can now separate to some extent the loss of voltage into its constituents. There will be a certain loss of voltage due to the direct current driving the converter, equal to (no-load current × armature resistance). In the present instance this was 2·5 amps. × ·145 ohm = ·36 volt. At no load this will be the only source of loss. As the armature begins to supply alternating current there will be an additional armature ohmic drop due to this current. The value of this drop will not be simply the product of either the alternating or direct current by the armature resistance, since the alternating current will not flow through the whole of the armature, but at certain positions of the armature will largely flow from the direct-current brushes to the slip rings. The value of the drop may be calculated from the following figures, where k is the constant by which the product (direct current × armature resistance) must be multiplied to obtain the true loss of voltage due to that part of the current which is transformed into alternating current.

Loss of Voltage in Rotary Converter.

	Single-phase.	Three-phase.	Four-phase.	Six-phase.
k	1·175	0·75	0·615	0·515

To find the drop in the armature of a rotary converter due to any load, we must use the following formula, the value of k being taken from the table above:—

$$e = k\, C_c\, R$$

Where e = drop in volts,
C_c = direct current taken or supplied,
R = armature resistance.

The dotted curve in Fig. 161 is obtained by adding the calculated ohmic drop at a number

of points to the voltage given on the observed curve. The dotted curve is seen to become almost horizontal after an initial drop from the true no-load value. The slight inclination of the straight part of the curve must be ascribed to armature magnetic reactions, or the screening effect of eddy currents in the pole faces. An example of the calculation of one point on the dotted curve will make the method of obtaining it clear.

Take the point corresponding to 12 amps. A.C. From Fig. 162 it is seen that the direct current supplied is 10·7 amps.

Of this 2·5 amps. are spent in driving the machine, and 8·2 are converted into alternating current. We therefore have for the total ohmic drop (referring to the table above), since the armature resistance is ·145 ohms, due to

driving current $2{\cdot}5 \times {\cdot}145 = {\cdot}36$ volts.
converted current $8{\cdot}2 \times {\cdot}145 \times 1{\cdot}175 = 1{\cdot}4$ „

Total drop .. .. 1·76 „

It is to be noted that the machine for which the curves are drawn was an experimental machine of small size. The losses due to resistance of the armature and friction of the machine appear unduly high in consequence. In a machine of commercial size when working under normal load, the losses due to the no-load currents would be entirely negligible, and the total loss in volts would be obtained directly by employing the formula on p. 325.

From Fig. 162, which shows the ratio of current transformation, it is seen that the ratio is a constant one, since the curve is a straight line. It cuts the vertical axis at 2·5 amps., showing that this was the current required to drive the converter when running light and supplied at the normal voltage on the direct-current side. The power taken under these conditions (= 2·5 × 100 = 250 watts in the present case) represents the power required to overcome friction, windage and no-load iron losses. The iron losses will increase somewhat under load, but approximately the no-load losses + the copper losses, which can be calculated for any load, will give the total losses at any load. The approximate efficiency of the converter when driven from the direct-current side could therefore be calculated from the no-load observation.

Effect of Varying Excitation of a Rotary Converter.—The effect of a variation in exciting current upon the behaviour of a rotary converter is different according as to the side from which the converter is driven.

When driven from the direct-current side a lessening of the exciting current will cause the machine to run more rapidly, since the driving side behaves similarly to any direct-current motor. The increase of speed will not, however, produce a corresponding increase in voltage on the alternating side, since the increase of speed is not more than enough to make up for the decrease in the strength of the field in which the armature rotates. The conductors will therefore rotate more rapidly, but in a weakened field, and, if the losses remained the same as before, it would appear that the voltage generated should be unchanged by change in excitation. It will be found in the experiment which follows that the voltage is affected to a considerable extent by changes in excitation.

EXPERIMENT XLVII.—DETERMINATION OF EFFECT OF VARIATION OF EXCITATION UPON RATIO OF TRANSFORMATION OF A ROTARY CONVERTER. (1) CONVERTER DRIVEN FROM DIRECT-CURRENT SIDE.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XLVI., Fig. 160.

Instructions.—Make connections as described in the case of Experiment XLVI., above.

With the alternating-current circuit open, vary the excitation, and for each value of the exciting current take readings of the direct and alternating voltages, the current spent in driving the machine, and the speed. The applied voltage should be kept constant during the test.

The alternating circuit should then be closed and a similar series of readings taken with one or two values of the load current. For each set of readings the resistance in the load circuit should be varied so as to maintain the current at a constant value.

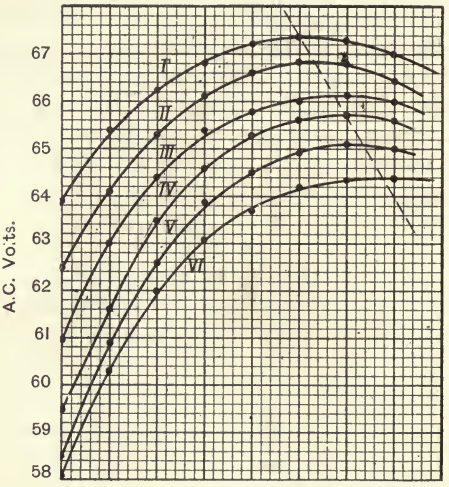
The readings should be entered in columns as shown for Experiment XLVI.

Curves showing the variation in secondary voltage should be plotted on a base of exciting current, each

curve corresponding to a definite value of the alternating-current output.

Fig. 163 shows a set of six curves obtained on the same machine as that employed for the result shown in the previous Figs., 159-162.

The direct voltage applied was maintained constant at 100, so that the alternating voltage gives directly its percentage value in terms of the direct voltage.



D.C. Volts = 100
Curve I. = 0 Amperes load.
II. = 1 ,, ,,
III. = 2 ,, ,,
IV. = 4 ,, ,,
V. = 8 ,, ,,
VI. = 14 ,, ,,

FIG. 163.—ROTARY CONVERTER DRIVEN FROM D.C. SIDE. RELATION BETWEEN A.C. VOLTS AND EXCITATION.

It will be seen that there is one value of the excitation which gives the maximum ratio of voltage-conversion, and further, that this value varies for different values of the load. The inclined dotted line passes through the maximum point of each curve, and shows the variation of exciting current which would be necessary in order that the converter should give the maximum voltage ratio for varying loads.

Fig. 164 shows the values of the direct current observed during the same experiment, at the same values of the alternating current. It is seen, as might have been expected, that within the limits taken, the currents decreased as the strength of field was increased and as the speed of the converter became less. Less power was consequently required to drive the converter with

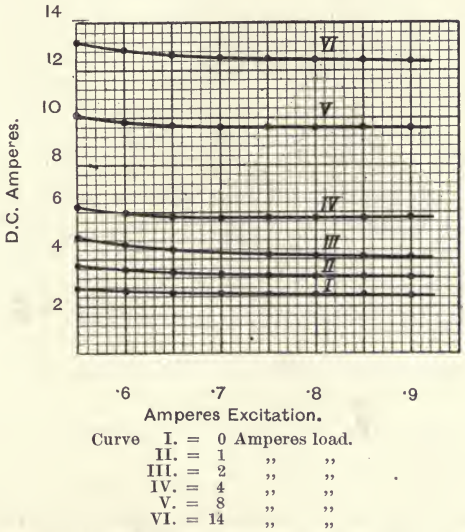


FIG. 164.—ROTARY CONVERTER DRIVEN FROM D.C. SIDE. RELATION BETWEEN CURRENT AND EXCITATION.

higher excitations. Within the limits taken in the experiment, the efficiency consequently increased with the excitation.

If the converter supplies an inductive circuit, the effect upon its voltage will be similar to that in the case of an alternating-current generator. A lagging current will tend to lower the A.C. voltage, while a leading current will tend to raise it.

Rotary Converter Driven from Alternating-current Side.—When used to convert alternating into direct currents, the converter is driven from the alternating side as a synchronous motor, and generates direct currents which are supplied to the load circuit from the commutator. This is the most usual application of rotary converters.

The field windings must be supplied with direct currents which may conveniently be obtained from the D.C. side of the converter itself.

EXPERIMENT XLVIII.—DETERMINATION OF CHARACTERISTIC AND EFFICIENCY OF A ROTARY CONVERTER. (2) WHEN RUN FROM THE ALTERNATING-CURRENT SIDE.

DIAGRAM OF CONNECTIONS.

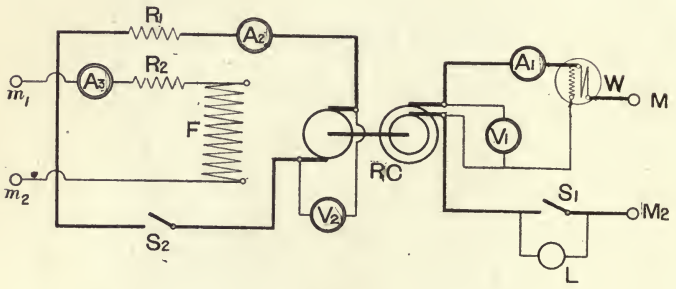


FIG. 165.

M_1 M_2 Source of alternating current.
m_1 m_2 Source of direct current.
$R.$ $C.$ Armature of rotary converter.
F Field windings.
A_1 Ammeter in alternating supply circuit.
A_2 Ammeter in direct-current load circuit.
A_3 Ammeter in field circuit.
V_1 Voltmeter reading alternating supply voltage.
V_2 Voltmeter reading direct-current voltage.
W Wattmeter reading power supplied.
S_1 S_2 Switches.
L Synchroniser.
R_1 Variable resistance in load circuit.
R_2 Field-regulating resistance.

Instructions.—Connect the alternating side of the machine to the source of alternating current in series with an ammeter, series winding of a wattmeter and switch. A synchroniser of some kind must also be provided. Connect a voltmeter and the volt coil of the wattmeter to the alternating terminals of the converter. Connect the field winding to a supply of direct current through an ammeter and regulating resistance. Connect the direct-

current machine terminals to a variable load resistance in series with an ammeter and switch.

First excite the field windings, then run up to speed and synchronise the rotary converter with the alternating supply, and then switch the armature direct on to the circuit, having previously adjusted the excitation, so that the voltage is the same as that of the supply. Usually the most convenient method of running the machine up to speed will be to use the D. C. side as a motor and to drive from a source of direct current. In order to do this, break the load circuit, shown on the left of the diagram, Fig. 165, and join the two free ends of the circuit thus obtained to the direct-current supply. On closing the switch S_2, the variable resistance R_1 may be made to serve the purpose of a starting resistance, and the speed of the machine may be regulated by varying this and the field-regulating resistance. If the direct and alternating supply voltages are not of suitable values, difficulty may be experienced in getting both speed and voltage on the alternating side to the correct value for synchronising. In such cases it is often best to run the motor up to a speed which is considerably above synchronism, then switch off the direct-current supply and switch on to the alternating circuit when the motor has slowed down to the correct speed. In this case exact voltage on switching in will usually not be of importance.

After synchronising, cut out the connection to the D. C. supply, then adjust the excitation until the motor takes the minimum current, and complete the direct-current load circuit.

First for no load, and then for a succession of increasing loads, take readings of the alternating and continuous currents and voltages, and also the power supplied as registered by the wattmeter. The speed should also be observed; it should, of course, be constant.

The results should be entered in tabular form, the power factor being calculated for each set of readings by dividing the watts supplied by the volt-amperes.

Several sets of readings should be taken, first one or two sets each with constant excitation, and then a set with the excitation varied to give the minimum current on the A.C. side.

TEST OF ROTARY CONVERTER DRIVEN FROM A.C. SIDE.

Rotary converter No. .....Type.....
A.C. supply.....volts.....amps.
D.C. output.....volts.....amps.

Speed.	Excitation.	Alternating.			Direct.			Primary Power Factor.	Efficiency.
		Volts.	Amps.	Watts.	Volts.	Amps.	Watts.		

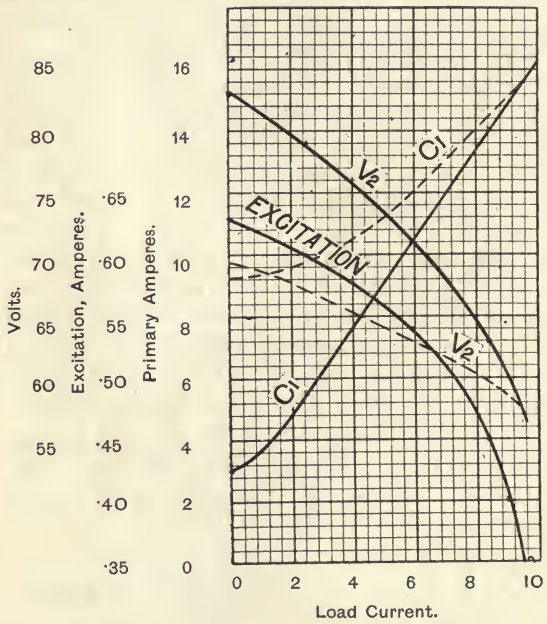


FIG. 166.—ROTARY CONVERTER DRIVEN FROM ALTERNATING CURRENT SIDE.
Primary Volts 58.
Full-line curves—Excitation for maximum power-factor.
Dotted curves—Constant excitation of ·35 amp.

Fig. 166 shows some curves obtained on the same machine from which the previous curves were obtained, showing the behaviour of a converter driven from the D.C. side.

The curves C_1 and V_2, drawn as continuous lines, show the variations of alternating current supplied, and direct

voltage given out, for various outputs of the converter, the field being adjusted throughout so as to give a minimum current. Under these conditions the power factor was found to be unity, except when the converter was working at no load, when the power factor was ·99.

The curve marked "excitation" shows the excitation supplied in order to maintain these conditions.

The dotted curves marked C_1 and V_2 show the primary current and secondary voltage for the same loads as before, but with a constant excitation of ·35 amps. A comparison of these curves shows a most important property of rotary converters, which will be further alluded to in connection with the compounding of converters. The upper curve V_2 shows that if the power factor of the supply circuit remains constant there is a considerable drop of voltage due to armature reactions as the load increases. Also, in order to maintain the power factor constant, a decrease of excitation is required.* Further, contrary to what might have been expected, with a constant exciting current, although the current supplied to the armature is much larger, the variation in voltage is much smaller. It is thus found that although the efficiency is greater with the excitation adjusted so as to maintain constant voltage, and the voltage generated is higher, the effect of thus decreasing the excitation is to increase the *variation* of voltage.

Since a diminution in excitation with increase of load produces an *increased* drop in voltage on the direct-current side, it appears at once to be likely that if the excitation were increased with the load, a *decreased* variation in voltage might result. This reasoning is supported by facts, and the idea is frequently carried out by adding a series magnet winding in the direct-current circuit, which automatically increases the ampere-turns on the field as the load increases.

Effect of Variation of Excitation.—In the case of a rotary converter driven from the alternating-current side, it is evident that no change in speed can follow from an alteration in exciting current, since the machine must continue to run synchronously at a speed depending only

* The percentage variation in voltage and excitation shown in the curves greater than would be obtained in practice in a large commercial machine.

on the frequency of the supply. It would at first appear that the direct voltage generated in the armature would consequently vary in the same proportion as the strength of the field, since the conductors always rotate at a constant speed in this field. This will be found from the experiment which follows not to be the case, for reasons given later in the discussion of the curve obtained from the results.

When driven from the alternating-current side, a rotary converter behaves in most respects similarly to a synchronous motor. It must be run up to speed and synchronised in a similar manner; and, generally, what has already been stated as to the conditions of running of a synchronous motor will apply in this case also. Thus, variation in the excitation will produce changes in the phase relations between the current and terminal voltage of the machine.

For the purpose of investigating the current taken by the converter from the line, it will be convenient to consider the direct current as being generated in a separate armature coupled to the armature receiving the alternating-current supply. Under these conditions we may consider three voltages as being in equilibrium in the alternating-current armature. These voltages are (1): The impressed voltage dependent only on the source of supply; (2) the induced back electro-motive force depending only on the strength of field for a given speed; (3) the voltage overcoming the impedance of the armature. This depends only upon the strength of the armature current.

The phase of the reactance voltage must always be such that the back electro-motive force and the impetance voltage together exactly balance the applied voltage, since the three voltages are the only ones acting. Hence, if the back electro-motive force is less than the impressed voltage, the impedance voltage must be so related in phase as to assist the back electromotive force and oppose the impressed electro-motive force, *i.e.*, it must have an energy component in phase with the back electro-motive force, and the current producing the impedance voltage will lag behind the impressed electro-motive force in phase. If the excitation of the machine is so great that the back electro-motive force becomes greater than the impressed voltage, the impedance voltage

will, on the whole, tend to assist the impressed voltage and oppose the back voltage. Under these conditions the current will lead the terminal voltage in phase. This action has been explained more fully in the discussion of synchronous motors.

EXPERIMENT XLIX.—DETERMINATION OF EFFECT OF VARIATION OF EXCITATION UPON RATIO OF TRANSFORMATION OF A ROTARY CONVERTER. (2) CONVERTER DRIVEN FROM ALTERNATING SIDE.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XLVIII., Fig. 165.

Instructions.—Connect up and start the machine as described in the case of Experiment XLVIII., page 330.

With the direct current circuit open, vary the excitation, and for each value of the exciting current take readings of both direct and alternating voltages and the

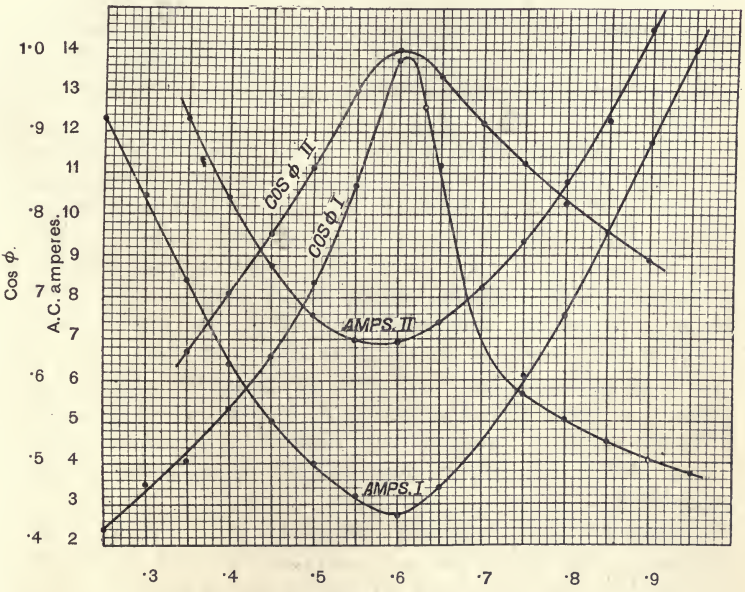


FIG. 167.—ROTARY CONVERTER DRIVEN FROM A.C. SIDE.
CURVES OF CURRENT AND POWER FACTOR.
Curves I. no load.
Curves II. 4 amperes load.
Primary volts = 58.

current supplied to the armature. The voltage and frequency of the alternating supply should be kept constant.

The direct-current load circuit should then be closed, and a similar series of readings taken for one or two values of the load current, which should be kept constant during each complete series of readings.

The results should be entered in tabular form, as shown in the case of Experiment XLVI., page 322.

Curves should be plotted showing excitation measured horizontally, and primary voltage, primary current, primary watts, and power factor vertically.

Figs. 167, 168, and 169 give some curves obtained in the manner just described on the same converter as that already experimented upon.

In Fig. 167 are shown the curves of alternating current supplied, and power factor for no load and for a load of 4 amperes, equal to about a quarter full load. The smaller the load on the converter, the greater is the current variation produced by a change in excitation, and the more steep and pointed at the bottom does the curve become. At heavy loads the current curve is comparatively flat and rounded, showing that the permissible variation of excitation is then much greater.

The power-factor curves show somewhat similar features, although the curves are inverted, since an increase of primary current corresponds to a decrease of power factor when the load is maintained constant. The no-load curve is very steep and pointed, whereas with heavy loads the curves become flat or well rounded at the top. When the converter is loaded, the power-factor curves will generally rise to 1, as shown in Curve II. At no load, on the other hand, it is not generally possible to obtain a point when the machine works non-inductively. This is due to the influence of harmonic variations in the wave form of the current which occur under these conditions.

The similarity of the curves in Fig. 167 to those already obtained with a synchronous motor is at once apparent. As explained in connection with the motor curves, the conditions to the left of the point of maximum power factor correspond to a lagging current, while points to the right indicate leading currents in the supply

circuit. In the present case it is apparent that with less than about ·6 amps. excitation the converter works with a lagging current, while with more than this excitation the current begins to lead. The importance of these V-curves in connection with compounding and automatic regulation of converters will be mentioned later.

In Fig. 168 is shown the ratio of voltage transformation at various excitations, at no-load and with a load of 4 amps.

Curve I. shows this ratio at no load, and Curve II. at about quarter load consisting of 4 amps.

The first thing that will be noticed in connection with

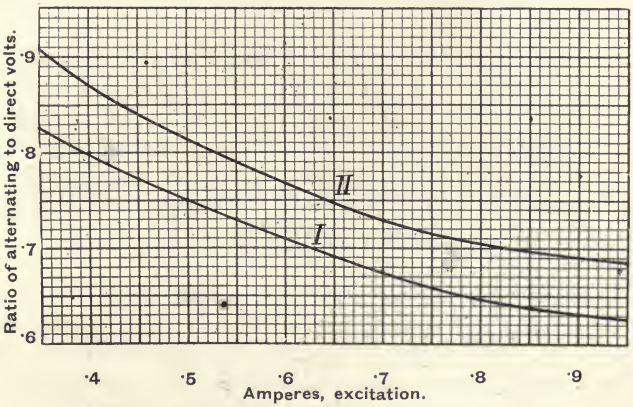


FIG. 168.—ROTARY CONVERTER DRIVEN FROM A.C. SIDE.
RELATION BETWEEN RATIO OF TRANSFORMATION AND EXCITATION.
Curve II. load 4 amperes.
Curve I. no load.
Primary volts=58.

these curves is the wide divergence they show from the true ratio of transformation, which was seen in Fig. 159 to be ·675 for this particular machine.

It must, however, be remembered that the volts lost in the armature are not simply the product of current and armature resistance, as was the case when the converter was driven from the d.c. side. The volts lost are now the product of current and armature impedance. Also the phase of the volts thus lost will be more nearly opposed to the applied voltage when the power factor of the circuit is low and the current lagging. There is thus a very great

armature drop at low excitations, both on account of the comparatively high armature current and also on account of the low power factor and lag in current.

Referring to Fig. 167, the power factor is at its maximum value at an excitation of ·61 amps., the power factor then being ·99. Under these conditions the current taken by the machine is 2·8 amps. The armature drop is then almost entirely due to armature resistance, and will be nearly in phase with the applied volts. Thus the drop will be 2·8 × ·145 volts = ·41 volts, the armature resistance being ·145 ohms. The a.c. volts actually converted are therefore 58 − ·41 = 57·59, and the ratio of conversion $\frac{57·59}{82·3}$ = ·7 (approx.), 82·3 being the actual voltage on the d.c. side.

In calculating this ratio we have not taken account of the fact that cos ϕ is not exactly unity. The difference between ·7 and the ratio of transformation ·675 previously obtained is to be attributed to this, to armature magnetic reactions, and the fact that the wave form of the alternating current supplied was considerably more "peaky" than the form of wave given by the converter when driven as a generator.

It is important to notice that the direct voltage continues to rise with increase of excitation after the point of maximum power factor is reached. This is due to the impedance voltage in the armature now becoming more nearly in phase with the impressed voltage, instead of the generator back voltage, as the current now leads the voltage in the supply circuit.

In Fig. 169 are shown the primary and secondary watts and primary current observed while the converter was supplying a constant direct current of 4 amps.

The very great variation of primary current and considerable variation in primary power taken are shown very strikingly in contrast to the nearly constant power output. The secondary watts increase slightly on account of the rise in the ratio of voltage transformation.

Compounding Rotary Converters.—It has been shown (see Fig. 166) that the *most constant* ratio of transformation is not obtained by maintaining the power factor of the circuit constant, as the fall in pressure on the secondary

side is considerable in that case. If a *constant* and not a *maximum* ratio of voltages is required, it would be possible by altering the power factor of the circuit to produce a lagging current at light loads, and thus reduce the secondary voltage and to make the power factor unity at full load, so as to get a maximum voltage in this case which might be equal to, or even higher than, the voltage at no load.

It has been shown by the results of the experiment just described how the power factor of the supply circuit

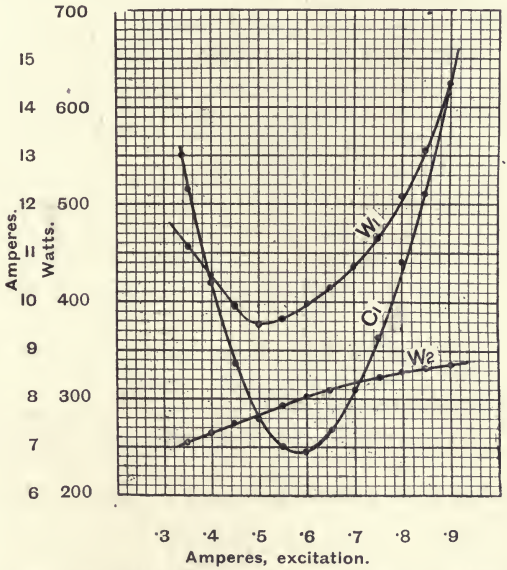


FIG. 169.—ROTARY CONVERTER DRIVEN FROM A.C. SIDE.
Primary and secondary watts and primary current for constant load of 4 amperes.

is altered by change of excitation—an increase of excitation produces an increase of lead in the current.

It would consequently be possible, by increasing the excitation of the converter in the correct proportion to the load, to obtain an approximately uniform ratio of conversion from no load to full load. The principle of this method of regulation is to be ascribed only indirectly to the change of field. The regulation is really due to change of power factor in the first instance.

By adding a series winding on the magnets, arranged to carry the direct current of the load circuit, an increase in excitation is produced proportional to the load. If this is correctly proportioned, the required increase of excitation to produce the necessary change in phase of the current may be obtained. The general action is then as follows: The field windings are so proportioned that at no load the converter is under-excited, and a lagging current is consequently produced in the supply circuit. This has the effect of lowering the output voltage, as can be seen from the curves in Fig. 168. It is arranged that the voltage thus obtained should be that required for all loads. If the converter is now loaded the excitation will increase on account of the series turns, and the current and voltage will come more nearly into phase, the ratio of conversion $\frac{\text{direct voltage}}{\text{alternating voltage}}$ being thus increased. It is thus possible to make the increase in the conversion ratio just counteract the armature drop due to increase of load, and thus to maintain a constant voltage, or, if desired, to produce a rise in voltage approximately proportional to the load, in order to make up for losses in the transmission line.

Sometimes the regulating action is assisted by the insertion of an inductance in the supply circuit, so as to produce a greater lag in the current, which can be counteracted by over-excitation of the converter fields at full load.

The power-factor corresponding to any required ratio of conversion for the rotary converter can be determined from the curves taken in the last experiment. From curves taken at full load and no load the required increase in excitation in order to maintain a constant secondary voltage can be at once obtained.

To maintain a constant voltage at all loads the excitation is usually adjusted to give unit power factor at full load. To give 10 per cent. over-compounding the fields are made to give unit power factor at about three-quarters of full load, and consequently a leading current at full load.

Chapter XI.

The Induction Motor.

General Principles of the Induction Motor.—Production of a Rotating Field.—Let the diagrams in Fig. 170 represent a ring-shaped iron core with two separate windings upon it in positions at right angles to each other. The two windings are each wound in halves on opposite sides of the ring. For briefness we will call the winding composed of the top and bottom coils the "vertical" winding, and the other the "horizontal" winding. A current sent through either winding will have the effect of magnetising the ring, producing north and south poles at points in the ring half-way between the two coils composing the winding.

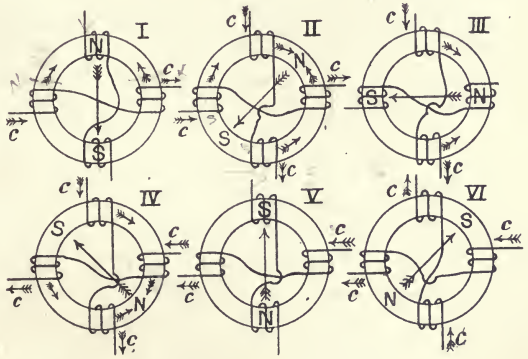


Fig. 170.—Production of a Rotating Field by 2-phase Currents.

Imagine a magnet needle to be pivoted in the centre of the iron ring so as to be free to rotate in the plane of the ring: This is represented by the long arrow in the centre of the ring.

If a current is sent through the horizontal winding only, as indicated in Diagrams I. and V., the ring will be magnetised with a north pole at the top or bottom, according to the direction of the current, and the needle will set itself at right angles to the lines of this page, as shown. If the same current is sent through both vertical and horizontal coils (Diagrams II. and VI.) the needle

OF THE UNIVERSITY OF CALIFORNIA

will take up a position 45° from its previous one. It will occupy this position whatever the strength of the current may be, as long as the same current passes through both coils; consequently, if the current in the two coils is an alternating one the field will always be formed along this axis, but the direction and strength of the magnetic field will undergo the same changes as the current. The magnet would in this case tend to set itself in the direction of this field, with its axis along a fixed line, but with the relative position of north and south poles rapidly reversed. The magnet would consequently not rotate (unless given an initial rotation by hand), but would receive rapid impulses in the nature of alternate pulls and pushes in the direction of its axis at every reversal of the current.

If the currents in the two coils do not vary simultaneously, but flow one after the other, the result will be that the magnet will tend to rotate in the manner illustrated by the six diagrams in Fig. 170.

Thus, suppose (Diagram I.) the current flows from left to right, through the horizontal coil, the needle will point downwards. On a current being started downwards in the vertical coil (Diagram II.) the needle is deflected to the left. If, now, the current in the horizontal coil ceases (Diagram III.), the needle will point horizontally to the left. As a current is started in a reverse direction, *i.e.*, from right to left, the needle inclines upwards (Diagram IV.). And so the changes may be followed further.

A step-by-step motion of rotation would thus be maintained by such a sequence of changes in the currents. The cycle of changes just suggested would be experienced if a *two-phase* current were supplied to the windings, one phase being connected to the vertical coil, and the other phase to the horizontal coil, except that in this case the changes would occur gradually, instead of step by step. It will be shown that the resultant field produced by a two-phase current flowing in two windings situated perpendicular to each other is *constant in strength*, and that its direction changes gradually and uniformly from the axis of one coil to the axis of the other. A two-phase current thus produces a *rotating magnetic field* of constant strength, which would make one complete revolution of the field formed within the two coils in the time of one period of the current.

A similar effect may be produced by means of three coils, situated at an angle of 120° apart, and supplied each with current from one phase of a three-phase system. In this case also, the current in the several coils will attain its maximum value successively, and the magnet would be attracted into a position normal to each coil in turn. The magnetic field in this case also makes one revolution in the time of one complete cycle of the current.

The rotating field is employed in the case of all induction motors, and the principle of its production is the same as that just described. The method of winding the wire upon the iron magnetic circuit of a motor is necessarily somewhat different from that shown in Fig. 170.

In an actual induction motor a laminated ring-shaped core is employed as typified by the ring in Fig. 170. In place of the pivoted magnet, however, a cylindrical iron core is used with conductors embedded in slots in its circumference. The nature of the magnetic field produced in the ring and inner core is illustrated by Fig. 171, where a single winding only is shown carrying

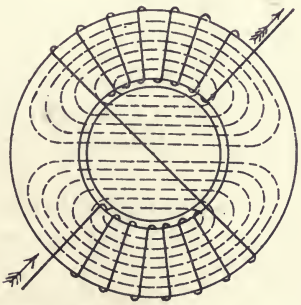


FIG. 171.—FIELD IN INDUCTION MOTOR.

current. Exactly the same magnetic field would be produced by applying the winding to the ring in any other position, so long as the winding embraces the lines forming the magnetic flux. Thus, in Fig. 172, the winding is shown as embracing the magnetic flux at the point where it comes to the air gap and passes from the ring to the inner core.

The advantage of putting the winding in the position shown in Fig. 172 is that the coil can be wound complete, and then applied to the inside of the ring, the projecting part of the winding being bent over, as indicated in the diagram.

Proof of the Constancy of the Rotating Field.—The simplest method of proving generally that a constant rotating field is produced by a polyphase current flowing in equally spaced coils is to resolve the field due to each phase into two components in two fixed directions at right angles to each other. The component fields in each of the two

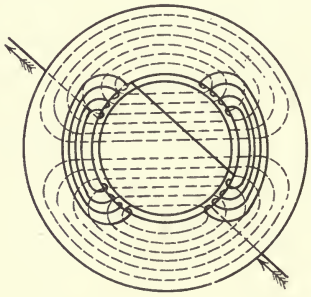


FIG. 172.—FIELD IN INDUCTION MOTOR.

directions are then added together, and the resultant of the two fields thus found is calculated as the square root of the sum of the squares of these two mutually perpendicular fields.

The method is generally applicable to currents having any number of phases. The following calculation applies to a three-phase circuit.

In this case there are three coils, situated at 120° from each other, and carrying varying currents, which are assumed to produce fields proportional at any moment to the strength of the current in the coil.

Let z_1, z_2, z_3 = the number of lines induced in the three coils I., II., and III., at the instant under consideration.

Z = max. value of the number of lines induced in the three coils in rotation.

Z_r = number of lines forming the resultant field.

The values of the fields due to the several coils may be written in the form $Z \sin \theta$, since they vary harmonically in the same manner as the current, and attain a maximum value Z.

Hence, at the instant under consideration, they will have values as follows, where θ can be chosen to have any value from 0 to 360°.

$z_1 = Z \sin \theta$ in the direction $0 Z_1$ in Fig. 173.

$z_2 = Z \sin (\theta - 120)$ in a direction $0 Z_2$.

$z_3 = Z \sin (\theta + 120)$ in a direction $0 Z_3$.

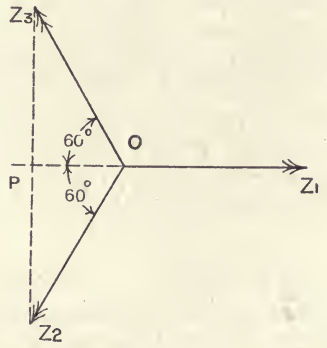


FIG. 173.—THREE MAGNETIC FLUXES DUE TO 3-PHASE CURRENT.

Resolve each of these fields in two directions respectively parallel and perpendicular to the direction of the field of coil I.

From Fig. 173 it is clear that the components are

	Horizontal.	Vertical.
for z_1, . .	$0 Z_1$ and	*zero*
„ z_2, . .	$0 P$ „	$P Z_2$
„ z_3, . .	$0 P$ „	$P Z_3$

Let us first evaluate the **horizontal** components :—

for z_1, $0 Z_1$ $= Z \sin \theta$.

„ z_2, $0 P = 0 Z_2 (- \cos 60°)$

$= Z \sin (\theta - 120°) (-\cos 60°) = -\frac{1}{2} Z \sin (\theta - 120)$.

„ z_3, $0 P = 0 Z_3 (- \cos 60°)$

$= Z \sin (\theta + 120) (-\frac{1}{2}) = -\frac{1}{2} Z \sin (\theta + 120)$.

By addition we get : $OZ_1 + OP + OP = Z \sin\theta - \frac{1}{2}\sin(\theta - 120) - \frac{1}{2}\sin(\theta + 120)$.

$= Z\{\sin\theta - \frac{1}{2}[\sin(\theta + 120) + \sin(\theta - 120)]\}$

$= Z\{\sin\theta - \frac{1}{2}\, 2\sin\theta\cos 120\}$

$= Z\{\sin\theta + \frac{1}{2}\sin\theta\}$ $\because \cos 120 = -\frac{1}{2}$.

$= \frac{3}{2} Z \sin\theta$.

On evaluating the vertical components we get :—

for z_1, *zero* *i.e.*, 0.

„ z_2, $PZ_2 = OZ_2(-\sin 60°) = -Z\sin(\theta - 120)\frac{\sqrt{3}}{2}$

„ z_3, $PZ_3 = OZ_3 \sin 60° = +Z\sin(\theta + 120)\frac{\sqrt{3}}{2}$

Adding,

$$0 + PZ_2 + PZ_3 = Z\left\{\sin(\theta + 120) - \sin(\theta - 120)\right\}\frac{\sqrt{3}}{2}$$

$$= Z\frac{\sqrt{3}}{2}\, 2\cos\theta\sin 120$$

$$= Z\sqrt{3}\cos\theta\frac{\sqrt{3}}{2}$$

$$= \frac{3}{2} Z\cos\theta.$$

The three fields are thus equivalent to two fields at right angles to each other, whose values are :

$$\frac{3}{2} Z\sin\theta \text{ and } \frac{3}{2} Z\cos\theta.$$

The resultant field formed of these two will be :

$$Z_r = \frac{3}{2} Z\sqrt{\sin^2\theta + \cos^2\theta} = \frac{3}{2} Z.$$

Hence the resultant field is constant in value and equal to $\frac{3}{2}$ the maximum value of each field taken singly.

Further, if α is the angle which this resultant field makes with the vertical

$$\tan\alpha = \frac{\text{sum of horizontal components}}{\text{sum of vertical components}} = \frac{\frac{3}{2} Z\sin\theta}{\frac{3}{2} Z\cos\theta} = \tan\theta.$$

hence $\alpha = \theta$, and the angle of the resultant field will vary at the same rate as the phase of the separate fields, and the resultant field will complete one revolution in the

time of one period of the alternating current, rotating with constant angular velocity.

It may be shown that if the system is that of a p-phase current, the magnitude of the resultant field would be $\frac{p}{2}$ × maximum strength of each individual field.

With regard to the direction of rotation of the field, it is evident that it will revolve in the direction in which the current in the successive coils attains its maximum value. For instance, if after the current in coil I. has its maximum current, the current in coil II. increases to its maximum value, the field will rotate in the direction from I. to II., and then to III., &c. If, on the other hand, the current in coil III. reached its maximum value next after coil I., the direction of rotation would be I., III., II., &c. Thus by interchanging the connections of coils II. and III. the direction of the rotation of the field is reversed. Consequently in a three-phase motor supplied with current at three terminals, the direction of rotation will be reversed by interchanging the connections of any two terminals to the supply circuit.

Rotation of Armature of Induction Motor.—It has been explained in the previous paragraph that the effect of polyphase currents passing through a polyphase winding on a circular magnetic field system is to produce a rapidly rotating pole, so that the result is similar to the rotation of a permanently magnetised ring with poles produced at fixed and equal intervals along the circumference.

In an induction motor, the field system surrounds an armature consisting of an iron core carrying a number of conductors round its circumference. These conductors are usually all connected together, so as to form a number of complete electrical circuits. The lines of force produced by the exciting current supplied to the ring pass from the circular field into the armature or rotor, and by this means complete their path to the nearest magnet pole of opposite polarity, as indicated for the case of a single pair of poles in Fig. 172. In passing from magnet to armature, and again from armature to magnet, the lines of force cross the layer of armature conductors. If the armature remains stationary, the revolving poles will generate a rapidly alternating electromotive force in

the armature conductors in the same way as the revolving field of an alternator induces currents in the conductors of its armature.

If an alternator armature carries no current, the electromotive force induced in the conductors exerts no retarding force on the field. When the armature circuit is closed, so that the conductors have a current formed in them, there is at once set up a force tending to retard the revolving field and to reduce the speed of the relative motion of armature and field. This force increases in direct proportion to the current carried by the armature conductors.

In the induction motor exactly the same action occurs, and, if the electromotive force generated in the armature conductors is allowed to produce a current, this current will produce a force acting on the rotating field opposing the relative motion of the field and armature, and tending at the same time to retard the rotating field, and to cause the armature to revolve with it.

Since the field always rotates at a fixed speed, depending only on the frequency of the current supplied to it, and the armature is mounted on a shaft, the force cannot retard the field, but may produce a revolution of the armature in the same direction as that of the field. The force producing the motion of the armature of an induction motor is, consequently, essentially the same as that which is sustained by the stationary armature of a revolving-field alternator, and its value is calculated in a similar manner.

Thus the torque of the motor is proportional to the product (field strength) × (armature current).

In an induction motor the armature is usually called the *rotor*, since it is the rotating member, while the circular magnet system is called the *stator*, since it forms the stationary part. These terms will in future be used.

Production of Current in Rotor Winding.—The diagram (Fig. 174) indicates the outline of the stator and rotor of an induction motor, a few conductors being shown to indicate the rotor winding. The diagonal arrow indicates the direction of the rotating field at the moment under consideration, and the curved arrow shows the direction of its movement.

Applying Fleming's rule for the production of a current (see Fig. 1, page 3), it will be seen that the direction of the current in the conductors in the upper part of Fig. 174 will be away from the reader (as indicated by the crosses), while in the lower conductors the current will flow towards him. The direction of the lines of force set up in the immediate neighbourhood of the conductors by these currents is shown by the small circular arrows. The

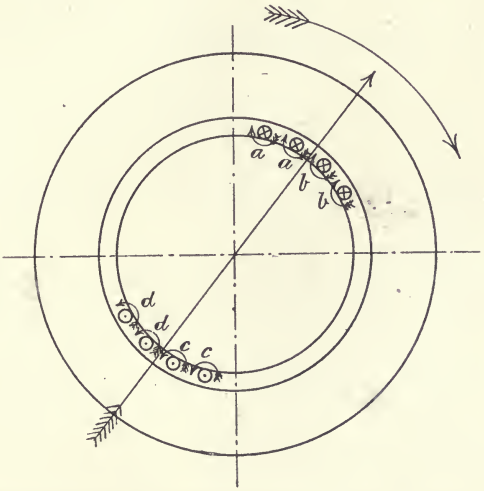


FIG. 174.—INDUCTION OF CURRENTS IN ROTOR ARMATURE.

direction of these arrows shows that immediately in front of the rotating lines of the main field the induced field in the rotor is in the same direction as, and consequently repelled by, the main field, while behind the main field the lines are oppositely directed, and are attractive.

The strength of the current in the rotor conductors is given by the usual rule—

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Impedance}}$$

$$\text{E.M.F.} = \frac{\text{rate of cutting lines of rotating field}}{10^8}$$

The impedance depends upon the resistance of the conductors, the method of connecting them together, and the self-induction of the rotor winding.

If the armature is stationary, the lines of the rotating field will cut each of the conductors $2n$ times per second (where n = frequency of current supplied to the motor), since the lines each cross the layer of conductors twice in opposite directions. If, however, the armature rotates in the same direction as the field n^1 times per second, the field will only be cut $2(n - n^1)$ times per second by each conductor.

Consequently, with a constant rotating field the electromotive force induced in the rotor conductors is proportional to the *difference in speed* of rotating field and rotor.

The difference between the speed of the conductors of the rotor and of the rotating field is called the *slip* of the motor, and is measured either in terms of revolutions per second or as a percentage of the speed of the rotating field.

Thus in a 2-pole motor : slip* $= n - n^1$

Where n = periodicity of current.

= revs. per second of rotating field.

n^1 = revs. per second of rotor.

The slip of a multipolar motor is $\frac{n - n^1}{p}$ where p = number of pairs of poles, and the synchronous speed of such a motor is $\frac{n}{p}$.

Thus the electromotive force induced in the rotor conductors by a constant rotating field would be a maximum when the rotor was stationary, *i.e.*, with a slip equal to n, and would be zero with the rotor revolving synchronously, *i.e.*, at the same rate as the field and slip $= 0$. A curve representing the dependence of induced rotor voltage upon speed would be a straight line, as in the case of an alternator or direct-current dynamo.

The wave-form of the voltage thus induced in the conductors of the rotor must depend on the distribution of the magnetic field in the air-gap between stator and rotor in which the conductors rotate. Usually, the field is so distributed that no serious error is introduced by assuming that the electromotive force produced is

* Sometimes the slip is defined as being equal to $\frac{n - n^1}{n}$, *i.e.*, the ratio of the difference of speeds to the frequency. This would be better defined as the "fractional slip," or, when multiplied by 100, as the "percentage slip."

sinusoidal, *i.e.*, the curve representing its variation is a sine curve.

If, for the present, the effect of magnetic leakage is neglected, we may say that this electromotive force will give rise to a current whose magnitude =

$$\frac{\text{E.M.F.}}{\text{resistance of rotor winding}}.$$

The direction of the induced currents is indicated on Fig. 174.

These rotor currents will produce a magnetic field in the rotor itself. The direction of this field is seen from the figure to be perpendicular to that of the rotating field produced by the stator. As in the direct-current motor,

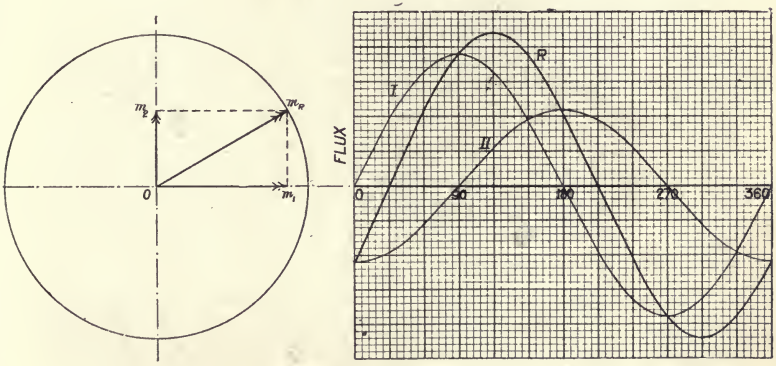


FIG. 175.—COMPONENT AND RESULTANT FLUXES IN MOTOR ARMATURE.

the actual field in the air-gap in which the conductors move is the resultant of the magnet (or stator) field, and the armature (or rotor) field. In this case also for a 2-pole motor the armature field will be perpendicular to the field of the stator. Consequently, the direction of the resultant field which acts upon the rotor conductors is intermediate between the directions of the main field and cross magnetising field. The resultant field will be of constant magnitude when the speed and load of the motor are constant, since the rotating field of the stator and the currents induced by it in the rotor are both constant when the motor speed is constant, and since both of these

quantities change their direction with the velocity corresponding to the alternating current supplied. It must be remembered that this resultant field rotates only with the velocity $n - n^1$ (= the slip) relatively to the conductors on the rotor.

The nature of the change of field experienced by each conductor of the rotor may be represented in the manner shown in Fig. 175, where the vectors Om_1 Om_2 represent the portions of the resultant field due respectively to the stator and rotor currents, and Om_r is the resultant rotating field. The wave form of the E.M.F. induced by these fields in the conductors is shown by the curves annexed.

These curves may be considered to represent two distinct relations in the rotor. (1) They represent the variations of the E.M.F. in each conductor of the armature, and consequently the current in an inductionless armature. (2) They also may be taken as representing at a given instant the strength of the field round the air-gap of the motor, and, consequently, the distribution of electromotive force and current in the conductors. The same diagram thus represents at a *fixed point* the variation with time of electromotive force and current, and at a *fixed time* the variation as regards position of the voltage and current induced in the rotor winding.

Thus the direction of the resultant line Om_r represents the *direction* and magnitude of the resultant rotating field relative to the two components, and also the magnitude and *phase* of the E.M.F. induced by the rotating field relative to the E.M.F. which would be produced by either component field acting alone.

Heating of Rotor and Turning Moment.—The induction motor consists primarily of two parts, viz., the means for obtaining a rotating field called the stator, and a rotor or armature, carrying conductors, which are cut by the rotating field and have currents induced in them.

In its essentials, it thus corresponds to the usual types of direct or alternating current generator.

In the case of an ordinary direct-current dynamo, the turning effort which has to be applied to the armature in order to produce its rotation, is the exact equivalent of the electro-magnetic reactions of the armature currents upon the field.

The power spent in driving the armature measured in mechanical units = $2\ \pi\ n\ T$, where n is the speed of rotation and T is the torque applied.

The electrical power to which this is equivalent = power output of the dynamo = $C^2\ R$ where C is the current of the armature, and R is the total resistance of the armature and external circuit. Thus we may write

$$2\ \pi\ n\ T\ =\ C^2\ R.$$

In this equation we may suppose only absolute C.G.S. units to be employed, so as to avoid multiplying by constants.

In the case of the induction motor a similar condition exists. If each conductor has induced in it a current c, and if there are in all N conductors on the rotor, each with a resistance r, in the short-circuited armature, the total power developed due to the relative rotation of the field and armature will be $N\ c^2 r$. This power must be the equivalent of the mechanical power exerted by the rotating field upon the armature as in the case of the direct-current armature just discussed. Consequently, in this case also we may write

$$2\ \pi\ n\ T\ =\ N\ c^2 r\ .\ .\ .\ .\ .\ .\ .\ (1)$$

Where n is the relative speed of the rotating field and the rotor, *i.e.*, the slip, and T is the torque exerted upon the rotor by the field of the stator.

In the case of the rotor conductors, however, the currents are not constant, but are alternating currents varying according to the E.M.F. induced by the rotating field.

In order to determine the value of these currents consider Fig. 176, in which the circle represents the circumference of the rotor, and the line $O\ X$ indicates

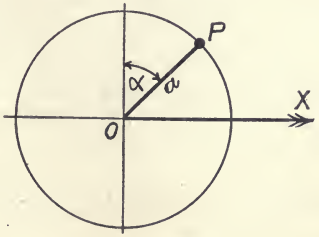


FIG. 176.

the direction of the main flux at the instant under consideration. The rotor is assumed to have no magnetic leakage, and the circuits formed by the conductors are consequently without self-induction.

P is the position of a conductor, $O\ P$ making an angle α with the vertical

B = maximum field density.
b = field density at P.
s = slip of rotor.
a = radius of rotor.
v = rate of motion of field relatively to conductors = $2\ \pi\ s\ a$.
l = length of rotor core.
N = number of rotor conductors.
r = resistance of each conductor.
c^1 = current induced at given moment in conductor at P.
c = virtual value of current induced in conductor.
e^1 = E.M.F. induced at given moment in conductor at P.
e = virtual value of E.M.F. induced in conductor.

Then

e^1 = lines cut per second by conductor = $lb\ v$.

Since distribution of field round the armature is sinusoidal and the maximum density is along $O\ X$ ($= B$).

$$b = B \sin \alpha$$

hence $e^1 = l\,v\,B \sin \alpha = 2\,\pi\,sal\,B \sin \alpha$

and $$c^1 = \frac{e^1}{r} = \frac{2\,\pi\,sal\,B \sin \alpha}{r} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The maximum value of c^1 will be $\frac{2\ \pi\ sal\ B}{r}$

Consequently the virtual current = $c = \frac{1}{\sqrt{2}}$ (maximum value of c^1) = $\frac{2\,\pi\,sal\,B}{\sqrt{2}.\,r} = \frac{\sqrt{2}\ \pi\ sal\ B}{r.}$

Instead of B it is more useful to state the flux in terms of the total number of lines forming the rotating field ($= Z$).

The average value of b or $B \sin \alpha$ is $\frac{2\,B}{\pi}$ since the average value of $\sin \alpha = \frac{2}{\pi}$

Hence $Z = \pi a l \times$ average value of b

$$= \pi a l \times \frac{2B}{\pi} = 2 B a l$$

and we may substitute for B the expression $\frac{Z}{2 a l}$

Thus $c = \frac{\sqrt{2}\pi s a l Z}{2 a l r.} = \frac{\pi s Z}{\sqrt{2} r.}$ ... ... (3)

Power developed in rotor

$$= W = N c^2 r = \left(\frac{\pi s Z}{\sqrt{2} r}\right)^2 N r$$

But from equation (1) $W = 2 \pi s T$

Consequently, $2 \pi s T = \frac{\pi^2 s^2 Z^2 N}{2 r}$

$\therefore$ rotor torque $= T = \frac{W}{2 \pi s} = \frac{\pi s N Z^2}{4 r}$ ... (4)

This expression applies to one N and S pole only. If the motor has p pairs of poles we must write $Z p$ instead of Z and the torque becomes

$$T = \frac{\pi s N p^2 Z^2}{4 r} \quad \ldots \quad \ldots \quad (5)$$

By substituting the value for s given in equation (3) we have since $s = \frac{\sqrt{2} c r}{\pi p Z}$

$$T = \frac{\pi \sqrt{2} c r N p^2 Z^2}{4 \pi r p Z} = \frac{c N p Z}{2\sqrt{2}} \quad \ldots \quad \ldots \quad (6)$$

The equations (3) and (6) above show the fundamental relations of the quantities affecting the performance of an induction motor having no magnetic leakage. From (3) we see that the current in the rotor having a given field is directly proportional to the slip. From equation (6) it appears that the torque depends only on the product of the current and field strength in a given motor, and is directly proportional to the current in the case of a motor having a constant field.

Both of these relations are exactly analogous to the case of a direct-current motor. In the first case, the current in a direct-current motor $= \frac{E - e}{R}$* when E = voltage applied to the armature and e is the back

* See the Author's "Practical Dynamo and Motor Testing," p. 106.

voltage induced by the rotation, and R = resistance of armature. In this case of the D.C. motor

$e = N\ v\ Z$ when measured in absolute units, and

$E = N\ v^1\ Z$ when v^1 is the speed of the motor armature when running without overcoming any resistance to its motion, *i.e.*, at its *critical* speed or without *slip*.

$$\text{Thus } E - e = N\ Z\ (v^1 - v),$$

and the current may be said to be proportional to the "slip" of the armature, *i.e.*, to the difference between the actual and the critical speeds.

In the second place, the torque of a direct-current motor is directly proportional to the product of total field strength and armature current. For a direct-current machine the torque may be written in absolute units

$T = \dfrac{Z\ N\ C}{2\ \pi}$ when C is the total armature current.

If we write, as in the case of the induction motor, c as the current flowing in each conductor, we have

$$T = Z\ c\ \frac{N}{2\ \pi}\ 2\ p = \frac{Z\ c\ N\ p}{\pi}.$$

Since, then, for a parallel-connected armature

$$c = \frac{C}{2p}$$

Thus we can compare the expressions for the torque in the two cases by writing

$$T_d = \frac{Z\ c\ N\ p}{\pi} \text{ for direct-current motors.}$$

$$T_a = \frac{Z\ c\ N\ p}{2\sqrt{2}} \text{ for alternating-current motors.}$$

$$\text{Thus } \frac{T_a}{T_d} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Also in the case of the direct-current machine, the armature efficiency is inversely proportional to the slip.

For the direct-current motor

$$C\ E = C\ e + C^2\ R,$$

but $C\ E$ is the power supplied to the motor and $C^2 R$ is the power lost in heating the armature conductors, and $C\ e$ consequently represents the useful power exerted by the motor.

Consequently the efficiency $= \dfrac{\text{power exerted}}{\text{power supplied to motor}}$ $= \dfrac{C\,e}{C\,E} = \dfrac{e}{E} = \dfrac{v^1 - \text{slip}}{v^1}$ in accordance with the notation employed above. By exactly similar reasoning the power lost in the alternating-current armature is $N\,c^2 r$ and the efficiency is $\dfrac{\text{speed of rotor}}{\text{synchronous speed}} = \dfrac{v^1 - \text{slip}}{v^1}$.

Making use of these results and equation (7) we see that for the same losses in the armature the torque, and consequently the power, exerted by the induction motor is 1·11 times as great as with a direct-current motor, and that the factors upon which the torque and power depend are similar for the two cases.

The E.M.F. in the rotor conductors increases in direct proportion to the slip. Thus with constant

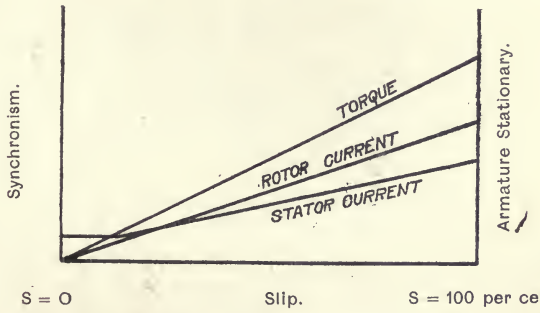


FIG. 177.—RELATION BETWEEN SLIP AND TORQUE AND CURRENTS IN MOTOR WITHOUT LEAKAGE.

field and rotor resistance, the current and the turning effort will increase in the same ratio as the slip, and curves showing the dependence of rotor current and rotor torque upon slip would both be straight lines as shown in Fig. 177.

Effect of Magnetic Leakage.—The leakage occurring in an induction motor is of two kinds, viz., that in the stator and that in the rotor.

The stator leakage field is formed of magnetic lines which do not enter the rotor, but pass across from tooth to tooth of the stator core without crossing the air gap

between stator and rotor, and so do not affect the rotor conductors.

Similarly the rotor leakage lines are formed in the rotor core and do not pass into the stator.

The leakage field consists of the magnetic lines which do not form part of the rotating field. The leakage in either stator or rotor is increased by making the slots nearly or entirely closed, since it then becomes relatively easier for the lines to pass from tooth to tooth in the same core rather than to pass twice across the air gap into the opposite core. The path of the leakage lines is illustrated in Fig. 178.

The effect of the stator leakage is that a certain number of the lines formed by the stator current have

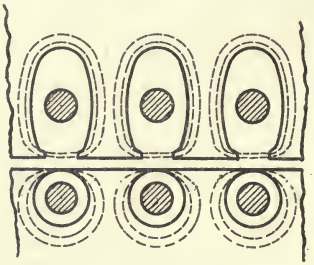


FIG. 178.—LEAKAGE LINES IN STATOR AND ROTOR.

no effect upon the rotor, and that a portion of the applied voltage is opposed by the back E.M.F. set up by this portion of the field. In other words, the leakage field of the stator appears in the form of self-induction of the stator winding. It does not produce any appreciable waste of power, but diminishes the output of the motor, since it practically has the effect of lowering the effective voltage applied to the machine.

The effect of the magnetic leakage in the rotor is somewhat more complicated. The leakage lines do not pass into the stator, and are consequently not neutralised by the stator field. On the contrary, these lines produce exactly the effect produced by the lines formed in the core of a choking coil when supplied with an alternating current, *i.e.*, they produce a back E.M.F. of self-induction in the rotor winding, and give rise to an increased apparent resistance in this circuit.

Hence the voltage induced in the rotor conductors by the rotating field will partly be spent in overcoming the self-induction of the rotor winding (*i.e.*, producing the rotor stray field), and part only will be spent in producing the currents which furnish the torque. The conditions may therefore be represented by a triangle of electromotive forces as in Fig. 179, in which the ratio of the sides

$$\frac{AB}{BC} = \frac{\text{E.M.F. due to self-induction or leakage lines}}{\text{E M.F. due to lines which cross the air gap}} = \frac{\text{leakage field}}{\text{useful field}} = \tan \phi,$$

where ϕ is the angle of lag of the current in the rotor circuit.

In this triangle CB represents the phase of the current so that it will be seen that the current lags behind the total E.M.F. of the rotor. This is an important matter,

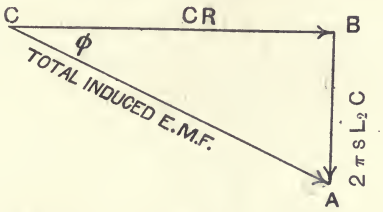


FIG. 179.—VOLTAGES IN ROTOR CIRCUIT.

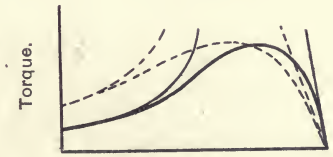


FIG. 180.—RELATION BETWEEN SLIP AND TORQUE OF MOTOR.

since the line AC represents the phase of the inducing field acting upon the conductor for which CB represents the phase of the current induced. The pull exerted upon the conductor is proportional to the product of strength of field and current in conductor. These quantities are, however, never at their maximum values at the same time, the average value of their product being $c.z. \cos \phi$, where c and z are the virtual values of the current and flux respectively. This may be proved in

exactly the same way as the average value of the power in a circuit ($c\ e\ cos\ \phi$) was determined (see page 95).

It is thus seen that the effect of the rotor leakage is two-fold, (1) the value of the current in the rotor conductors instead of being $\frac{AC}{R}$ (see Fig. 179) is only $\frac{BC}{R}$, *i.e.* the current is reduced by the factor $\frac{BC}{AC} = \cos\phi$. (2) Owing to the phase of the current not being the same as that of the main field, the *effectiveness* of the current is further diminished by the factor $\cos\phi$.

Hence the torque is reduced in amount to $\cos^2\phi \times$ (value of torque without leakage).

The calculation previously given for the value of the torque on the shaft may be employed for a motor with leakage, the final expression being multiplied by the factor $\cos^2\phi$.

Thus for a motor with leakage we must write the equations (3) and (6) for current and torque given previously in the following form :—

$$c = \frac{\pi\ s\ p\ Z}{\sqrt{2}\ r} \cos\phi \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

$$T = \frac{c\ n\ p\ Z}{2\sqrt{2}} \cos^2\phi \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Referring to Fig. 179, we may write,

$$\cos^2\phi = \frac{1}{1 + \tan^2\phi} = \frac{1}{1 + \left(\frac{\text{leakage volts}}{\text{useful volts}}\right)^2}$$

The value of the reactance voltage due to leakage is $2\pi\ s\ L_2\ C$, where L_2 is the coefficient of self-induction of the rotor circuit and s is the slip per second, which is also the frequency of the currents induced in the rotor.

Thus the value of $\tan\phi$ is $s\ \frac{2\pi\ L_2}{R}$

and we may write this in the form

$$\tan\phi = s\ k, \text{ where } k = \frac{2\pi\ L_2}{R}$$

$$\text{and } \cos^2\phi = \frac{1}{1 + (s\ k)^2}$$

Thus in the actual motor with leakage the torque is

$$T = \frac{c\ n\ p\ Z}{2\sqrt{2}\ (1 + s^2\ k^2)}$$

in which the factor $(1 + s^2 k^2)$ in the denominator shows the modification in the value of torque produced by leakage.

At light loads the slip will be very small, and the factor $1 + s^2 k^2$ will be practically equal to 1.

At very heavy loads $1 + s^2 k^2$ will be almost equal to $s^2 k^2$, since 1 will be small compared with the value of $s^2 k^2$. Also the current c in the numerator of the fraction is proportional to the slip. Hence at light loads the torque is proportional to the slip, and at heavy loads the torque is proportional to $\frac{1}{\text{slip}}$. Thus the speed torque curve begins as a straight line, coinciding with the theoretical curve corresponding to nc leakage, and ends by becoming a hyperbola as the slip of the motor increases up to the point of still-stand. The actual shape is that shown as the result of a later experiment; its general form is shown in Fig. 180.

It is important to notice that the value of $k = \frac{2 \pi L}{R}$ can always be altered by inserting resistance in the rotor circuit so as to change R. By doing this we should vary both the current and slip of the motor corresponding to a given torque, *i.e.*, a given load overcome by the shaft.

Thus it is always possible with any given load to alter the slip at which the motor will overcome this load by altering the resistance in the rotor circuit.

This is of advantage especially at starting, since with suitable variation of the rotor resistance the motor can be made to give any desired torque at low speeds. This will become more evident from the experiments which follow.

It must be remembered that in the formulæ just given only "absolute" or C.G.S., units have been used, and that consequently the torque is expressed in dyne-centimetres, and the current and voltages in units, which are respectively 10 ampères and $\frac{1}{10^8}$ volts.

The following relations may be found convenient for reference in converting the expression just obtained into other units.

To bring		divide by
Dyne-centimetres	to gramme-centimetres	981
,, ,,	,, kilogramme-metres	981×10^5
,, ,,	,, pound-feet	$13{\cdot}56 \times 10^6$

Power expressed as the product of the absolute units of force and velocity, *i.e.*, dynes × cm. per second is in ergs per second. This also holds good if the power is expressed in torque of dyne-centimetres multiplied by angular velocity in radians per second.

To bring			divide by
ergs per sec.	to watts		
,, ,,	,,	kg. cm. per sec.	,, ,, $9{\cdot}81 \times 10^7$
,, ,,	,,	ft.-lbs. per sec.	,, ,, $1{\cdot}356 \times 10^7$
,, ,,	,,	horse-power	,, ,, 746×10^7

The discussion given above is intended to give an idea of the relation between the various factors in the induction motor in order to explain its behaviour. The formulæ are therefore not put in the form in which they would be used by a designer, and the torque or power of a motor would not in the ordinary course be obtained from the expressions given. The complete mathematical determination of the leakage coefficient and other factors involves the consideration of too many matters to be suitable for discussion here.

Measurement of Slip.—Since the slip of a motor is only a small percentage of its speed, it cannot be accurately measured by a tachometer of the usual type, as the readings of such an instrument cannot usually be read with a closer approximation than about 1 per cent. on account of the large range of the scale. Even this degree of accuracy can hardly be relied upon when two belt-driven tachometers are employed—one on the generator and one on the motor. As the slip of an induction motor usually varies between 2 and 5 per cent. of its speed, it is evident that other methods of measurement must be employed.

The simplest method is to apply simultaneously a speed counter to the shaft of both motor and generator for one minute. If both machines are in the same room, two observers can easily signal to each other, so as to obtain exact coincidence in the times of reading. If the generator is at a distance, it is often possible to run a temporary pair of wires for a signal bell or lamp from

one to the other. In such a case a code of signals for "ready," "on," "ready," "off," must be arranged, the warning "ready" before either applying or removing the counter being essential for reliable working.

In such a case, if n_1 and p_1 are the revolutions per second and number of poles of the generator, and n_2 and p_2 are the revolutions per second and number of poles of the motor, the slip is given by

$$S = n_1 p_1 - n_2 p_2,$$

or when stated as a percentage of synchronous speed

$$S \text{ per cent.} = 100 \frac{(n_1 p_1 - n_2 p_2)}{n_1 p_1}$$

In some cases it may be possible to arrange a gong on the shaft of the generator and a hammer on the shaft of the motor in such a way that the hammer strikes the gong each time the generator shaft makes one revolution more than the motor. If the two machines have the same number of poles, the number of blows per minute divided by 60 equals the slip of the motor. If the number of poles is not equal, the method is not applicable, since the sounds would be too rapid to be distinguishable.

Instead of a bell and hammer, discs may be fixed to the shafts of the machines with sectors cut out of them, bearing a definite ratio to the number of poles. By viewing the discs in line with one another, the number of times one disc makes one more revolution than the other can be seen.

Another simple method is to insert an ammeter in one of the phases of the rotor circuit. The needle of the ammeter will vibrate as the currents induced in the rotor vary. The number of deflections may be counted, and will equal the number of times the rotor makes a half-revolution less than the rotating field. A Morse recorder may be employed to mark the number of changes of current on a strip of paper. By employing a moving-coil direct-current ammeter, the number of vibrations will be equal to the rotor slip, since the needle will only respond to currents in one direction. It will consequently be easier to count the vibrations than if an alternating-current instrument were used, which would give a deflection for the currents induced in both directions. It is hardly possible to count much more than

150 vibrations per minute, which, with a frequency of 50, would correspond to a maximum slip of 2·5 per cent.

If the rotor is of the squirrel-cage type, an ammeter cannot be inserted in its circuit. In this case a revolving contact maker may be employed. Any form of contact maker described in Chapter IV. might be employed, but a small stud on the rim of the motor coupling will serve the purpose. In this case one of the mains is connected to the shaft of the motor and another to a light spring (which is touched by the stud as it revolves) in series with a dead-beat voltmeter. The voltmeter will then show a full deflection each time the moment of contact coincides with a maximum voltage in one direction. If there are as many contact points as there are pairs of poles, the deflections of the voltmeter will equal the slip. If only one contact is employed, the deflections must be multiplied by the number of pairs of poles.

Another method of determining the slip when this is not great is to employ an alternating arc lamp. This lamp is supplied with current from the same source as the motor, and the light given out by it will flicker with the velocity of the alternations of this current. If white sectors are painted on the motor pulley, or on a piece of card attached to the motor shaft, these sectors will rotate with a speed slightly less than that of the flickering of the arc. At each period of maximum illumination the sectors will consequently be slightly behind the position which they occupied at the last time of illumination. The sectors will thus appear to rotate slowly backwards, and will make the same number of apparent revolutions as the slip of the motor, if the number of segments is equal to the number of pairs of poles.

A very simple device which may be employed to measure the slip in an induction motor provided with external starter is a small pivoted compass needle, provided with stops to prevent the needle swinging through more than a small angle.

This needle is placed immediately above, or below, one of the connections to the rotor starting resistance, this wire being placed in such a direction that the needle normally points along the wire, so that the alternating currents induced in the rotor will deflect the needle. Thus, as in the case of the ammeter just described, the

number of deflections in either direction in one minute, divided by number of pairs of poles, gives the value of the slip in revolutions per minute.

The slip at any load can be approximately predetermined without actual measurement from a knowledge of the rotor current and resistance, as given in the following formula for a 3-phase rotor :—

$$\frac{S}{v} = \frac{3\,c_2^2\,r_2}{W + 3\,c_2^2\,r_2}$$

the symbols having the following significations :—

S = Slip in revolutions per minute.
v = Speed of rotating field in revolutions per minute.
c_2 = Current in rotor winding per phase.
r_2 = Resistance of rotor winding per phase.
W = Output of motor in watts.

The above formula may be arrived at from the following considerations :—

$$\text{The torque on a generator armature} = \frac{\text{watts output}}{\text{speed relative to field}}.$$

The watts generated in the rotor armature are $3\,c_2^2\,r_2$, and hence torque on motor shaft $= \dfrac{3\,c_2^2\,r_2}{2\pi S}$.

Also torque exerted by the rotor is equal to $\dfrac{\text{output of motor}}{\text{speed of rotation of shaft}} = \dfrac{W}{2\pi v_2}$ where v_2 = speed of rotation, hence torque $= \dfrac{3\,c_2^2\,r_2}{2\pi S} = \dfrac{W}{2\pi\,v_2}$.

Adding the two ratios together we obtain

$$\frac{W + 3\,c_2^2\,r_2}{2\pi\,(v_2 + S)} = \frac{3\,c_2^2\,r}{2\pi S}$$

or since $v_2 + S = v$ we obtain

$$\frac{S}{v} = \frac{3\,c_2^2\,r_2}{W + 3\,c_2^2\,r_2}$$

as in the equation above.

Measurement of Resistance of Windings.—The measurement of the resistance is usually carried out by sending a measured direct current through the windings, and noting the drop in voltage. In 3-phase motors the resistance per phase will not be obtained by this method, since in a star-connected stator there will be two windings between each pair of terminals, and in the case of a mesh-connected winding there will be two paths in parallel between the terminals.

In order to get over the necessity for special calculations for each case, the method often adopted is the following :—

The resistance from terminal to terminal between each pair of terminals is measured, the three resistances being then added together; the result when divided by two is called the "equivalent" resistance of the winding. It is evident that for a star-connected stator the equivalent resistance

$$= R = \frac{3 \times 2r}{2} = 3r$$

where r = resistance of each phase winding.

In the case of a mesh-connected winding

$$R = \frac{3}{2} \frac{1}{\frac{1}{r} + \frac{1}{2r}} = \frac{3}{2} \times \frac{2}{3} r = r$$

The special convenience of the equivalent resistance is that when multiplied by the square of the current supplied to the motor it gives the copper losses in watts directly.

Thus in the case of the star winding losses $= 3\ c^2r$, and since line current = current per phase and $3r = R$

losses $= C^2R$.

Similarly with mesh-connection phase current $= \frac{1}{\sqrt{3}}$ line current, and losses $= 3 \left(\frac{C'}{\sqrt{3}}\right)^2 r = C^2r$.

And since in this case $r = R$

We have watts lost in resistance $= C^2R$.

Speed Regulation of Induction Motors.—The speed of an induction motor depends primarily on the speed of the rotating field, *i.e.*, on the frequency of the supply. The revolutions per minute of the motor when running synchronously $= v = \frac{n \times 60}{p}$, where n = periodicity of supply circuit and p = number of pairs of poles in the motor.

The actual speed under working conditions will be slightly less than this on account of the slip, but will generally be between the speed as given above and a speed about 5 per cent. lower.

It is not generally possible to vary the periodicity of the supply current in order to regulate the speed of the motor.

In special cases, where motors are required to run at two different speeds, the number of poles of the motor is changed by altering the connections of the windings in the stator by means of a special switch. This is possible, since the number and position of the poles depend only on the connections between the windings, and not on the structure of the stator.

A resistance in the primary circuit is not an admissible method of regulation. Such a resistance produces no appreciable effect upon the speed unless it is sufficiently high to produce very great effect upon the primary voltage. This would then be not only very wasteful, but would seriously diminish the torque which the motor could exert.

By inserting resistances in series with the rotor windings, and thus practically increasing the rotor resistance, it is possible to produce a certain amount of variation in the speed. This is owing to the fact that the torque exerted by the motor is proportional to the product of the rotating field strength and the rotor current. For a given load to be overcome, this product must, therefore, remain constant. Suppose the motor to be running at a certain speed and to be driving a certain load, and that an increase in the resistance of the rotor is suddenly produced, the rotor current will at once become weaker, and the torque exerted by the motor will diminish and the speed will drop: that is, the slip will increase. This at once produces the result of an increased electromotive force in the rotor circuit and a consequent rise in rotor current. The loss in speed of the rotor will, consequently, continue until the rotor current has attained its previous value, enabling the motor to overcome the load. The possible speed variation attainable by this method may be seen from the curves given later.

Speed variation by insertion of resistance in the rotor circuit is necessarily accompanied by loss of efficiency, since the losses C^2R will increase in the same proportion as the increase in the resistance of the rotor circuit. A further disadvantage is that the current in the rotor is different for every value of the load. Consequently, for a given variation in speed, the amount of resistance inserted is also different for each load. This method of regulation is, consequently, analogous to and suffers from

the same disadvantages as the regulation of a direct-current motor by a resistance in its armature circuit, while the degree of variation which is possible is much smaller than with a direct-current motor.

A method of enabling two motors to be run either at full speed or half speed is that known as the cascade system. It will be remembered that the periodicity of the currents in the rotor circuit of an induction motor is equal to the slip. Usually this frequency is extremely small. If, however, the motor runs at a low speed, the slip will be high and the periodicity of the rotor currents may be sufficient to drive a second motor if connected to the rotor circuit of the first. In the cascade arrangement, the stator of one motor is connected to the supply, while the windings of its rotor, instead of being connected to the usual starting resistance, are connected to the stator of the second motor, the rotor of which may be either short-circuited or connected to a starting resistance. The speed of the second motor will be approximately equal to the slip of the first motor, while the speed of the first motor will be equal to the frequency of the supply minus the slip. If the motors are coupled together or compelled to run at equal speeds, we shall have the condition

$$\begin{aligned}(\text{frequency} - \text{slip of motor 1}) &= \text{speed of motor 2} \\ &= \text{slip of motor 1}\end{aligned}$$

hence the speed of both motors will be approximately equal to half the frequency of supply, and would be accurately half this speed but for the slip of the second motor.

Braking and Return of Power to Line.—A 3-phase motor is reversed by interchanging any pair of supply connections, which has the effect of reversing the direction of rotation of the rotating field.

A reversing switch is usually made in the form of a 3-pole throw-over switch, the inter-connections being so arranged that with the switch thrown over to one side the motor receives current and runs in one way, and with the switch in the opposite position the motor runs in the opposite direction.

If, while the motor is running at full speed, the reversing switch be suddenly thrown over, so as to reverse

the direction of the rotating field, the rotor will at the moment after reversal rotate in the opposite direction to the rotating field, and the slip will be practically 200 per cent., and large currents will be induced in the rotor, since this will be running with the starting resistance cut out. It is, consequently, not permissible to brake any except small motors with squirrel-cage rotors by reversing the field, although in case of serious danger it might be resorted to for larger motors.

It is not possible to brake an induction motor by disconnecting it from the line and short-circuiting the terminals through a resistance, as is frequently done with direct-current motors. As soon as the stator of an induction motor is disconnected from the supply, the rotating field ceases to exist, and no force exists to retard the motor, which will continue to rotate until gradually stopped by the friction of the shaft or gearing.

It is important to notice what happens if, while the rotor is driven by external means, the motor remains connected to the supply, and the rotating field consequently continues to rotate.

If the speed of the rotor is below that of synchronism, currents will be induced in the rotor in the usual manner, and will flow in such a direction that the motor tends to rotate with the field, and the external driving force will be less than that required to drive the motor without current. As the speed of the rotor is increased the currents induced in it will fall until at the synchronous speed no currents will be formed, and the external force required to maintain the rotation of the rotor will exactly balance the friction and other forces opposing the rotation.

If the speed is now still further increased the rotor conductors will begin to cut the lines of the rotating field in the opposite direction. This will cause rotor currents to flow in the reverse direction, and their action will be to *oppose* instead of to *assist* the rotation. Under these conditions the machine becomes a generator and supplies current to the line through the primary winding. It may thus be made to act as a brake, preventing the speed of the rotor rising much above synchronism.

No-load Curves of an Induction Motor.—No-load curves may be obtained on an induction motor under either of two conditions, viz.: (1) with the armature shaft free to rotate, so

that the motor runs (except at very low voltage) at almost the speed of synchronism, and (2) with the rotor rigidly clamped, so that it remains stationary and the slip is 100 per cent.

From the curves obtained under these two conditions several important properties of motors may be observed. The power supplied to the motor in either case is spent in internal losses because the output is nil, and the power measured will in the one case be the power spent in driving the motor with (practically) no slip, and in the other case with the maximum slip which the machine can have as a motor.

The discussion of the conclusions to be derived from the tests will be given after the method of procedure has been explained.

Experiment L.—No-load Test of an Induction Motor at Varying Voltages.

Diagram of Connections.

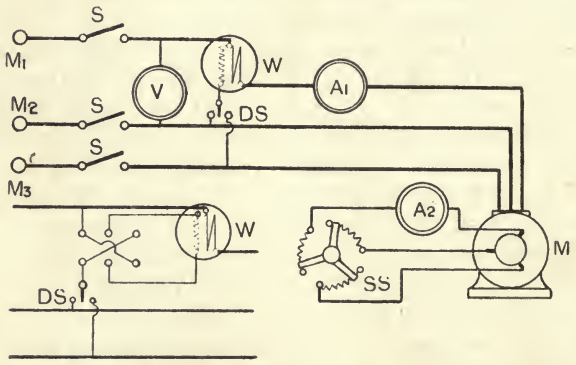


Fig. 181.

$M_1 M_2 M_3$	Source of three-phase alternating current.
M	Induction motor.
A_1	Ammeter in supply circuit.
A_2	Ammeter in rotor circuit.
V	Voltmeter reading supply voltage.
W	Wattmeter.
$S\,S\,S$	Three-pole switch.
$D\,S$	Two-way voltmeter switch.
$S\,S$	Motor starting switch.

Connections. — Connect the stator windings of the motor to the source of alternating current, inserting in one main an ammeter and the series coil of a wattmeter. Connect the shunt coil of the wattmeter to a two-way switch so that it may be connected alternately to the two mains not containing the series coil. Connect also a voltmeter across one pair of mains to read the supply voltage. Connect the rotor windings to the usual starting switch after inserting an ammeter of low resistance in one of the phases. The starting switch and resistance may be dispensed with, and the rotor may be short-circuited directly through the ammeter if the motor is always started at a reduced voltage. If the motor is of the squirrel-cage type, of course, no starter is to be employed.

Means must be provided for varying the primary voltage through a considerable range. This may be done either by inserting a variable three-phase resistance in the primary circuit, or by varying the voltage of the alternator supplying the circuit by regulation of its shunt regulator. In either case the periodicity of the circuit is to be kept constant.

In carrying out the experiment it will probably be found necessary to add a reversing switch for the shunt coil of the wattmeter, as indicated in the left-hand lower corner of Fig. 181. This necessity is owing to the fact that in throwing over the shunt coil from one main to the other the reading of the wattmeter will be found to be reversed in some cases, although not in all, since the relative direction of the readings will depend upon the value of the power-factor. If the reversing switch has to be thrown over between the readings on the two mains one value observed must be considered *negative*, and must be subtracted from the other in order to give the total watts supplied. In other cases the *sum* of the readings gives the total watts. If a three-phase wattmeter is employed, it reads the power of the whole circuit directly without any throw-over switches.

In many cases it will be found advisable to read the current and watts in all three phases, to avoid errors due to unsymmetrical winding. By means of throw-over

switches this may be accomplished without increasing the number of instruments required.

Instructions.—(1) After starting the motor in the usual way, light and with belt off, adjust the primary voltage to the full working pressure of the motor. Take readings of the stator and rotor currents, watts supplied (by readings on the wattmeter with the shunt coil connected first across one pair of mains and then across the other). Observe also the primary voltage and the slip (by any of the methods described on p. 362).

Repeat the readings for gradually decreased values of the primary voltage, the periodicity of the supply being maintained constant.

Readings should be continued in this manner until a point is reached when the motor speed falls rapidly. The motor should then be stopped and the voltage reduced to its minimum value. With the rotor winding short-circuited, close the switch in the primary circuit and take similar readings to those already taken for gradually *increased* values of the applied volts up to the point where the motor speed rapidly rises to approximate synchronism.

(2) Tie or clamp the rotor shaft in such a way that it cannot rotate. Apply first a low voltage, and then gradually increasing voltages to the stator windings with the rotor short-circuited as before. Take the readings as in the previous case, continuing the experiment until the rotor current becomes excessive.

From the readings of the wattmeter and primary ammeter and voltmeter, the value of the power-factor $\cos \phi$ should be calculated. The power in a three-phase circuit $= \sqrt{3}\, C\, E \cos \phi$ where C and E are the line current and voltage as measured on A_1 and V. Consequently the power-factor is calculated from the formula

$$\cos \phi = \frac{\text{true watts}}{\sqrt{3}.\ \ C \times E}$$

The following table shows a few sample readings from a test on a 2½-h.p. Brown-Boveri motor, and illustrates the manner of entering up the results. The readings are plotted on the curves shown in Fig. 182, the armature being free to rotate. In Fig. 183 are shown curves obtained with the armature clamped.

No-load Test of Three-phase Induction Motor.
Motor No..... Type: Brown-Boveri four-pole.
Output 2½ h.p., at 1,460 revs. per minute.
Voltage 200 50 ∿.

Primary Volts = E	Primary Current = C	Speed. Revs. per min.	Secondary Current.	Primary Watts.			Apparent Watts $\sqrt{3}\,EC$	Power Factor $\cos\phi$
				Volt Coil Phases I. & II.	Volt Coil Phases I. & III.	Total.		
Volts Decreasing from Maximum Value.								
234	8·1	1450	1·2	1000	−496	504	3280	·153
124	3·8	1450	2·3	300	− 7	293	817	·359
25	6·0	1260	6·0	144	+ 84	228	260	·878
Volts Increasing from Minimum Value.								
20	11·5	20	16·7	210	− 20	190	398	·478
30	17·4	107	25·1	500	− 70	430	895	·482
36	21·0	600	30·3	780	−100	680	1310	·519

In Fig. 182 it will be noticed that there are two completely separate curves for each of the quantities plotted—one portion to the extreme left of the diagram and another more extended portion to the right. We meet here for the first time with an illustration of the two conditions under which an induction motor will run. It is found in many experiments that under certain conditions, the motor will run at either of two speeds, one of these speeds being more stable than the other. In the present case, for instance, at 30 volts (see Fig. 182), the motor would run either at 1,320 revs. per minute, as indicated on the upper curve, or at 110 revs. per minute, shown on the lower curve. Corresponding to either speed is found a set of internal conditions of currents, watts, &c., quite distinct from those existing at the other speed. The external condition, *i.e.*, the nature of the voltage applied, was identical in the two cases. At points where the two sets of curves overlap, the curves corresponding to the higher speed in all cases represented the more stable conditions, *i.e.*, the motor after running a short time at the lower speed would generally speed up until the conditions of the right-hand curves were reached. The unstable curves are not of much practical value, since

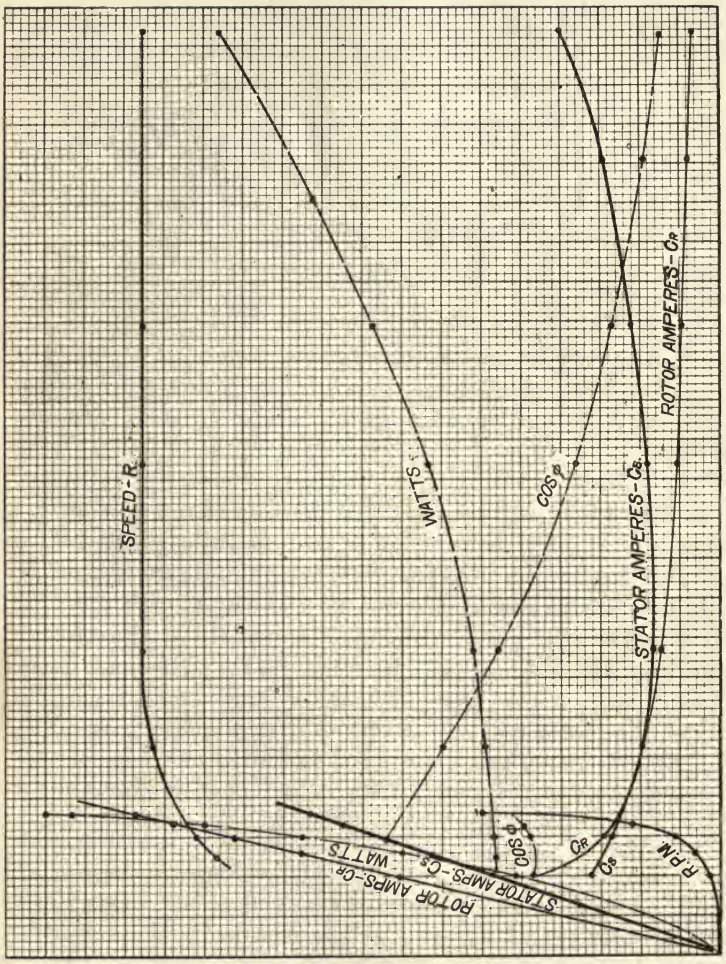


FIG. 182.—NO-LOAD TEST OF INDUCTION MOTOR.

they do not, of course, represent the usual running conditions. To obtain them, the voltage was raised very gradually from zero, readings on the instruments being taken as rapidly as possible at points where the motor tended to increase its speed to the values of the upper curve. In order to get the overlapping part on the higher speed curves, the voltage was gradually reduced, care

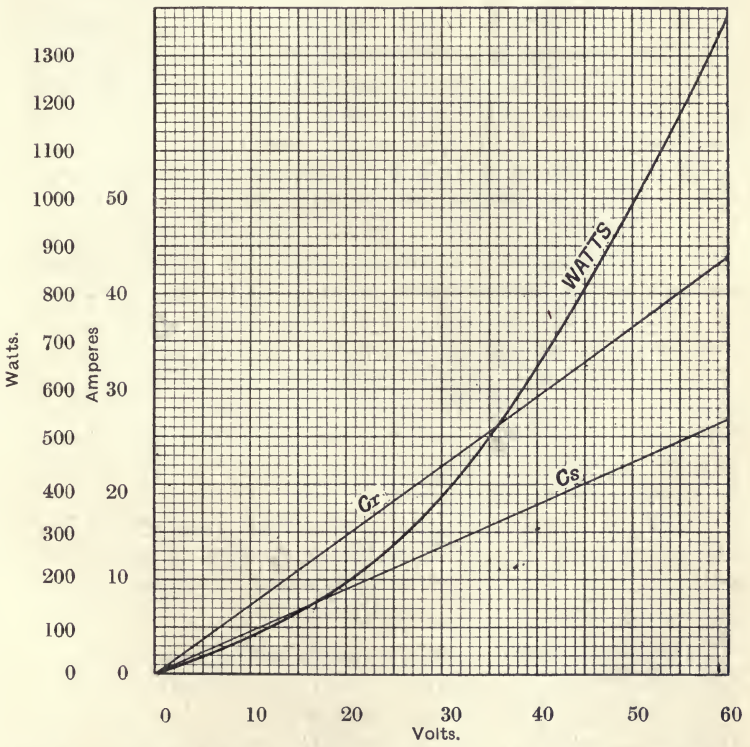


FIG. 183.—CURVES OF STATOR AND ROTOR CURRENTS AND WATTS IN STATIONARY MOTOR.

being taken to avoid sudden changes of any kind, which would result in the speed of the motor falling to its lower value when nearing the lower end of the curve.

It is interesting to trace the variation in the normal current curves. At low voltages the strength of the rotating field is very weak, and in order to overcome the resistance to turning of the rotor, the rotor current is necessarily high, since the torque is the product of field

strength and rotor current. In order that this rotor current may be produced by such a weak field, the slip is considerable. The energy component of the stator current must always increase in proportion to the rotor current, and is consequently also fairly high, although the true magnetising current is low, and most of the primary current is spent in neutralising the rotor ampere turns.

As the voltage increases, the strength of the rotating field increases, and, as the torque on the shaft remains constant, the rotor current decreases in practically the same proportion as the field becomes stronger. The slip required to produce this current decreases. Also the portion of the primary current spent in neutralising the field due to the secondary current diminishes, while the true magnetising current increases. The primary current is consequently seen to decrease on the whole, though not so rapidly as the rotor current. The same process is seen to continue, the rotor current decreasing as the field becomes stronger. The stator current continues to decrease with the rotor current and to increase with the increased voltage and magnetising current. The two tendencies neutralise one another, and then the stator current, as a whole, begins to increase as the rotor current becomes more and more nearly constant.

The power absorbed is seen to increase gradually, being almost entirely due to iron losses, except at the lower voltages, when the saturation is low and the currents comparatively high.

The power factor decreases as the voltage and magnetising current rise. It is seen from the power-factor curve how large a part of the primary current at low voltages must be spent as "energy current," in neutralising the secondary ampere turns, since the power-factor is so high with the large initial current.

The curves in Fig. 183 were taken with the rotor held stationary. In this case stator and rotor currents and power absorbed all increase with the voltage, and assume much higher values than when the rotor is free to rotate. The power in this case increases on account of the heavier currents, and very slightly because the rotor core is now subjected to the magnetising action of the rotating field at

its full periodicity. Under normal running conditions the iron losses in the rotor core are small because the frequency of rotation of the magnetic field relative to the core is only the frequency of the *slip* and not of the *supply current.*

The total amount of the iron losses will, of course, be small, because the maximum voltage applied during the experiment was only about one-fourth of the working voltage.

In the conditions under which the curves shown in Fig. 183 were taken, viz., with stationary rotor, the motor is practically a static transformer, the stator and rotor

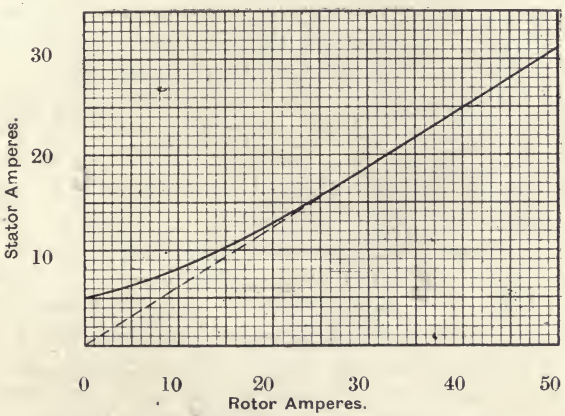


FIG. 184.—RATIO OF CURRENTS.
Full line = Full voltage, motor running under load.
Dotted line = Motor clamped, voltage varied.

forming the primary and secondary windings. The test is consequently similar to the curve taken on a short-circuited transformer in order to separate the copper losses (see Fig. 70, p. 153).

It will be noticed on comparing the two curves that the shape of the watts curve, showing the copper losses, is similar in the two cases.

The current curves are straight lines. In the case of the rotor this is easy to understand, since the current will naturally increase in proportion to the increased voltage induced in it by the transformer action. The primary current increases in proportion to the secondary current, as in a transformer, in order to counteract the

demagnetising action due to the rotor currents. Practically the only field which the stator windings can maintain (with stationary short-circuited rotor) is a leakage field between the teeth of the stator core, since any flux entering the rotor is at once neutralised by the rotor currents which it induces. The experiment represents, consequently, the condition of maximum leakage.

In Fig. 184 the values of the currents shown in Fig. 183 are plotted in another way, viz., as the ratio of stator to rotor current. The partly-dotted straight line passing through zero is the curve referred to, and shows that the ratio of the currents is constant. This curve is similar to the curve of ratio of primary to secondary current in a transformer, and gives the ratio of transformation of the induction motor when considered as an alternating-current transformer.

It is interesting to note that the ratio of the two currents always approximates to this straight line, quite irrespective of the load on the motor, primary voltage, or resistance in the rotor circuit, since these do not affect the ratio of transformation of the two windings.

As a result of this experiment it would always be possible to calculate the value of the rotor current from a curve of stator currents obtained in a load test. This is of special value, as it is not usual to measure the rotor current when carrying out a test. The full-line curve in Fig. 184 shows the ratio of primary to secondary currents when the motor works under load (as obtained in the next experiment), and shows that when fully loaded the ratio of the currents approximates to the theoretical ratio of transformation. This ratio would be maintained at all loads if it were not for the considerable no-load current of the motor when working at normal voltage.

The special practical application of the two measurements just described will be further alluded to later. They are of importance, because in one case the motor works (when near full voltage) practically without slip, and without magnetic leakage. In the other case, both slip and magnetic leakage have their maximum values.

Experiment LI.—Load-Test of an Induction Motor.

Diagram of Connections.

(Same as for Experiment L., Fig. 181, p. 370.)

Instructions.—The instruments and connections required will be the same as those given in the connections for Experiment L. In this case, however, both voltage and periodicity of the supply are to be maintained constant, The motor must be provided with a brake on its pulley, or must be coupled to a generator which has been

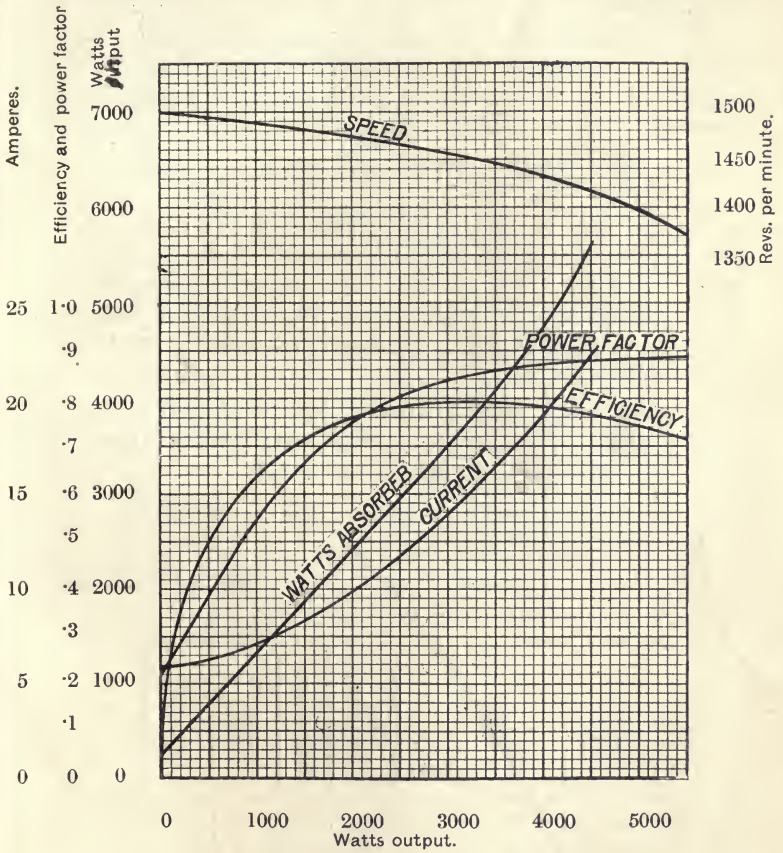


FIG. 185.—LOAD CURVES OF INDUCTION MOTOR.

previously calibrated, so that it may serve as a brake or absorption dynamometer. The power given to a generator employed as load in this way will be the measured output of the generator divided by the efficiency of the generator at this output.*

* For a description of a very convenient form of eddy-current brake for such experiments, see paper by Dr. D. K. Morris, read before the Birmingham section of the Institution of Electrical Engineers, April, 1905.

The test is carried out by observing the primary and secondary amperes of the induction motor, the watts supplied, and the slip, first at no-load, and then for a series of increasing values of the load up to about 25 per cent. overload for an ordinary commercial test, or up to the point of stoppage of the motor in the case of a more complete experiment.

The results should be entered under headings similar to those shown below, and curves plotted with either watts or horse-power output horizontal, and primary and secondary current, slip, efficiency, apparent efficiency $\left(=\frac{\text{watts output,}}{\text{volt-amperes input,}}\right)$, and power factor vertical.

In the tests of motors usually carried out, it is not the general practice to measure the current in the secondary, as its exact value is not of first importance in the behaviour of the motor, and its approximate value can be deduced from the other quantities observed, as already explained on page 378.

Several other useful curves may be plotted from the results of this experiment, such as: watts absorbed and primary current on a load base.

LOAD TEST OF THREE-PHASE INDUCTION MOTOR.

Motor No.....Type.....

Output.....h.p., at.....revolutions per minute.

Voltage..... ∿ .....

Supply.					Slip, Per Cent.	Rotor Current.	Apparent Watts Supplied.	Power-factor.	Efficiency.
Current.	Volts.	Watts.							
		a.	b.	Total.					

The three columns for watts are to be employed when readings are taken in two phases, so that their sum gives the total power. Where a 3-phase wattmeter is employed only one column will be required.

The nature of the curves to be obtained from this experiment may be judged from Fig. 185, taken from tests of a 3½ h.p. 3-phase Oerlikon motor running at a frequency of 50, and having a synchronous speed of 1,500 revs. per minute.

As a further illustration, some additional curves obtained in another experiment in a rather different manner are shown in Fig. 186. The motor employed this case was a 2½ h.p. Brown-Boveri motor with slip rings having a synchronous speed of 1,500 revs. per minute on a 200-volt 50-cycle circuit. These curves are given as an

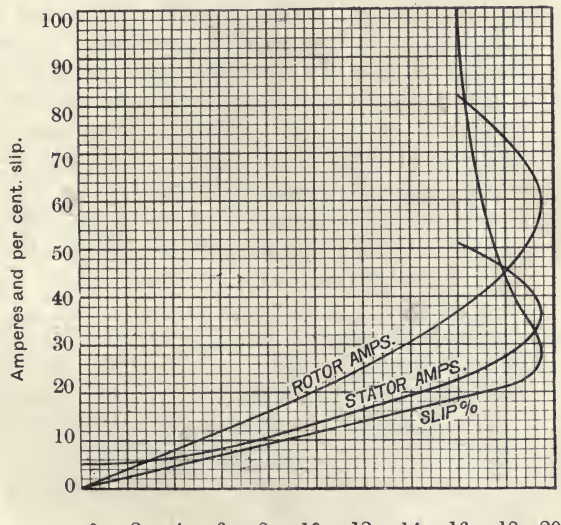


FIG. 186.—LOAD TEST OF INDUCTION MOTOR COUPLED TO DIRECT-CURRENT GENERATOR.

illustration of a simple method of obtaining load curves of a motor by coupling it to a direct-current generator.

The test was carried out with a voltage supply of 120 volts instead of 200, as it was found that the currents flowing in the motor windings when heavily loaded at the full voltage produced dangerous overheating. The load on the motor shaft was a direct-current generator driven by a belt, and excited with a constant current. The currents plotted horizontally in Fig. 186 are the currents generated by the direct-current machine + the no-load

current required to drive both machines at each speed. In this way the horizontal scale is a scale of torque, but measured in terms of the armature current of the direct-current generator. If the generator field remained constant during the experiment for all currents, the armature current would have been exactly proportional to the amperes thus plotted. In order to make this as nearly true as possible, the excitation of the generator was carefully adjusted to a constant value, and the brushes were fixed in the neutral position to avoid much weakening of the field at higher current densities. Some weakening of the field due to magnetic distortion must have taken place; but as the machine was not worked up to its maximum output this was probably not serious. This assumption was made after testing the proportionality between speed and voltage of the generator, when on open circuit and when fully loaded, and finding that the observed loss of voltage determined with the heaviest armature current at the reduced speed was nearly all accounted for by the resistance of armature and brushes. In any experiment carried out in the same manner this should be similarly tested.

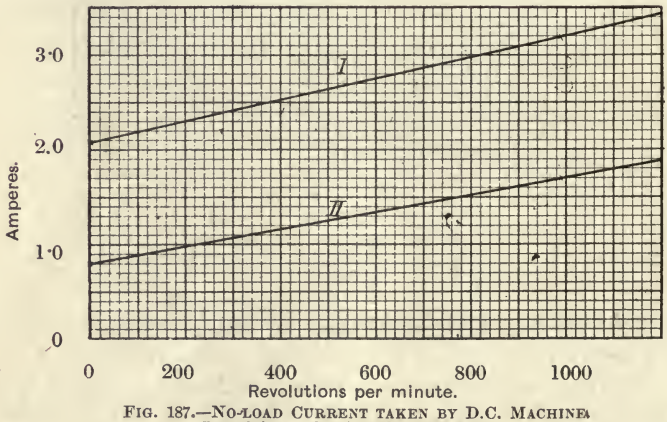


FIG. 187.—NO-LOAD CURRENT TAKEN BY D.C. MACHINE.
I. Driving Induction Motor by belt.
II. Belt thrown off.

The method of obtaining the curves in Fig. 186 was briefly as follows :—

The motor was connected to the supply of three-phase current exactly as described in the instructions given

above. A direct-current generator was coupled to the motor by a belt, and its armature was connected to a variable resistance in series with an ammeter; a voltmeter read the terminal voltage. This generator was separately excited, the excitation being kept constant by means of a regulating resistance. The motor was supplied at a constant voltage of 120 throughout the test, and the generator was allowed to send gradually increased currents, so as to increase the load on the motor. For each value of the load on the generator, watts supplied, rotor and stator currents, speed and generator voltage were read.

The final readings, at and near the point of stoppage of the motor, were taken by supplying the direct-current machine armature with current from an external source in series with a resistance and reversing the fields, so as to make it tend to drive the alternating-current motor in a reverse direction. By this means the armature current corresponding to the torque at stopping was obtained.

A test on the direct-current generator was then made to ascertain the current equivalent to the frictional and other losses at various speeds. The machine was run as a motor at the same excitation as before, and the current taken by the armature at various speeds was noted, both with the belt coupling it to the motor and with the belt thrown off. The readings taken with the belt on included the friction of the induction motor and belt as well as of the direct-current generator.

The results of this test are shown in Fig. 187. In plotting the curves of Fig. 186 the no-load current required to overcome the total losses of both machines (as shown on the upper curve) were added to the current actually given out by the generator. The frictional losses of the induction motor itself are thus counted as torque exerted by the rotor.

In order to determine the relation between the horizontal scale of amperes in Fig. 186 and torque, measured in lb.-ft. or kg. cm., it is only necessary to observe the voltage generated in the direct-current machine when running unloaded, and its speed. In the present case the no-load voltage at 1,320 revs. per minute was 210.

Thus, at an output of 10 amperes, and the speed observed of 1,160, the power generated (including

armature copper losses) was $\frac{1,160 \times 210}{1,320} \times 10$ watts.

The no-load driving current at this speed is (see Fig. 187) 3·4 amps. Thus the power developed by the motor was

$$\frac{1,160 \times 210}{1,320} \times 13{\cdot}4 \text{ watts} = 2,480 \text{ watts.}$$

This power is equivalent to a torque of 15lb.-ft.

This may be arrived at as follows :—

$$\text{H.P. developed} = \frac{\text{watts}}{746} = \frac{2,480}{746} = 3{\cdot}32.$$

$$\text{Also, H.P.} = \frac{2\,\pi\,n\,T}{33,000}$$

where T = torque in lb.-ft.

and n = revolutions per minute.

$$\text{Hence torque} = T = \frac{\text{H.P.} \times 33,000}{2\,\pi\,n} = \frac{3{\cdot}32 \times 33,000}{2\,\pi \times 1,160}$$

= 15 approximate.

It will now be convenient to give a short discussion of each of the quantities shown in the curves of Figs. 185 and 186, taking the stator current, power absorbed, efficiency, power factor, and slip separately, and pointing out some of the chief conclusions and reasons for their variation as shown on the curves.

Primary Current.—The current supplied to the stator is determined by two factors as in the case of a loaded transformer. It has, firstly, to supply the varying magnetising current in each phase required to maintain the rotating field, corresponding to the no-load current of a transformer which we may call the no-load component. Secondly, the current has a component which produces a field equal and opposite to that formed in the stator by the rotor currents, corresponding to the component of the primary transformer current which overcomes the demagnetising action of the secondary circuit. This we may distinguish as the load component.

As already explained, the component of the rotating field due to the rotor currents differs in direction from the main primary field by 90°; that is, it will act upon the conductors one-quarter period later than the stator component of the rotating field. Consequently, the load component of the primary current will be one-quarter period in phase behind the no-load component.

The no-load component, spent in exciting the magnetic circuit, is usually not more than quarter to one-third of the full-load current of the motor. Consequently, the primary current will be nearly proportional to the secondary current when the motor is loaded, and appears as a nearly straight line in Fig. 186, within the usual working range, bending to the right and becoming horizontal for very small values of the slip.

In Fig. 177 (page 357) were drawn the curves of primary and secondary current and torque on a slip base for the ideal case of a motor without magnetic leakage.

If a motor could be constructed without leakage, it would have a very strong turning moment, and would operate without resistance in the rotor circuit with a definite slip and speed for each value of the load, but would, at starting and at low speeds, take an excessively heavy current.

It must be remembered that the total flux produced by the primary current is constant, as in the case of a transformer. The relation between primary flux and primary voltage is given by the same formula as that on page 126, for the transformer

$$e_1 = 4{\cdot}44\, n Z N_1 10^{-8}$$

When Z = maximum flux produced.
n = periodicity.
N_1 = turns in winding.
e_1 = primary voltage.

The flux consequently depends only on the construction of the motor and the voltage applied, and is independent of the load.*

In a three-phase motor it may be shown that current in each phase winding must produce a flux equal to two-thirds of the total strength of the rotating field. The value of the no-load current, or magnetising current, will be given by applying the same formula as that given for the magnetising current in a transformer, account being taken of the fact that the current in each phase is that required to produce two-thirds of the rotating flux. For the same reasons as exist with the transformer, a short magnetic circuit, with as small an air-gap as possible, and a low saturation are employed.

*This is neglecting the loss of voltage in the resistance of the primary winding $= c_1 R_1$

The dependence of the primary current on the secondary or rotor current is exactly similar to that existing in the case of a static transformer. We may illustrate the relation by referring to Fig. 188.

In the case of the unloaded motor, the primary current will be the magnetising current necessary to produce the rotating field. This rotating field we may write equal to $Z_1 = k_1\ c_0\ t_1$ where k_1 is constant, and $c_1\ t_1$ represents the primary ampere turns. The rotor current which is produced when the motor is loaded gives rise to a magnetic flux, perpendicular to Z_1, having a value $Z_2 = k_2\ c_2\ t_2$, as illustrated in Fig. 175 (page 351). The ampere turns supplied to the motor will have a value sufficient to maintain the resultant flux. Consequently, if Z is the resultant flux, the primary current will be given by the expression $\frac{Z}{k_1\ t_1}$. The varying relation between the primary and secondary currents may be represented by the diagram Fig. 188, in which the horizontal line

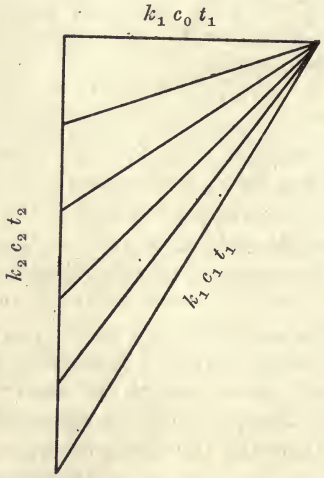


FIG. 188.—RELATION BETWEEN PRIMARY AND SECONDARY CURRENTS.

represents the constant no-load ampere turns required to produce the primary flux; the vertical side represents a series of values of the secondary ampere turns and the third side represents the resultant ampere turns of the primary winding. Fig. 189 shows the relations between primary and secondary ampere turns given in Fig. 188 plotted in

the form of a curve. This curve should be compared with the relation between primary and secondary currents actually obtained in the experiment, and plotted as the full line in Fig. 184.

If the uppermost point in the curve in Fig. 189 be taken as the point of normal full load, it will be seen that the no-load current is about one-half of the full-load current. This condition corresponds roughly to the case of small motors. With larger machines the no-load current may be about one-third of the full-load current,

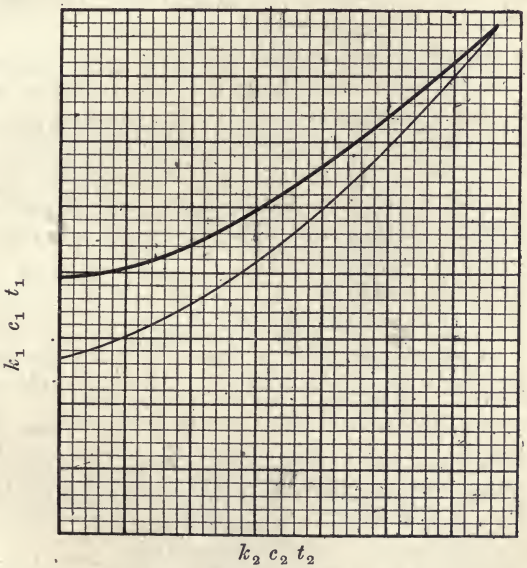


FIG. 189.—CURVE SHOWING RELATION BETWEEN PRIMARY AND SECONDARY CURRENTS.

and the relation between primary and secondary current is that shown by the lower curve.

Thus, the no-load *current* of an induction motor is considerably higher than that of a direct-current motor, although the no-load *power* absorbed is less. From Fig. 188 it is easy to see that the phase of the primary current becomes more and more nearly one-quarter period in front of the no-load current, and consequently approaches more nearly the phase of the applied voltage. The power-factor thus increases rapidly with increasing loads. These

relations are seen to hold in the case of the experimentally-obtained curves.

As the rotor current is practically proportional to the load within the usual working range, the curves in Fig. 189 will be similar to curves of primary current plotted on a load base. The similarity to these curves of the curves of stator current shown in Figs. 185 and 186 confirms this statement.

Power Absorbed and Efficiency.—The losses occurring in the motor are practically of three kinds. (*a*) The loss in the stator and rotor windings; (*b*) iron losses in stator and rotor cores; (*c*) frictional losses.

(*a*) In a three-phase motor, the watts lost in the stator winding are equal to $3c_1^2 r_1$ where c_1 is the primary current per phase, and r_1 is the resistance of the winding per phase. If the rotor winding is a short-circuited three-phase winding the watts lost in the rotor are $3c_2^2 r_2$ where the symbols c_2, r_2 denote the current and resistance per phase of the rotor winding. Both losses increase together as the load on the motor is increased.

(*b*) The iron losses in the stator will depend on the saturation of the iron and the frequency of the magnetic changes. It has already been shown that the strength of the primary field is constant at all loads, depending only on the supply voltage. The rotating field makes $\frac{n}{p}$ revs per min. when n = frequency of supply and p = number of pairs of poles. Thus the iron of the stator passes through a complete magnetic cycle n times per second. This source of loss is consequently independent of the load.

The iron losses in the rotor will be proportional to the slip, since the polarity of the rotor core will change p times for each rotation of the rotating field relative to the rotor. The slip under ordinary working conditions is so small that the maximum rate of magnetic reversal is only about 5 per cent. of the speed of the rotating field. The iron losses in the rotor may consequently be safely neglected in comparison with the other losses.

(*c*) The frictional losses consist of bearing friction and windage. Since these depend almost entirely on the speed, which is practically constant, and not appreciably

on the load, they may be taken as being constant for any motor.

From the consideration just given we see that the friction and iron losses are practically constant at all loads, while the copper losses are very small (not more than 1-2 per cent. and due to the magnetising current in the stator) at no load, but increase in proportion to the square of the current taken by the motor.

If we group together the useful output of the motor and the power spent in overcoming friction and iron losses, and denote their sum by W_t, calling the power supplied to the motor W_s, we may write

$$W_s = W_t + 3c_1^2 r_1 + 3c_2^2 r_2.$$

If there were no copper losses, we should have

$$W_s = W_t,$$

and the curve showing the relation between total watts output and watts supplied would be a straight line passing through zero and inclined at an angle of 45° to the horizontal, if the same horizontal and vertical scales are chosen. The effect of the copper losses is to make the watts supplied increase more rapidly at higher loads, the increased power being proportional to the square of the load, so that the line will bend upwards in a curve.

Referring to Fig. 185, we see that the curve of watts agrees with the statements just made exactly. The line does not appear to pass through zero, because the horizontal scale is *useful* output only. If the curve were prolonged backwards to meet the horizontal axis, the distance to the left of the vertical axis would measure the power spent in overcoming friction and iron losses at no load This distance is the same as the height of the point where the line cuts the vertical axis, since the line is inclined at 45 deg., as may be seen. In the present case, therefore, the iron and friction losses may be taken to be 270 watts.*

By drawing a tangent to the watt curve inclined at 45 deg. we might measure the power spent in copper losses at any load by determining the vertical distance between this tangent and the curve.

The curve of efficiency is of the usual shape. If the

* These losses are not *accurately* constant at all loads, but sufficiently nearly so for most practical purposes.

copper losses are neglected, the equation to the curve would be

$$\text{efficiency} = \eta = \frac{\text{watts output.}}{\text{watts output} + \text{constant losses.}}$$

This would represent a hyperbola approaching its asymptote $\eta = 1$ more nearly as the load increases. Owing to the copper losses the curve fails to reach this value and begins to bend downwards at higher loads, thus departing from the shape of the hyperbola. The curve should reach its maximum value at normal load, and being very flat near this point, the efficiency does not vary much between three-fourths and five-fourths of full load.

The Power Factor.—The power factor is very low at no-load, which accounts for the fact that induction motors take a relatively high no-load current, although absorbing little power.

The shape of the power-factor curve is similar to that of the efficiency curve, but does not pass through zero. It rises sharply at first, then becomes horizontal, and finally falls slightly. The general shape of the curve follows from the curves of power and stator current, already discussed. Since $\cos \phi = \frac{\text{watts supplied}}{\text{amps} \times \text{volts} \sqrt{3}}$ and the voltage of supply is constant, the fraction $\frac{\text{watts}}{\text{current}}$ is proportional to the power factor. Referring to Fig. 185, it is seen that the watts rise rapidly from the first, whereas the current increases only slowly. The fraction $\frac{\text{watts}}{\text{current}}$ must, consequently, rise rapidly at first. As the current curve becomes steeper, the increase of the fraction becomes slower.

When the motor works at a low power factor the current taken by it is chiefly magnetising or idle current. The disadvantage of large idle currents in a supply system is less due to its effect on the operation of the motors than to the fact that considerable useless currents flow in the supply circuit, producing losses in them and in the generator armature, and tending to produce bad regulation of the generator voltage, through causes already discussed, in connection with the characteristic curves of alternators.

In order to increase the power factor of a motor, there are two ways which it is possible to adopt, both of which diminish the no-load currents. These are, either to increase the number of windings of the primary, or to reduce the reluctance of the magnetic path by reducing the air-gap. The first method has the effect of increasing the resistance of the winding, and, consequently, diminishing the efficiency of the motor. This disadvantage is not possessed by the other method of improving the power factor.

Relation Between Slip and Torque.—It has already been shown on page 361 that with light loads the torque is proportional to the slip. The curve of slip in Fig. 186 is seen to confirm this statement, since it is a straight line rising gradually from the point corresponding to zero torque.

In the same place, it was further shown that, when heavily loaded, the torque is approximately proportional to $\frac{1}{\text{slip.}}$ The curve representing this relation would be a hyperbola. The two theoretical curves, to which the conditions approximate at no load and full load, are indicated by the straight line and portion of a hyperbola on the curves plotted in a slightly different manner in Fig. 180 (page 359). The actual shape indicated by a thick line in Fig. 180 passes gradually from one curve to the other. This curve, which is again seen in Fig. 190, shows the characteristic form of the relation always existing between the slip and torque.

The straight first portion of the curve is the same as would be obtained in a motor without magnetic leakage. It is, therefore, the same line as the one drawn from theoretical considerations in Fig. 177. It is, further, the most practically important portion of the curve, since the working conditions always correspond to points upon it. Thus, although there are two values of the slip corresponding to the higher values of the torque, it is only the lower values of the slip which are reached under normal working conditions, *i.e.*, the motor works upon the straight portion of the curve.

In Fig. 180 is shown a second dotted curve, which applies to the same motor and conditions as the first one, except that a resistance is supposed to have been intro-

duced into the rotor circuit virtually doubling its resistance. The effect is seen to be that the inclination of the straight portion of the curve is halved, and at the same time the hyperbolic portion is raised so as to cut the axis at twice the previous height from the horizontal axis. The height above the horizontal axis, at which the hyperbola cuts the vertical axis, represents the starting torque of the motor, since it is the torque of the motor

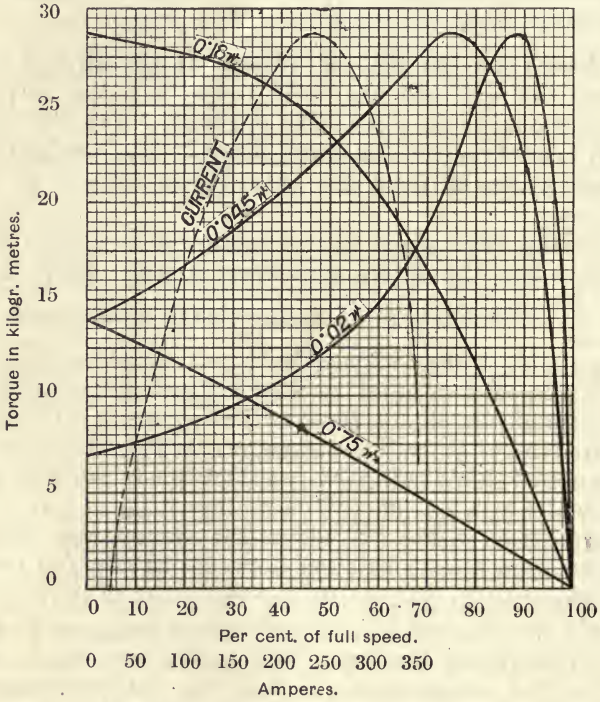


FIG. 190.—CURVES OF TORQUE, SPEED, AND CURRENT WITH VARYING ROTOR RESISTANCE.

when stationary. The starting torque is, therefore, increased by increasing the rotor resistance up to the point when it is the top part of the torque curve, which cuts the vertical axis. A further increase of rotor resistance will only again diminish the torque.

It is of great importance to remember that the maximum torque of the motor is constant and independent

of the rotor resistance. The value of the slip at which it occurs is determined by the rotor resistance.

The starting current is thus proportional to the rotor resistance, as is also the slip for a given torque when under load.

This is discussed more fully in connection with the next experiment.

Effect of Increase of Resistance in Rotor Circuit.—The next point to be considered is the effect of increasing the resistance of the rotor. This is best studied by using a motor with wound rotor and slip rings, so that a variable resistance can be put in series with each phase of the windings. It is, of course, necessary to make the resistance of all the phase circuits equal. For the purpose of the experiment, the usual three-phase starting resistance may be conveniently employed, provided that its carrying capacity is sufficient to enable it to carry continuously the heavy currents induced in the rotor when the motor is overloaded.

Experiment LII.—Test of an Induction Motor with Variable Resistance in the Rotor Circuit.

Diagram of Connection.

As for Experiment L., Fig. 181, page 370.

Instructions.—The experiment may be carried out exactly as described in the last case, but with a series of different values of the resistance in the rotor circuit. The series of curves are then plotted, each for a different value of the rotor resistance. From these curves a fresh curve may then be obtained, showing the variation of the primary current, torque, efficiency and power factor with the slip.

For the particular purpose of determining the effect of the rotor resistance on the behaviour of the motor, it is, however, sufficient to take measurements only of the stator and rotor currents, slip and torque, for each of a series of values of the rotor resistance. Complete sets of readings of each of these quantities should be taken and entered up as shown for the previous test, curves being then plotted on a torque base as before.

A set of torque-slip curves obtained in this way is shown in Fig. 190.

From such a set of curves, each drawn for a particular value of the resistance, it is useful to plot curves showing the starting currents and torques of the motor with various rotor resistances, *i.e.*, to plot curves of starting current and starting torque on a base of rotor resistance.

By the starting current and torque are here meant the values of these quantities as shown on the curves when the motor comes to rest, *i.e.*, when the slip is 100 per cent.

These curves may be obtained more directly by loading the motor with a brake and weights or spring balance until it comes to rest, the current and retarding couple being then observed. By repeating this process with different rotor resistances, the desired curves are obtained.

A modification of the plan just suggested is to attach a lever to the motor pulley and load this with weights, or by a spring balance until the turning effort of the motor is exactly balanced.

Fig. 190 shows several curves of torque and slip obtained with different resistances in the rotor circuit. It will be seen that the effect of increasing the resistance is to make the curve slope upwards less steeply on the right, so that for small values of the torque the slip is increased All the curves reach ultimately the same maximum value of the torque so that the maximum torque exerted by the motor is not affected by the rotor resistance. The maximum torque is, however, reached with a different value of the slip in each case.

One of the most important results to be drawn from the curves is the variation in the starting torque, that is, the torque at still stand, due to a change in the resistance of the rotor. The point at which the curves cut the vertical line, corresponding to a slip of 100 per cent., gives the turning effort of a motor when started from rest. Evidently the greatest starting torque will be obtained by so choosing the rotor resistance, that the curve has its maximum value on this vertical line. In the case of the motor for which Fig. 190 is drawn, this resistance of rotor winding and starter per phase is seen to be about ·18 ohms. A resistance greater than this would give a maximum torque at a slip of over 100 per cent., that is, with the motor rotating in a reversed direction. Resistances less than ·18 ohms enable the motor to exert its greatest turning effort, at speeds intermediate between still-stand

and synchronism. By suitably choosing the successive steps of the starting resistance, and moving the starter handle over at the correct rate, as the motor increases its speed it is possible to insure that the motor is started and got up to speed under the most favourable conditions, that is, under conditions at which it can exert the maximum torque.

When the starting torque is plotted as a function of the rotor resistance, as has been done for a different

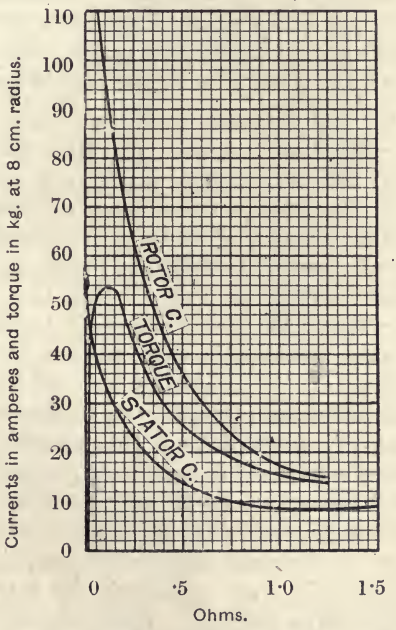


FIG. 191.—RELATION BETWEEN STARTING CURRENT AND TORQUE AND ROTOR RESISTANCE.

machine in Fig. 191, the value of the resistance giving a maximum starting torque is seen at once, being the resistance corresponding to the peak of the curve. This would correspond to the value of the resistance giving the curve of ·18 ohm in Fig. 190. The points to the right of this are lower, because the maximum turning effort is only reached with a slip, greater than 100 per cent., as explained above. Points on the curve to the left of the peak indicate values of the resistance, for which maximum

torque is only reached after the motor has attained some speed.

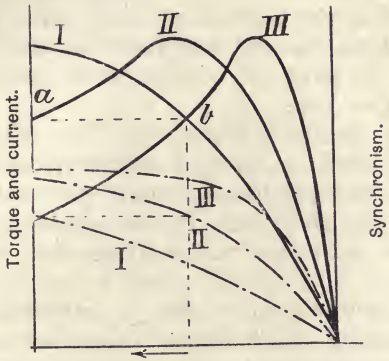
The lower the resistance of the rotor circuit the nearer to synchronous speed is the point at which the motor can exert its greatest torque. Since the greatest slip usually occurring in practical working is about 5 per cent., it is an advantage to have an extremely low resistance of the secondary winding. In cases where a separate resistance is not employed for starting—that is, in the case of squirrel-cage motors—it is necessary to sacrifice some of the starting torque which might have been attained when running at full speed, in order to get a sufficient torque at starting, and when running at the initial low speeds.

In Fig. 190 will be seen a dotted curve of current. This curve shows the value of the stator current (measured on the horizontal scale) corresponding to each value of the torque (measured vertically). It is possible thus to draw a single curve representing the relation between current and torque independently of the rotor resistance. This is due to the fact, which has already been alluded to, that for a given torque the rotor will always take constant current, the slip automatically adjusting itself to enable this rotor current to be induced. Since the stator current depends only on the rotor current and the constant no-load current, there will be a definite value of the stator current for each torque independent of the rotor resistance. As seen from the current curve, Fig. 190, there are actually two values of the current instead of a single one, as just stated. This is because there are two values of the slip for each value of the torque, as seen from the torque-slip curves. The running conditions always correspond to the lower slip, and, consequently, lower current.

The variation of rotor current with torque may be easily followed from Fig. 192, where rotor and current and torque are plotted for three different values of the armature resistance.

If two points on corresponding arms of two of the curves be taken, each corresponding to the same torque, *e. g.*, *a*, and *b*, and the corresponding currents be noted on the current curves, the currents are seen to have the same value.

Attention may be called to one further point in connection with the starting curves, Fig. 191. For a given value of the starting torque there are evidently two possible values of the rotor resistance. In designing the



S = 100 per cent. Slip. S = 0.

———— torque.
— . — . — . current in rotor.

Curves I.—Rotor R = ·1 ohm.
II.— ,, = ·05 ,,
III.— ,, = ·025 ,,

FIG. 192.—ROTOR CURRENT AND TORQUE COMPARED WITH SLIP.

starting resistance, the greater value of the resistance should be chosen, since this corresponds to a smaller starting current. Consequently the rotor will *start* upon the *flatter* right-hand limb of the torque curve (Fig. 191). It will run when at full speed on the *steeper* limb of the running curves shown in Fig. 190.

Determination of Losses and Efficiency.—From the tests already given, it will be apparent that the losses in an induction motor may be determined in a manner very similar to that employed in the case of a transformer.

Corresponding to the determination of the copper losses in a transformer, we have the test on the motor while held stationary, described on page 370. The power supplied to the motor when locked is nearly all C^2R loss, since the iron losses are small at the low saturation employed. The losses due to the resistance of the windings increase with the load, and are thus easily determined for any value of the stator current.

The core losses and friction losses are practically

constant at all loads. Their value is determined from the no-load running test (see page 370). The power taken by the motor when running light is nearly all spent in iron losses, and running friction. A small part will be due to copper losses in the windings, and this should be subtracted from the wattmeter readings to give the true iron and friction losses. The value of the copper losses is easily derived from the results of the determination of the total copper losses.

From the results of these tests the efficiency of the motor at any load can be approximately calculated, since the iron and friction losses remain practically constant at all loads, and the losses due to resistance are at once seen from the curves obtained in the first test (see Fig. 183, page 375).

The *power factor* may also be calculated from the results of the two loss tests just described.

The idle magnetising current is determined from the readings of the wattmeter, ammeter, and voltmeter taken in the no-load running test.

From these readings $\cos \phi^1 = \dfrac{\text{watts}}{\text{volts} \times \text{amps.}}$, where $\cos \phi^1$ is the no-load power factor. The magnetising current is then (no-load amps) $\times$ sin ϕ^1). This idle current will remain approximately constant at all loads. Thus the total current at any load will be given by

$C^2 =$(energy load current + energy loss current)^2 + (idle current)^2.

In a 3-phase motor the total output $= 3\ C_1 E$ when C_1 and E are current and volts per phase. Thus total current at any load

$$= C = \left\{ \left(\frac{W}{3E} + \frac{w}{3E} \right)^2 + C_o^2 \right\}^{\frac{1}{2}}.$$

When $W =$ total watts output.
$w =$ total calculated losses.
$C_o =$ magnetising current.

$$\text{The power factor is } \frac{\text{energy current}}{\text{total current}} = \frac{\dfrac{W + w}{3\ E}}{C} = \frac{W + w}{3\ E\ C}$$

The *percentage slip* at any load can be also approximately calculated from the results already obtained, since

this will be practically equal to the percentage drop of voltage in the windings.

The method of separating the mechanical losses of the motor from the other losses has already been described on page 389.

The graphical representation of the various losses at all loads is discussed in connection with the Heyland diagram page 403 and seq.

Single-phase Motors.

If a three-phase motor is running while connected to the supply circuit by three wires in the usual way, and one of these wires is broken, the motor will continue to rotate and to perform its work. The current supplied to the machine under these conditions is a single-phase alternating current, and, although an increase of current in the two remaining wires will have occurred in order to counterbalance the interruption of current in the third wire, the motor will apparently operate quite satisfactorily when supplied with current in a single phase only.

Further investigation will show that when supplied with single-phase current only, the properties of the motor are somewhat different from those of the ordinary three-phase motor. Thus, if the motor be stopped and supplied with single-phase current while at rest, it will not start itself. If, however, it be started by hand it will probably continue to rotate and gradually attain its normal speed. Further, it will be found that the motor may be started equally well in either direction, and will gain speed and continue to rotate with complete indifference as to its original direction of rotation.

The explanation of this behaviour is to be found in the fact, to be explained immediately, that, when stationary, there is an oscillating field (instead of a rotating field) set up by the single-phase alternating current in the rotor. Let Fig. 193 represent diagrammatically the stator coils of a two-pole motor with squirrel-cage rotor, the phase supplied being shown by a firm line, the idle coils being shown by dotted lines. The flux due to the winding A A^1 will pass through the rotor in such a way as to induce currents in the conductors a, b, c, in one direction, and in the conductors a^1, b^1, c^1 in the opposite

direction, and these conductors may be imagined to form circuits $a\ a^1$, $b\ b^1$, $c\ c^1$, &c., through which the stator flux is threaded, and in which it will produce currents. The whole of the rotor conductors may be imagined to be divided up in this way into separate circuits, the axis

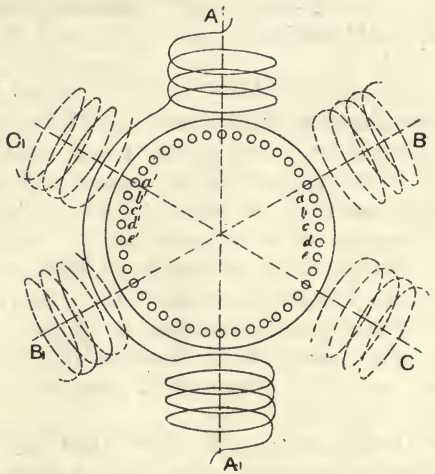


FIG. 193.—PRINCIPLE OF SINGLE-PHASE MOTOR.

of each circuit coinciding with the axis of the stator coil $A\ A^1$. These currents will continue to produce a magnetic field in the rotor along the axis $A\ A^1$. This field will be an alternating field having the periodicity of the stator current and being 90° out of phase with it (since the rotor currents are induced by the *change* of stator field and are greatest when the *change* is greatest, *i.e.*, when the actual field is zero).

If now the rotor in Fig. 193 is moved round fairly rapidly by hand, the conductors under $A\ A^1$ will move across the flux at $A\ A^1$, and will have currents induced in them, the strength of which will depend upon the strength of the flux and the speed of rotation. The conductors in the neighbourhood of $b\ b^1$ will move *along* the direction of the flux, instead of across it, and will have little or no current induced in them by the rotation. The rotation will consequently produce currents in the rotor winding which will set up a flux in the direction

perpendicular to $A\,A^1$, and *in phase* with the flux generated by the stator current.

When the rotor rotates there are consequently two separate effects, each tending to produce currents in the rotor winding and consequently each tending to produce an alternating rotor flux. The relation between these fields will be as follows, from what has just been said: One field, having the same direction as the stator flux and 90° out of phase with it, and a second field perpendicular to the axis of the stator flux, but in phase with it. The second field is due to the rotation of the rotor, and will depend for its direction upon the direction in which the rotation occurs.

As already explained in the discussion of the production of a rotating field, the necessary conditions for the existence of such a field are fulfilled by the formation of two alternating magnetic fluxes in two directions perpendicular to each other, the phase of the fluxes differing by $\frac{1}{4}$ period. These conditions therefore will exist in the rotor of the induction motor supplied with single-phase current as soon as it is rotated with sufficient speed to produce the current which forms the second field. At slow speeds the currents produced by the motion of the conductors across the stator field will be weak and the field produced by them will also be weak. The rotating field will consequently be unequal in strength and will have its maximum strength as it rotates into the direction of the axis of the stator field, and will be weakest when it has moved into the position at right angles to this. We shall consequently obtain at first a field rotating with the full speed corresponding to the periodicity of the supply, but varying in strength during each revolution. As the speed rises, the field will become more uniform until ultimately, when the speed attains its steady value, we shall have a rotating field differing only slightly from the uniform field to be found in a two-phase motor.

The increase in current in the single phase winding is due to the fact that this now has to supply the whole of the magnetising current of the motor, as well as the current necessary to balance the demagnetising ampere-turns of the rotor.

It is interesting to note that the rotating field will now be capable of acting on the idle windings of the stator which we assumed to be cut out of circuit. Thus, if one wire of a three-phase motor be interrupted, while the rotor is revolving, so as to convert the motor into a single-phase motor, and voltmeters are connected across the two idle phases, they will be found to have voltages induced in them of practically the same value as that of the supply. Further, the phase of the voltages induced in these windings by the rotating field will be exactly 120° respectively in advance and behind the phase of the voltage in the winding which is still supplied with current.

Starting Devices of a Single-phase Motor.—It has been explained that a single-phase motor with a short-circuited rotor is not self-starting, since no rotating field is set up in the stationary rotor. The various methods employed for starting such motors consist in the addition of an extra winding to the stator between the main windings, *i.e.*, so situated as to produce a field in a direction perpendicular to the field of the main winding. This extra winding must then be supplied by some means with an alternating current *out of phase* with the current of the main winding, since this is the condition for the production of a rotating field. The chief difference in the starting devices employed by different makers lies in the method of producing the phase-difference in the "starting phase." The usual arrangements consist of either an inductive resistance or condenser in the starting circuit, or a non-inductive resistance in the main circuit. By any of these means a difference in the relative inductance of the main and auxiliary circuits is produced, and a difference of phase in the currents in them is brought about.

The starting torque of a single-phase motor is usually low and the starting current is relatively high; it is consequently desirable whenever possible to start up under light loads only. The difference in phase produced between the currents in the main and auxiliary phases is only small, and consequently the flux perpendicular to the main flux is weak. It is not of much use to make the auxiliary winding itself highly

inductive by the shape given to the slots in which it is wound, since the rotor currents will largely neutralise the leakage lines and so destroy the self-induction. Hence, it is usual to have a large choking coil in the starting circuit external to the motor, and to make the auxiliary winding itself of only a few turns, so that its self-induction forms only a small part of that of the auxiliary circuit. In this case, it is evident that the higher the self-induction of the coil, the less will the current be in the circuit. Consequently the greater the lag obtained, the smaller will be the strength of the flux.

In the case of the Heyland motor, the starting coil is wound in square holes, so as to give a large leakage and great lag, but the winding is connected directly to the mains and a very strong flux is obtained and a large starting torque. The impedance of the starting phase is thus kept smaller than that of the running phase, although the self-induction is greater, and by the special method of winding the leakage flux is counteracted to a comparatively small extent by the rotor currents. The current is greater in the starting phase than in the main phase, but the total current at starting under full load does not exceed twice the normal full-load current.

The measurements to be made on single-phase motors are practically identical with those already described for polyphase motors, except that special attention must be paid to starting current and torque, as it is chiefly in this respect that different types vary. It will be found that the maximum torque which can be overcome by a single-phase motor is usually lower than a three-phase or two-phase motor of the same rated output.

Induction Motor Diagrams.

Graphic Representation of the Induction Motor.—As in the case of the static transformer, the relation between the various electrical quantities may be shown graphically by vector diagrams. This method of representation is of great importance practically, as well as theoretically.

Adopting the same general method of procedure as in the transformer diagrams, Figs. 85-89, we begin the construction as follows :—

For simplicity, assume no losses in the stator winding.

Then, for a given applied voltage, the primary back voltage will be constant and equal to the applied voltage. Consequently, the primary flux, producing the back voltage, will be constant and at right angles to it in phase.

Let Z_s = stator flux.
Z_r = rotor flux.
z = leakage flux.

In Fig. 194 draw $O\,E_s$ = primary voltage to scale of volts.
= back E.M.F.
$O\,A$ = stator flux = Z_s to scale of flux.
$O\,C_s$ = stator current to scale of amps.,
making an angle ϕ with $O\,E_s$ equal to the angle of lag of current in primary winding. The line $O\,A$ is, of course, perpendicular to the voltage line $O\,E_s$, since the voltage

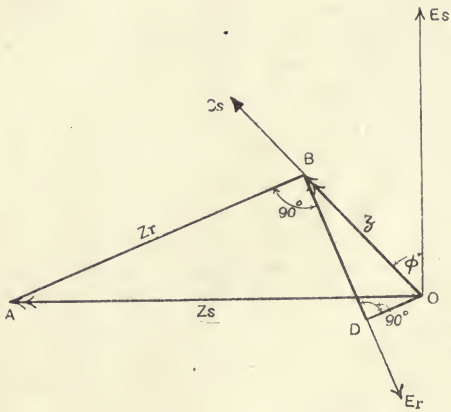


FIG. 194.—SIMPLE DIAGRAM OF INDUCTION MOTOR.

is proportional to "rate of change of flux," and is consequently perpendicular in phase to the flux.

As in the case of a transformer, the primary flux partly enters the secondary winding, producing the secondary voltage, and partly forms the stray field. For constant primary volts the total primary flux is constant, and we may write

$$Z_s = Z_r + z = \text{constant}.$$

Also z is proportional to $\dfrac{\text{magnetising current}}{\text{reluctance of leakage path}}$

hence z will always be in phase with the magnetising current. Mark off a length $O\,B$ along $O\,C_s$, to represent

the leakage flux on the scale of flux (the leakage flux must evidently be in phase with the primary current, since it is unaffected by the secondary winding, whereas the rotor flux is due to the resultant action of both primary and secondary currents). Then the triangle $A\ O\ B$ will be the triangle of fluxes for the motor, since the two sides $O\ A$, $O\ B$, represent the primary and leakage fluxes, which must have as their resultant the third side $A\ B$, which will, consequently, represent the rotor flux.

z is the total leakage flux of the motor, and therefore includes the rotor leakage. Hence, we can assume the rotor winding to be non-inductive, as its self-induction is already taken account of in the length of $O\ B$. The rotor current will, consequently, be in phase with the E.M.F., which produces it; that is, perpendicular in phase to the rotor flux Z_r. Draw $B\ E_r$ perpendicular to $B\ A$ to represent the induced rotor voltage. Draw $O\ D$ perpendicular to $B\ E_r$. The triangle $O\ B\ D$ will then be the triangle of primary, secondary, and magnetising currents, drawn to a reduced scale, since the line $O\ B$ is in phase with the stator current, the line $B\ D$ is in phase with the rotor current, and the line $O\ B$, being perpendicular to $B\ D$, and parallel to the line $B\ A$ = rotor flux, is in phase with the motor magnetising current. $O\ D$ will always be perpendicular to $B\ D$ and parallel to $B\ A$ at all loads, and represents the current which maintains the flux in the magnetic circuit of the machine.

The diagram obtained in this way represents the relation between the most important quantities in an induction motor.

Heyland Diagram.—By means of a Heyland diagram the relations between the quantities at all loads can be simply obtained. In Fig. 194, the line $A\ B = Z_r \propto \frac{\text{primary magnetising ampere turns}}{\text{reluctance of magnetic circuit}} \propto \frac{\text{magnetising current}}{\text{reluctance}}$

$\propto \frac{O\ D}{R}$ where R = reluctance of magnetic circuit.

Therefore $\frac{A\ B}{O\ D} \propto \frac{1}{R}$ = constant. That is to say, the line $O\ A$ will always be cut at a definite point by the line $B\ D$; calling this point F, the line $B\ D$ will

always pass through F, no matter how the load varies. Further, the angles at D and B are right angles, and consequently the points D and B will always lie upon the semi-circles $A\ B\ F$ and $F\ D\ O$, drawn upon $A\ F$ and $F\ O$. Their positions will depend upon the magnitude of the primary and secondary currents, that is, upon the load on the motor. Thus we may re-draw the diagram of Fig. 194 as shown in Fig. 195, which is the simplest form of Heyland's diagram.

There will be two extreme conditions, viz., when the stator current has its maximum and minimum values. The maximum current will flow when the motor is so heavily loaded that the rotor is pulled up, and the rotor currents have their maximum value owing to the large slip. In this case the motor becomes practically a static transformer with short-circuited secondary winding. The primary flux which cuts the rotor conductors will produce rotor currents of such a magnitude that the rotor flux will exactly counteract the primary flux (if we neglect the losses in the rotor winding), and the resultant flux through the rotor becomes nil. Under these conditions the only portion of the primary flux which remains unneutralised is the leakage flux, and consequently we may write—stator flux = leakage flux or $Z_s = z$. At the same time rotor flux = $Z_r = O$.

This condition would be represented in the diagram if B were made to move to the left until it coincided with A, when $B\ O$, the leakage flux, would become equal to $A\ O$.

The other extreme case possible is when the motor is unloaded and runs synchronously (again the losses being neglected), in which case the rotor current becomes zero, and the primary current is only the magnetising current, as in the case of an unloaded transformer. The rotor and stator fluxes are then equal, the leakage flux being practically zero. This case would be represented in the diagram if B were considered to have moved round to the right until it coincided with the point F. In this condition $A\ F = A\ B$ and $F\ B = O$.

In both limiting cases the stator flux Z_s is the same, since it depends only on the voltage applied to the motor, and in both cases the primary current is entirely spent in producing this flux, since the motor is not doing useful

work in either limiting condition. The amount of the primary current is consequently determined in either case by the reluctance of the magnetic path (which is different for the two cases, being in one case the usual path through rotor and stator cores, and in the other case

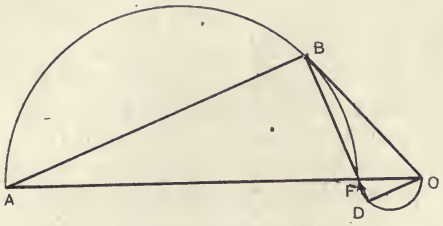


FIG. 195.—SIMPLE HEYLAND DIAGRAM.

the leakage path), through which the primary current has to force the constant flux of Z_s lines. Consequently, if R = reluctance of main magnetic circuit through stator and rotor, and r = reluctance of leakage paths

$$\frac{\text{Primary current at still-stand}}{\text{Primary current at synchronism}} = \frac{\text{reluctance of leakage path}}{\text{reluctance of main circuit}} = \frac{r}{R} = \frac{O\,A}{O\,F} \text{ in Fig. 195.}$$

The reciprocal of this ratio is often called the *leakage factor*, and is denoted by the letter v, since it determines the magnetic leakage of the motor.

The importance of the diagram given in Fig. 195 for our present purpose is that two simple measurements on a motor enable us to determine the primary current at still-stand and at synchronism, and from these measurements the diagram can be drawn.

Diagram for Commercial Motor.—In the simple diagram, Fig. 195, the losses in the motor are neglected. In the commercial motor the losses to be considered are—

(*a*) Iron and mechanical losses ;

(*b*) Copper losses due to resistance of rotor and stator windings.

The sum of losses (*a*) is approximately constant at all loads, for a constant voltage applied. They form the

no-load losses of the motor. The losses (*b*) increase in proportion to the square of the current in each winding. In the stator, the effect of the copper losses is a reduction in the voltage made use of by the motor. Thus, effective voltage is $E - C_s^2 R$, instead of E.

In the rotor, the effect of the copper loss is to diminish the current, and thus to diminish the output.

The application of the diagram to the case of a complete motor must now be briefly given, and will then be further explained by an example from an actual motor. Referring to Fig. 196, which is the diagram Fig. 195 extended to apply to the more complete conditions, we can represent the various quantities in the following manner:—

(1) **Power Supplied to Motor.**—The apparent watts supplied to a 3-phase motor are $1 \cdot 73 \times E \times C_s$, where E and C_s are the line current and voltage supplied. These can therefore be obtained from the diagram by multiplying the length of the line $O\,B$ measured on the scale of amperes by $1 \cdot 73$ and by the terminal voltage. The true power supplied will be due to the energy current, *i.e.*, the component of $O\,B$ in phase with $O\,E_s$, represented by the line $B\,G$ parallel to $O\,E$. Thus power supplied $= B\,G \times 1 \cdot 73 \times E$, and the power supplied to motor will always be proportional to the length $B\,G$.

(2) **Torque.**—This is proportional to the product of the rotor flux Z_r and the rotor current C_r. Hence, except for copper losses in the stator, the torque will be proportional to $A\,B \times B\,F$ in the diagram. In the actual motor, however, the rotor flux is diminished by the voltage lost in the resistance of the stator. $A\,B$ is the direction of the rotor flux, hence in the triangle of fluxes $A\,B\,O$ (where $A\,B = Z_r$, $A\,O = Z_s$, $O\,B = z$) we must diminish the length $A\,B$ by a portion $B\,H$ equivalent to the copper drop in the primary. Mark off a length $B\,H$ on the line $A\,B$ to represent this drop at the particular value of the primary current for which $A\,B$ has been drawn. Draw a semi-circle through the three points A H F, and call its centre *f*, which must evidently lie on the line *g q* bisecting $A\,F$ at right angles. Then the length $H\,B$ will be the copper drop corresponding to any position of $A\,B$ and the rotor flux will be represented by $A\,H$, and the length of $B\,H$ will vary,

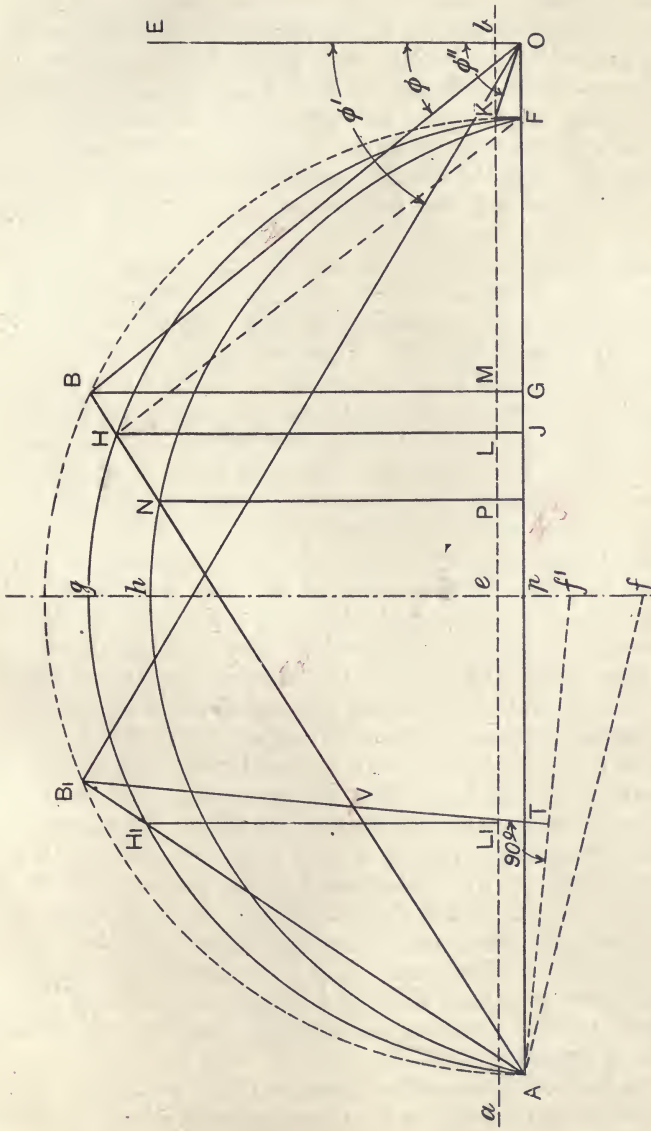


FIG. 196.—COMPLETE HEYLAND DIAGRAM.

as H moves along the circle according to the value given to $A\,B$.

The torque is consequently proportional to $A\,H \times B\,F$, that is to the area of triangle $A\,H\,F$, or since the base is constant, torque $\propto$ height of triangle $A\,H\,F$, which $\propto H\,J$. This is the total torque exerted and includes that spent in overcoming friction, &c.

In order to separate the torque expended in overcoming the friction of the motor itself from the total torque exerted by the rotor conductors we proceed as follows :—

$O\,F$ represents the no-load magnetising current. There will, however, be an energy component of the no-load current $F\,K$ in phase with $O\,E$. Hence, draw $O\,K$ the total no-load current. The component $F\,K$ perpendicular to $O\,F$ will then represent the energy component spent in overcoming friction losses, &c., and must be subtracted from the total torque current $H\,J$ to give the useful torque on the shaft. Thus all lines $H\,L$ may be taken to represent the effective torque of the motor, on some scale not yet determined.

(3) **Output of Motor.**—This is the product of the torque exerted by the rotor shaft and the speed. It would thus be proportional to the line $H\,L$, which represents the torque exerted by the rotating field on the rotor (since the speed is practically constant), except for the fact that some of this turning effort is taken up in generating the power spent by the rotor current in heating the rotor conductors. This is practically equivalent to a portion of the rotor flux being spent in overcoming the rotor resistance, and may be taken into account by reducing the rotor flux line $H\,A$ by a piece $H\,N$ proportional to the electromotive force spent in overcoming the rotor resistance. Then all such lines as $N\,P$ perpendicular to $a\,b$ are proportional to the output of the motor. The point N will lie on a circle drawn through the points $A\,N\,F$, and having its centre at some point f on the line $g\,q$. This circle may be called the circle of output.

(4) **Slip of Motor.**—The slip is directly proportional to the rotor current, and inversely proportional to the rotor flux, hence

$$\text{Slip} \propto \frac{H\,F}{H\,A}.$$

When the motor is overloaded until it stops, the slip becomes equal to 1, and under these conditions the output of the motor is nil, and the line $N\,P$ must become a point on the line $a\,b$. The line $O\,B$ will then be in a position $O\,B^1$ such that the line $B^1\,A$ is a tangent to the output semi-circle $A\,N\,F$, and $B^1\,A\,f$ is consequently a right angle (see Fig. 198).

Draw from B^1 a line $B^1\,T$ perpendicular to $A\,f^1$, which is the radius of the circle $A\,H\,F$, T being its intersection with $A\,O$, and so obtain the line $V\,T$. This line will be proportional to the slip, because angle $B^1\,T\,A$ = angle $A\,H\,F$, and consequently in the similar triangles $A\,H\,F$, $A\,T\,V$

$$\text{Slip} \propto \frac{H\,F}{H\,A} \propto \frac{V\,T}{A\,T}, \text{ or since } A\,T \text{ is constant}$$

$$\text{slip} \propto V\,T.$$

The various quantities represented by the diagram may be summarised as follows :—

Phase of terminal pressure by line $O\,E$.

No-load magnetising current by line $O\,F$.

Stator current by line $O\,B$.

Power factor by $\cos \varphi$ which is a maximum when $O\,B$ is a tangent to the semi-circle $A\,B\,F$.

No-load current by the line $O\,K$.

No-load power factor $\cos \varphi''$.

Energy component of no-load current by $F\,K$.

Rotor current proportional to $F\,B$.

Input of motor ($C_s \times E \times 1{\cdot}73 \times \cos \varphi$) directly proportional to $B\,G$.

Torque of motor by line $H\,L$, maximum torque which motor can exert before pulling up being represented by $e\,g$.

Output of motor proportional to $N\,P$, the maximum output being represented by $e\,h$.

Percentage slip of motor by line $V\,T$, line $B^1\,T$ represents 100 per cent. slip, in position shown Fig. 198.

Starting torque proportional to line $H^1\,L^1$ in Fig. 198, the corresponding starting current being the line $O\,B^1$ and the power factor at starting $\cos \varphi^1$.

EXPERIMENT LIII.—EXPERIMENTAL DERIVATION OF HEYLAND DIAGRAM FOR AN INDUCTION MOTOR.

(*a*) Run the motor without load with current of normal voltage and frequency. Measure the current

taken by the stator = C_0 and the power by means of a watt-meter as described in experiment L.

(*b*) Then make a fresh experiment with the shaft clamped so that it cannot rotate and the rotor conductors short-circuited. Again apply the normal voltage to the stator, and measure the current and watts supplied. If the current taken by the motor under these conditions is too high for safety or convenient measurement, reduce the primary voltage, and multiply the current observed by the ratio $\frac{\text{normal volts}}{\text{volts applied}}$ in order to obtain the true static current, and multiply the watts observed by the square of this fraction in order to get the static watts. This is done, because the no-load watts will be nearly all copper losses, and proportional to C^2.

(*c*) After these tests measure the resistance of the stator winding while still hot by passing a measured direct current through the windings, and observing the drop of potential in them.

These measurements are sufficient for the construction of the complete diagram from which the performance of the motor at all loads can be predetermined.

Construction of Diagram from the Observations.—Draw vertical and horizontal lines $O\ E$, $O\ A$ (see Fig. 198). Calculate the angle of lag ϕ'' at no load from the readings (*a*) and set off the line $O\ K$ representing the no-load current making an angle $E\ O\ K = \phi''$. Make $O\ K$ equal to the no-load current to a scale of amperes, and complete the no-load triangle of currents $K\ F\ O$ by drawing $K\ F$ vertical. Draw the horizontal dotted line $a\ b$ through K. From the readings (*b*) calculate the angle of lag ϕ' for the stationary motor, and hence draw the line $O\ B_1$ equal to the measured static current C_s and making the angle ϕ' with $O\ E$.

Draw the semi-circle $A\ B\ F$ through the points B_1 and K, the centre of the circle being a point p lying on the line $O\ A$. The point A is thus determined. Join $B^1\ A$ and draw $A\ f$ perpendicular to $B^1\ A$. This cuts the vertical through p at the point f, which is the centre of the output circle $A\ N\ F$, drawn through the points A and F. $A\ f$ is drawn perpendicular to $A\ B^1$ because $A\ B^1$ must be a tangent to the output circle, since the

output of the stationary motor is zero under the conditions represented by the position of B^1 (*i.e.*, when the motor is stationary).

In order to find the "torque" circle $A\ H\ F$, the scale to which $B^1\ A$ represents volts must be determined. It is known that $O\ A$ is proportional to the stator flux N_s, and that this is proportional to the impressed voltage E.

Hence, $\frac{\text{volts represented by } B^1\ A}{B^1\ A} = \frac{E}{O\ A}$. Thus the line $O\ A$ represents the primary voltage to the same scale as $B^1\ A$ represents volts. Hence a scale of volts can be constructed by dividing the line $O\ A$ into E parts, each corresponding to one volt.

From B^1 mark off the length $B^1\ H^1$ equal to the drop in the stator windings, due to the static current. This length is equal to $C_s \times R_s$ (*i.e.*, product of stator resistance and static current measured on the scale of volts just determined). The point H^1 divides the line $B^1\ A$ into two parts, which are proportional respectively to the stator and rotor resistances. Draw the "torque" circle $A\ H\ F$ through the points A_1, H^1, and F with its centre f^1 on the line $p\ q$.

In order to obtain the slip line join $A\ f^1$, and draw $B^1\ T$ perpendicular to $A\ f^1$, cutting $A\ O$ at T. The scale of slip is obtained from the fact that the slip corresponding to the position B^1, *i.e.*, the length $B^1\ T$, is 100 per cent. slip.

The following example of an actual test of a three-phase induction motor is taken from Mr. Eborall's paper, read before the Society of Arts in 1901.

The motor was rated to give 80 b.h.p. at 600 revs. per minute at 40 cycles and 350 volts.

Fig. 197 gives the results of the static and no-load tests from which the diagram Fig. 198 was constructed.

The upper curve gives the relation between voltage and static current with the rotor rigidly clamped, and short-circuited; the second curve shows the relation of no-load current to voltage, the motor running light; the third curve gives the corresponding watts input, with the motor running light.

From the curves we see that at a pressure of 202 volts per phase (*i.e.*, a terminal pressure of 350 volts with

a star-connected stator), the static current C_s is 532 amperes, the no-load current C_0 is 40 amperes and the power absorbed at no-load is 1,680 watts per phase. The power taken by the motor at rest with a static current of 532 amperes was estimated from a measurement at a lower pressure to be 25,100 watts per phase at 202 volts. The stator resistance per phase was ·032 ohm.

From these figures,

$$\text{Cos } \phi'' = \frac{1{,}680}{202 \times 40} = 0{\cdot}218$$

$$\text{or } \phi'' = 77 \text{ deg. (approx.)}$$

$$\text{Cos } \phi' = \frac{25{,}100}{532 \times 202} = 0{\cdot}233$$

$$\text{or } \phi' = 77 \text{ deg. (approx.)}$$

The values for the static and the no-load power factor happen to be practically the same in this case.

Referring to Fig. 198 take two lines at right angles $O\,E$ and $O\,A$. From O draw a line $O\,B^1$ making an angle of 77° with the vertical line $O\,E$; this line $O\,B^1$ represents then the phase of the static current and of

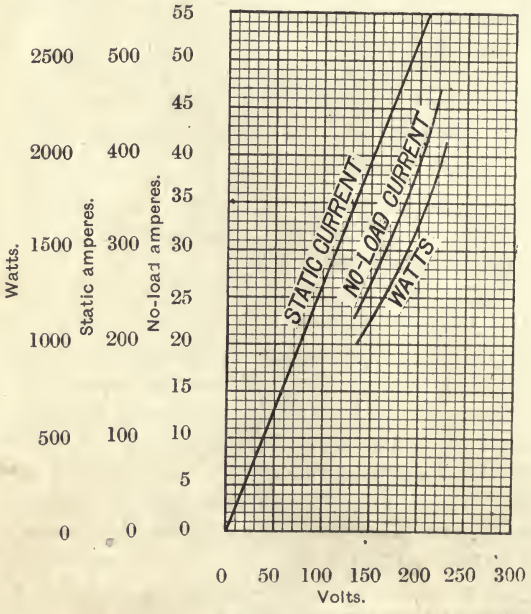


FIG. 197.—NO-LOAD CURVES OF INDUCTION MOTOR.

the no-load current. Next, select a suitable ampere scale for the diagram, say one centimetre corresponds to 20 amperes. Make, therefore, the piece $O\,K$ = 2 cms. = C_o, and make the piece $O\,B^1$ = 26·6 cms. = C_s. Now draw the semi-circle $A\,B\,F$, having its centre along the line $O\,A$ (viz., at point p), and passing through the points B^1 and K already found. Through point K draw the line $a\,b$ parallel to $O\,A$.

To get the circle of the output, the output of the motor is zero for the point B^1, as the motor is then at rest. Therefore join this point to point A, and from the latter draw the line $A\,f$, so that the angle $B^1\,A\,f$ is 90° ; the point f so found is the centre of the "output" semi-circle which can now be drawn at such a radius that it passes through the points A and F. The line $B^1\,A$ is thus a tangent to the circle $A\,N\,F$, so that the value of the output for the point B^1 corresponding to the stator current of 532 amperes, is zero.

To find the circle for the torque, scale off the values $B^1\,A$ and $O\,A$ from the diagram, which will be found to be 6·4 and 27·5 cms. respectively ; the latter corresponds to a pressure per phase of 202 volts. Hence

$$\text{Value of vector } B^1\,A = \frac{6{\cdot}4}{27{\cdot}5} \times 202 = 47 \text{ volts.}$$

The copper drop per phase of the stator for the current C_s = 532 amperes is (532 × 0·032) = 17 volts. Hence, to get the desired point, H^1 on the torque circle, we mark off along $B^1\,A$ from the point B^1 a piece equal to $\frac{17 \times 6{\cdot}4}{47}$ units of length, thus equal to 2·31 cms. Hence the torque circle is fixed by finding the point H^1 ; from a suitable centre along $p\,f$ (viz. f^1) draw it through points A, H^1, and F.

Finally, to get the slip line, drop a perpendicular from the point B^1 on the radius $A\,f^1$, thus getting the line of $B^1\,T$, which turns out to be 6·25 cms. in length. This is equal to a slip of 100 per cent. as the motor is not running ; at the load corresponding to the stator current $O\,B$, torque $H\,L$, and output $N\,P$, the slip is equal to the piece $V\,T$ cut off on the slip line $B^1\,T$, which scales 0·25 cm. Consequently the full load slip is 4 per cent. ; at the maximum load the motor will carry (156 b.h.p.) the slip is 16 per cent.

Several ways of calibrating the diagram will suggest themselves—for instance, we can proceed thus: The diagram having been constructed in the manner indicated above, take any point B which is evidently (from the appearance of the diagram) in the neighbourhood of full load—we could, for instance, draw a line from the point O which would be a tangent to the semi-circle $A\ B\ F$, thus giving the value of stator current at which the power factor has its maximum value. In the present case draw a vector of stator current, $O\ B$ (Fig. 198) 6·4 cms. in length, thus representing 128 amperes. The watt component of this current is the vector $B\ G$ which is found to be 5·6 cms. in length, and consequently 112 amperes. At this particular load on the motor, therefore, the watt input is

$$(112 \times 202 \times 3) = 68{,}000 \text{ watts (approximate).}$$

Now at this load the total motor losses are as follows :—

(1) The light-load losses are 1,680 watts per phase (from the measurement already made), or 5,040 watts total.

(2) The stator copper loss for the current $O\ B = 128$ amperes is given by the resistance per phase and by the component $F\ B$ of the stator current, for the C^2R loss caused by the component $O\ F$ has already been reckoned in with the no-load losses above, seeing that $O\ F$ is the (no-load) magnetising current of the motor. From the diagram we scale off $F\ B$ as 5·7 cms. thus 114 amperes. The stator copper loss is therefore

$$(3 \times 114 \times 114 \times 0{\cdot}032) = 1{,}250 \text{ watts.}$$

In the diagram, the line $B\ H$ is proportional to this loss and scales 0·6 cm. Thus we have the scale from which we can determine the losses at any other load.

(3) The rotor copper loss is proportional to the line $H\ N$ in the diagram, which scales 0·9 cm. Consequently we get the rotor loss from the known stator loss by proportion or

$$\text{rotor copper loss} = 1{,}250 \times \tfrac{2}{6} = 1{,}875 \text{ watts.}$$

The total losses in the motor at the load corresponding to the point B are therefore $5{,}040 + 1{,}250 + 1{,}875 = 8{,}165$ watts. The output is thus $68{,}000 - 8{,}165$ watts $= 59{,}835$ watts $= 80$ b.h.p. (approx.)

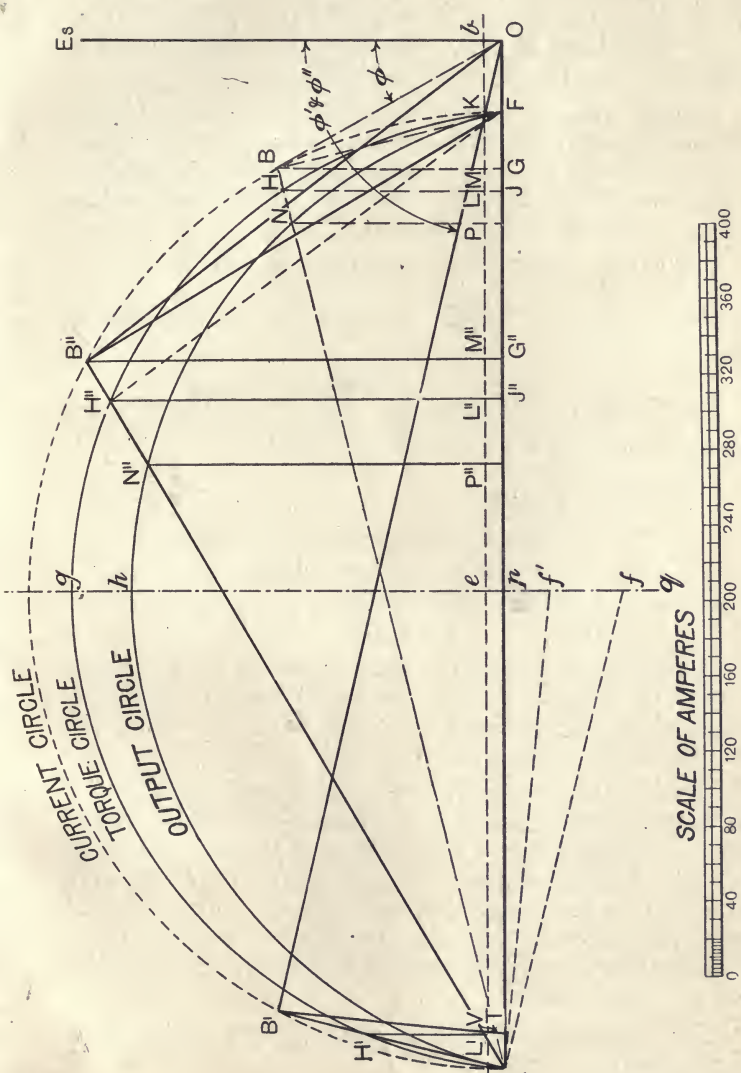


FIG. 198.—COMPLETE HEYLAND DIAGRAM FOR INDUCTION MOTOR.

It happens that the motor gives its rated output when the stator current = 128 amperes; the line $N\ P$ in the diagram thus represents 80 b.h.p. and the value of all such lines as this, now that the value of one of them has been found, is definitely known.

Taking into account the known slip of the motor we could, from the known output, find the value of the torque vector $H\ L$, in pound-feet, and having one such value, the values for all the other lines representing torque would be definitely fixed.

The value of the torque is obtained as follows :—

H.P. developed = $\frac{\text{watts}}{746}$ = say 80 h.p. in the present case.

The speed at full load is $\frac{600 \times 96}{100}$ = 576 revs. per minute, since there is a slip of 4 per cent.

Since h.p. = $\frac{2\pi n T}{33{,}000}$

where T = torque in pound feet.

and n = revs. per minute.

$$T = \frac{\text{h.p.} \times 33{,}000}{2\pi n} = \frac{80 \times 33{,}000}{2\pi \times 576} = 730 \text{ lb.-ft.}$$

If the line $O\ B$ had not turned out to be the stator current for exactly 80 h.p. we could have immediately found where the vector for 80 h.p. lies in the diagram with the help of this value.

Thus if the point B'' had been taken instead of B the output corresponding to the line $B''\ G''$ could be found exactly as described. From this a scale of watts could be plotted corresponding to the length of $B''\ G''$ and, after finding the length corresponding to 80 h.p. on this scale, the point B would be determined by drawing a horizontal line at the right height and finding B where this line cut the " output " circle.

The complete performance of the motor is thus given by the diagram for all loads.

In practice the diagram is accurate enough for all practical purposes, there being a slight error on the right side, *i.e.*, the actual performance of the motor is rather better than that given by the diagram. There are slight errors

which appear to cancel one another out, except for very small motors. The diagram can be made accurate for these also, if the line of no-load losses $a\,b$ is slightly inclined, being drawn with a downward slope of about 30 per cent. from the point K to a. This is not necessary for motors greater than 5 b.h.p. output, but with smaller motors than this the relatively greater no-load losses slightly affect the accuracy of the diagram and render this empirical correction advisable.

Chapter XII.

The Analysis of Alternator Curves.

Composition of Curves.—All curves of current or electromotive force may be considered to consist of a principal sine wave (called the *fundamental*), which is the regular curve to which the actual curve approximates, together with other sine waves of smaller amplitude and having frequencies differing from that of the fundamental.

The frequency of these minor waves will always be some multiple of the frequency of the fundamental, *i.e.*, of the periodicity of the supply. Further, there will be only one sine curve having a given frequency, since two sine

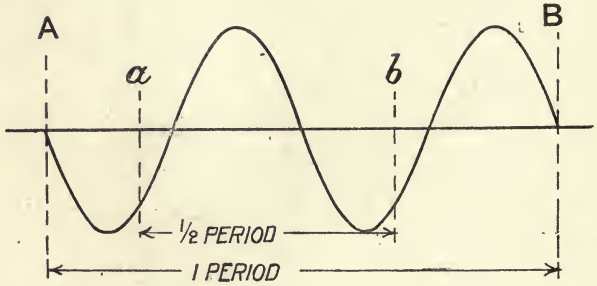


Fig. 199.—Even Harmonic.

waves of equal period may always be added together to give a single resultant sine wave. These small waves, which have each a periodicity which is a multiple of that of the fundamental, are called *harmonics*.

The general expression for a simple harmonic electromotive force may be written in the form

$$e = E \sin (2\pi nt + \theta)$$

where e = instantaneous value of the electromotive force.
n = frequency of the alternator.
t = time counted from any given moment.
θ = phase of the voltage at moment from which time is counted.
E = maximum value of the voltage.

The instantaneous value of the harmonic which has a frequency of $2n$ will be

$$e_2 = E_2 \sin (4\pi nt + \theta_2)$$

where E_2 is the maximum value of this harmonic, and θ_2 is its phase at the same moment as θ is the phase of the fundamental.

The instantaneous value of the voltage composed of fundamental and harmonics will consequently be

$$e = E_1 \sin (2\pi nt + \theta_1) + E_2 \sin (4\pi nt + \theta_2) + E_3 \sin (6\pi nt + \theta_3) + E_4 \sin (8\pi nt + \theta_4) + \ . \ . \ .$$

In this equation the constants E_1, E_2, &c., give the maximum value of the corresponding harmonic.

We may write $\sin (2\pi nt + \theta) = \sin 2\pi nt \cos \theta + \cos 2\pi nt \sin \theta$, and since θ is a constant term, each of the sine functions in the equation may be written as the sum of a sine and a cosine term, and the equation then takes the form

$$e = a_1 \sin 2\pi nt + a_2 \sin 4\pi nt + a_3 \sin 6\pi nt + \ . \ . \ .$$
$$+ b_1 \cos 2\pi nt + b_2 \cos 4\pi nt + b_3 \cos 6\pi nt + \ . \ . \ .$$

where a_1, a_2, a_3, b_1, b_2, b_3 are constants.

This form of the equation is sometimes more convenient.

Harmonics of a Symmetrical Curve.—The wave forms produced by the electromotive force of an alternator are

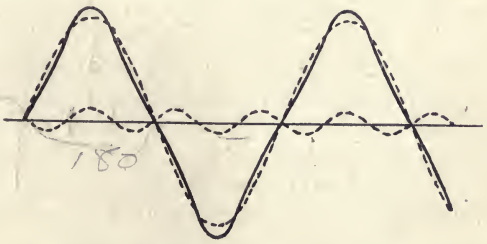


FIG. 200.—THIRD HARMONIC IN OPPOSITION. WAVE PEAKED.

always symmetrical about the horizontal axis, *i.e.*, the negative half waves are exactly similar to the positive half waves. This follows from the fact that the north and south poles of an alternator are similar, and therefore generate similar electromotive forces. From this follows the important fact that the wave forms met with in an alternating circuit contain only "odd harmonics"—that is to say, that the periodicity of any harmonic is 1, 3,

or 5, &c., times the periodicity of the fundamental. That this must be so follows from the consideration that any two points on the curve which are exactly half a period apart must be equal values of the varying quantity, but having opposite signs, since they are values of the electromotive force generated at similar positions under opposite poles. This is only true in the case of curves having a frequency which is an odd multiple of that of the fundamental. Thus, on referring to Fig. 199, if the distance between the two vertical lines A B is one full period of the fundamental, the frequency of the curve will be two per cycle of the main curve, so that the curve is the second (*i.e.*, an even) harmonic. Taking any two vertical lines, such as those shown dotted at a, b,

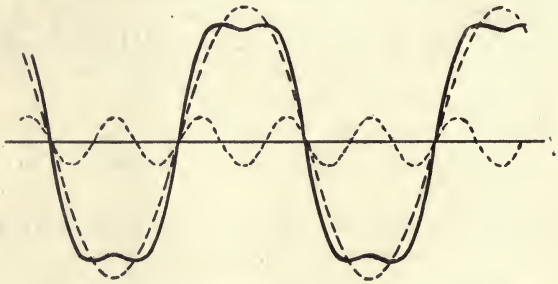


FIG. 201.—THIRD HARMONIC IN PHASE. WAVE FLATTENED.

half a period apart, it is evident that the values separated by half a period are equal, but *similar* in sign. A curve having this as one of its harmonics would, consequently, not be symmetrical in the sense of its having its negative half wave an exact repetition of its positive half wave, although it would be symmetrical in the sense that each wave would be similar to the preceding one.

The expression for the harmonics of a symmetrical wave form obtained from an alternator may consequently be written in the following form, in which the even harmonics are omitted:—

$$e = E \sin (2 \pi nt + \theta_1) + E_3 \sin (6 \pi nt + \theta_3) + E_5 (10 \pi nt + \theta_5) + \ . \ . \ .$$

A curve which contains only *even* harmonics will be symmetrical in a different sense from the curve containing only odd harmonics. Each half-wave wave will be similar

to the previous half wave, but with the *same* sign instead of the *opposite* sign. That is, any two ordinates of the curve half a period apart will be equal, and on the *same* side of the horizontal axis. Examples of such curves are sometimes met with, as in curves of variation induced in the exciting circuit of an alternator by the armature current. (See Fig. 104, page 224.)

Curves containing both odd and even harmonics are not symmetrical in any ordinary sense of the term. Such a curve is the one shown in Fig. 203.

Form of the Wave.—It is important to notice the effect of the addition of a harmonic to a fundamental wave upon the shape of the resulting curve. If the two curves are coincident in phase when the fundamental harmonic passes through its zero value, as in Fig. 201, the resultant curve is flattened at the top. If, on the other hand, the two curves are in opposition of phase at this point, *i.e.*, they cross the axis in opposite directions, as shown in Fig. 200, the resultant curve becomes peaky. In either case the half waves are symmetrical about a vertical line. If the harmonic is intermediate in phase, like that shown in Fig. 202, the curve becomes irregular, having one side flattened and the other raised to a peak.

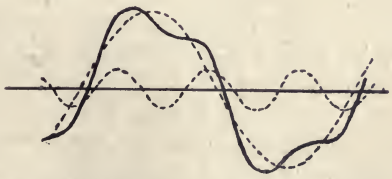


FIG. 202.—THIRD HARMONIC OUT OF PHASE. WAVE DISTORTED.

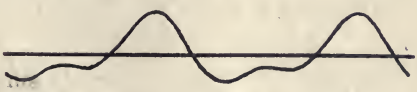


FIG. 203.—CURVE CONTAINING BOTH ODD AND EVEN HARMONICS.

In analysing a curve, the number of harmonic waves which must be considered will depend on the extent to which the wave form differs from a simple sine curve, and the degree of accuracy with which it is necessary to express the true shape. Usually it is sufficient to

consider the third and fifth harmonic in addition to the fundamental. This will, however, be largely governed by the number of slots per phase in which the armature is wound, the shape of the poles, &c.

Example of Curve Analysis.—The following examples of the analyses of a curve obtained experimentally will enable the student to analyse for himself any curve which he obtains, even if he is not able to follow the mathematics on which the methods are based. He may omit the mathematical proof if he is not able to follow it.

Outline of Mathematical Proof of Method.—Denoting the variable quantity by e,

$$\text{Let } e = A_0 + A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \ldots + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \ldots \quad (1)$$

Multiply both sides of the equation by $\sin\theta$, then

$$e \sin\theta = A_0 \sin\theta + A_1 \sin^2\theta + A_2 \sin 2\theta \sin\theta + A_3 \sin 3\theta \sin\theta + \ldots + B_1 \cos\theta \sin\theta + B_2 \cos 2\theta \sin\theta + B_3 \cos 3\theta \sin\theta + \ldots \quad (2)$$

Integrating both sides of this expression between the limits 0 and 2π, we have :—

$$\int_0^{2\pi} e \sin\theta \, d\theta = \int_0^{2\pi} A_1 \sin^2\theta \, d\theta \quad \ldots \quad (3)$$

the other terms all vanishing with the limits chosen. Hence, mean value of $e \sin\theta$ = mean value of $A_1 \sin^2\theta$

$$= \frac{A_1}{2}$$

i.e., $A_1 = 2 \times$ mean value of $e \sin\theta \quad \ldots \quad (4)$

Similarly by multiplying equation (1) by

$\left.\begin{matrix}\sin 2\theta, \sin 3\theta \ldots \\ \cos\theta, \cos 2\theta, \cos 3\theta \end{matrix}\right\}$ the values of $\left\{\begin{matrix} A_2, A_3 \ldots \\ B_1, B_2, B_3 \ldots \end{matrix}\right.$ may be determined.

Thus $A_2 = 2 \times$ mean value of $e \sin 2\theta$,

$B_1 = 2 \times$ mean value of $e \cos\theta$.

$B_2 = 2 \times$ mean value of $e \cos 2\theta$,

And so on.

Since all symmetrical wave forms will consist of odd or even terms only, only half of the constants need to be determined. In any ordinary alternating circuit the wave form will contain odd harmonics only.

Method of Analysing Curve (Summation).—Plot one half wave of the curve to a large scale on squared paper, the horizontal scale being chosen to enable the curve to be divided conveniently into a number of equal parts. These parts may conveniently correspond to 30 of 6° each, 15 divisions of 12° each, or 12 divisions of 15° each, according to the accuracy desired.

In order to obtain the value of the fundamental curve proceed as follows: Measure the height of the curve at each of the horizontal divisions decided upon, tabulating the values as shown in Table I., column 2, entering at the same time the angle corresponding to each ordinate in column 1. In column 3 enter the sines of the angles given in column 1, as found in any set of mathematical tables. Column 4 is then obtained by multiplying together the figures in columns 2 and 3.

Column 5 gives the cosines of the angle in column 1, and column 6 is the product of these cosines by the numbers in column 2.

Column 7 is obtained by writing down the values of the sines of 3 × angles in column 1. Multiplying these sines by the figures in column 2 we obtain column 8. Column 9 contains the values of the cosines of the angles whose sines are in column 7, and the tenth column gives the products obtained by multiplying them by column 2. These columns of figures will give the fundamental and the third harmonic. If the fifth or higher harmonics are also present, further similar columns for $\sin 5\,\theta$, $e \sin 5\,\theta$ and $\cos 5\,\theta$, $e \cos 5\,\theta$, &c., must be added.

After filling in the columns as described, add together all the figures in column 4 and divide by half the number of these figures, thus obtaining twice the mean value. Carry out the same operation for columns 6, 8, 10, &c. In this way the constants A_1, B_1, A_3, B_3, &c., are obtained.

The expression for the curve is then given by substituting the values found for these constants in the equation

$$e = A_1 \sin\theta + B_1 \cos\theta + A_3 \sin 3\,\theta + B_3 \cos 3\,\theta + \ldots.$$

This equation may be put into a simpler form by making the following substitutions:—

$$F_1 = \sqrt{A_1^2 + B_1^2} \qquad F_3 = \sqrt{A_3^2 + B_3^2}, \quad \&c.$$

$$\tan\phi_1 = \frac{B_1}{A_1} \qquad \tan\phi_3 = \frac{B_3}{A_3} \qquad \&c.$$

TABLE I.

1	2	3	4	5	6	7	8	9	10
θ	e	Sin θ	$e \sin \theta$	Cos θ	$e \cos \theta$	Sin 3 θ	$e \sin 3\theta$	Cos 3 θ	$e \cos 3\theta$
	2 Mean $= A_0$ $= 0$ for 360°.		2 Mean $= A_1$		2 Mean $= B_1$		2 Mean $= A_3$		2 Mean $= B_3$

TABLE II.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
θ	$f(\theta)$	$\log f(\theta)$	$L \sin \theta$	$\log \frac{f(\theta)}{\sin \theta}$	$\frac{f(\theta)}{\sin \theta}$	$\log \frac{f(\theta)}{\cos \theta}$	$\frac{f(\theta)}{\cos \theta}$	$L \sin 3\theta$	$\log \frac{f(\theta)}{\sin 3\theta}$	$\frac{f(\theta)}{\sin 3\theta}$	$\log \frac{f(\theta)}{\cos 3\theta}$	$\frac{f(\theta)}{\cos 3\theta}$	$\sin 5\theta$	$\frac{f(\theta)}{\sin 5\theta}$	$\frac{f(\theta)}{\cos 5\theta}$
0	26·5	1·4232	$-\infty$	$-\infty$	0	1·4232	26·50	$-\infty$	$-\infty$	0	1·4232	26·50	0	0	26·50
6	37·9	1·5786	9·0192	0·5978	3·96	1·5762	39·69	9·4900	1·0686	11·71	1·5568	36·04	·5	18·95	32·82
12	45·6	1·6590	9·3779	0·9769	9·48	1·6494	44·61	9·7692	1·4282	26·80	1·5670	36·90	·866	39·49	22·80
18	49·6	1·6955	9·4900	1·1855	15·33	1·6737	47·18	9·9080	1·6035	40·14	1·4647	29·16	1·000	49·60	0
24	51·0	1·7076	9·6093	1·3169	20·74	1·6683	46·59	9·9782	1·6858	48·51	1·1976	15·76	·866	44·17	−25·50
30	50·2	1·7007	9·6990	1·3997	25·10	1·6382	43·47	10·0000	1·7007	50·20	−∞	0	·5	25·10	−43·47
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
162	25·3	1·4031	9 4900	0·8931	7·82	1·3813	−24·06	9·9080	1·3111	20·46	1·1723	−14·87	1·00	25·30	0
168	6·2	0·7924	9 3179	0·1103	1·29	0·7828	− 6·06	9·7692	0·5616	3·64	0·7004	− 5·02	·866	5·37	− 3·10
174	−11·5	1·0607	9·0192	0·0799	−1·20	1·0583	+11·44	9·4900	0·5507	−3·55	1·0389	10·93	·5	−5·75	9·96
					Sum of 30 terms = 990·00 $2 \times \text{mean} = \frac{990·00}{15} = 66·00 = A_1$		Sum of 30 terms = − 1·09 $2 \times \text{mean} = \frac{-1·09}{15} = -0·073 = B_1$			Sum of 30 terms = 344·43 $2 \times \text{mean} = \frac{344·43}{15} = 22·96 = A_3$		Sum of 30 terms = 310·79 $2 \times \text{mean} = \frac{310·79}{15} = 20·72 = B_3$		Sum of 30 terms = − 23·19 $2 \times \text{mean} = \frac{-23·19}{15} = -1·546 = A_5$	Sum of 30 terms = 86·65 $2 \times \text{mean} = \frac{86·65}{15} = 5·776 = B_5$

Equation to curve : $f(\theta) = 66 \sin \theta - ·073 \cos \theta + 22·96 \sin 3\theta + 20·72 \cos 3\theta - 1·546 \sin 5\theta + 5·776 \cos 5\theta$

or $f(\theta) = 66 \sin \theta + 30·93 \sin (3\theta + 42) + 5·97 \sin (5\theta + 105)$

The function then becomes finally :—

$$e = F_1 \sin(\theta + \phi_1) + F_3 \sin(3\theta + \phi_3) + \ldots$$

As an illustration of this method of analysing a curve, its application to the curve in Fig. 204 may be given.

This curve was obtained by the method described in Experiment XV. from a small Pyke & Harris inductor alternator, having 7 inductors. The armature was connected to a condenser having a capacity of 6 micro-farads in series with a single incandescent lamp. The curve was plotted from readings taken at the terminals of the lamp. The R.M.S. volts of the alternator were 80, those across the lamp 51·8, and across the condenser 61·5.

Table II. shows the method adopted for entering up the observations and calculated figures. It will be seen that there are several modifications from the form of Table I.; these were made in order to enable logarithms to be used for the multiplication.

In the case of the fifth harmonic, logarithms were not used on account of the comparative simplicity of the values of sin 5 θ and cos 5 θ.

A separate column for values of the cosines was not considered necessary, since the column of logarithmic sines may be used, taking values of $90 - \theta$, instead of θ, &c.

The method of obtaining the first form of the equation given at the foot of the table will be clear from the description already given. The following calculation shows the steps for arriving at the second, simplified, form of the expression :—

As seen from the first form of the equation

$$A_1 = 66 \qquad B_1 = -0{\cdot}073$$

$$A_1 \sin\theta + B_1 \cos\theta = \sqrt{A_1^2 + B_1^2}. \quad \sin\left\{\theta + \tan^{-1}\left(\frac{B_1}{A_1}\right)\right\}$$

$$= \sqrt{66^2 + (0{\cdot}073)^2}. \quad \sin\left(\theta + \tan^{-1}\frac{-{\cdot}073}{66}\right)$$

$$= 66 \sin(\theta - 0^\circ\ 4') \text{ since } \tan 0^\circ\ 4' = {\cdot}00112 = \frac{{\cdot}073}{66}$$

$$= 66 \sin\theta, \text{ practically.}$$

similarly,

$$A_3 = 22{\cdot}96 \qquad B_3 = 20{\cdot}72$$

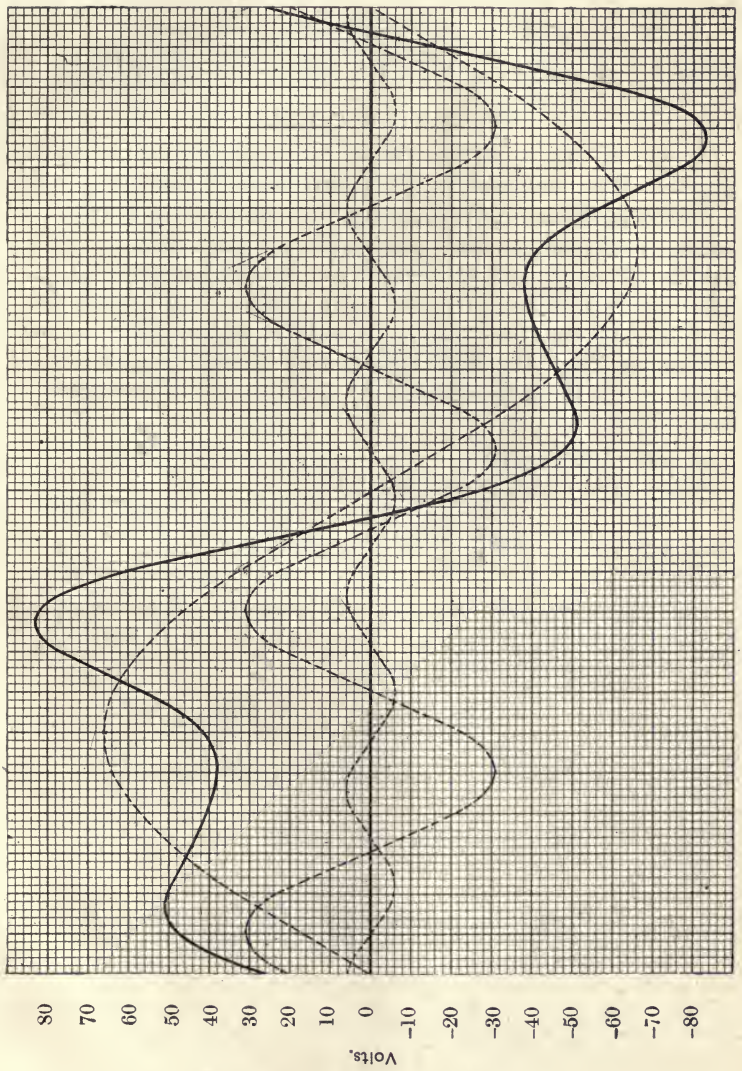


FIG. 204.—EXPERIMENTALLY DETERMINED CURVE AND HARMONICS.

$$A_3 \sin 3\theta + B_3 \cos 3\theta = \sqrt{A_3^2 + B_3^2}. \quad \sin\left\{3\theta + \tan^{-1}\left(\frac{B_3}{A_3}\right)\right\}$$
$$= 30{\cdot}93 \sin (3\theta + 42°)$$

also

$$A_5 = -1{\cdot}546 \qquad B_5 = 5{\cdot}776$$

$$A_5 \sin 5\theta + B_5 \cos 5\theta = \sqrt{A_5^2 + B_5^2}. \quad \sin\left\{5\theta + \tan^{-1}\left(\frac{B_5}{A_5}\right)\right\}$$
$$= 5{\cdot}97 \sin (5\theta + 105).$$

A method of analysis which, although not quite so simple, is less laborious to carry out, is the following :—

Analysis by Method of Superposition.—Again, a short mathematical summary of the principles of this method of analysis will be given, followed by a detailed description of the method of carrying out the calculation in practice. The non-mathematical reader should be able to carry out an analysis from the description and illustration of the method, even if he is obliged to omit the mathematical introduction.

Lemma.—Let $f(\alpha, \beta)$

$$= \sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots$$
$$\ldots + \sin(\alpha + \overline{n-1}\,\beta)$$
$$= \frac{\sin\frac{1}{2}\beta}{\sin\frac{1}{2}\beta}\left\{\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots \right.$$
$$\left. \ldots + \sin(\alpha + \overline{n-1}\,\beta)\right\}$$
$$= \sin\tfrac{1}{2}\beta\left\{\begin{array}{l}\cos(\alpha - \frac{1}{2}\beta) - \cos(\alpha + \frac{1}{2}\beta) + \cos(\alpha + \frac{1}{2}\beta) - \cos \\ (\alpha + \frac{3}{2}\beta) + \ldots \cos(\alpha + \overline{n-1}\,\beta - \frac{1}{2}\beta) - \\ \cos(\alpha + \overline{n-1}\,\beta + \frac{1}{2}\beta)\end{array}\right\}$$
$$= \frac{1}{2\sin\frac{1}{2}\beta}\left\{\cos(\alpha - \tfrac{1}{2}\beta) - \cos(\alpha + \overline{n-1}\,\beta + \tfrac{1}{2}\beta)\right\}$$
$$= \frac{1}{2\sin\frac{1}{2}\beta} \times 2\sin\left(\alpha + \frac{n-1}{2}\beta\right)\sin\left(\frac{n-1}{2}\beta + \tfrac{1}{2}\beta\right)$$
$$= \frac{\sin\frac{n}{2}\beta}{\sin\frac{1}{2}\beta} \cdot \sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

Now if $\frac{n}{2}\beta = 0$, and $\sin\frac{\beta}{2}$ does not equal 0, then $f(\alpha, \beta) = 0$.

But in the special case when $\frac{\beta}{2} = \pi$, $\beta = 2$

and the original expression becomes

$$f(\alpha, \beta) = \sin\alpha + \sin(\alpha + 2\pi) + \sin(\alpha + 4\pi) + \ldots$$
$$\ldots + \sin(\alpha + \overline{n-1}\,2\pi)$$
$$= \sin\alpha + \sin\alpha + \sin\alpha + \ldots$$
$$= n \sin\alpha$$

i.e., an expression of the form
$\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \quad \ldots \sin(\alpha + \overline{n-1}\,\beta)$
becomes equal to

$$n \sin \alpha \qquad \text{when } \beta = 2\pi.$$

Let the curve to be analysed be written in the following form

$$f(\theta) = y_1 = a_1 \sin(\theta + \alpha_1) + a_2 \sin 2(\theta + \alpha_2) + a_3 \sin 3(\theta + \alpha_3) + \ldots$$

Then we may write :

$$f\left(\theta + \frac{2\pi}{n}\right) = y_2 = a_1 \sin\left(\theta + \alpha_1 + \frac{2\pi}{n}\right) + a_2 \sin 2\left(\theta + \alpha_2 + \frac{2\pi}{n}\right) + a_3 \sin 3\left(\theta + \alpha_3 + \frac{2\pi}{n}\right) + \ldots$$

Similarly

$$f\left(\theta + \frac{4\pi}{n}\right) = y_3 = a_1 \sin\left(\theta + \alpha_1 + \frac{4\pi}{n}\right) + a_2 \sin 2\left(\theta + \alpha_2 + \frac{4\pi}{n}\right) + a_3 \sin 3\left(\theta + \alpha_3 + \frac{4\pi}{n}\right) + \ldots$$

Adding these three quantities together,

$$\begin{aligned} y_1 + y_2 + y_3 = a_1 &\left\{\sin(\theta + \alpha_1) + \sin\left(\theta + \alpha_1 + \frac{2\pi}{n}\right) + \sin\left(\theta + \alpha_1 + \frac{4\pi}{n}\right) + \ldots\ldots\right\} \\ + a_2 &\left\{\sin 2(\theta + \alpha_2) + \sin 2\left(\theta + \alpha_2 + \frac{2\pi}{n}\right) + \sin 2\left(\theta + \alpha_2 + \frac{4\pi}{n}\right) + \ldots\right\} \\ + a_3 &\left\{\sin 3(\theta + \alpha_3) + \ldots\ldots\ldots\right\} \end{aligned}$$

The expression thus consists of a number of terms of the form :—

$$a_m \left\{\sin m\left(\theta + \alpha_m\right) + \sin m\left(\theta + \alpha_m + \frac{2\pi}{n}\right) + \sin m\left(\theta + \alpha_m + \frac{4\pi}{n}\right) + \ldots\ldots\right\}$$

By the previously given lemma each of the expressions within the brackets will be zero, unless

$$\sin \frac{2\pi m}{2n} = 0$$

i.e., unless $\frac{m}{n}$ is an integer,

in which case the general term

$$= a_m n \sin m(\theta + \alpha_m)$$

Let $n = 3$; then since $\frac{m}{n}$ must be an integer, all the terms will cancel except those for which m is a multiple of 3, *i.e.*, all harmonics will vanish except the third, sixth, ninth, &c., and

$$y_1 + y_2 + y_3 = 3\,a_3 \sin 3\,(\theta + a_3) + 3\,a_6 \sin 6\,(\theta + a_6) + 3\,a_9 \sin 9\,(\theta + a_9) + \ldots$$

If the curve is symmetrical, even harmonics will not exist, hence the term containing $\sin 6\,(\theta + a_6)$ may be neglected. Further harmonics above the fifth are seldom present of sufficient amplitude to be worth determination. Hence we have for practical purposes

$$y_1 + y_2 + y_3 = 3\,a_3 \sin 3\,(\theta + a_3).$$

By choosing the value of θ such that $3\,(\theta + a_3) = 90°$, we can determine a_3 from observed values of y_1, y_2, and y_3. The method of doing this is, then, as follows :—

From the curve, measure the value of the ordinate for the angle $\theta = \frac{90}{n} = 30°$. Call this y_1.

Similarly measure the ordinate at $\theta + \frac{2\pi}{n} = 30 + 120 = 150$

Call this y_2

also the ordinate at $\theta + \frac{4\pi}{n} = 30 + 240 = 270$

Call this y_3

Add the three measured ordinates together and divide by $n = 3$. The result is the coefficient of $\sin n\,\theta = \sin 3\,\theta$. Similarly by starting from the angle O proceed to obtain the coefficient of $\cos n\,\theta = \cos 3\,\theta$.

The analysis may be further simplified by separating sine and cosine terms, as follows :—

Let

$$f(\theta) = Y_1 = a_0 + a_1 \sin\theta + a_2 \sin 2\,\theta + a_3 \sin 3\,\theta + a_4 \sin 4\,\theta + \ldots$$
$$+ b_1 \cos\theta + b_2 \cos 2\,\theta + b_3 \cos 3\,\theta + b_4 \cos 4\,\theta + \ldots$$

$$f(2\pi - \theta) = Y_2 = a_0 + a_1 \sin(2\pi - \theta) + a_2 \sin 2\,(2\pi - \theta) + a_3 \sin 3\,(2\pi - \theta) + \ldots$$
$$+ b_1 \cos(2\pi - \theta) + b \cos 2\,(2\pi - \theta) + b_3 \cos 3\,(2\pi - \theta) + \ldots$$

$$= a_0 - a_1 \sin\theta - a_2 \sin 2\theta - a_3 \sin 3\theta \ldots .$$
$$+ b_1 \cos\theta + b_2 \cos 2\theta + b_3 \cos 3\theta + \ldots .$$
$$\therefore\ Y_1 + Y_2 = 2\,(a_0 + b_1 \cos\theta + b_2 \cos 2\theta + b_3 \cos 3\theta + \ldots)$$
$$Y_1 - Y_2 = 2\,(a_1 \sin\theta + a_2 \sin 2\theta + a_3 \sin 3\theta + \ldots)$$

Thus $\frac{Y_1+Y_2}{2}$ will consist of the cosine terms of the original expression alone, and $\frac{Y_1-Y_2}{2}$ will be the sine terms

Method of Procedure.—Plot the curve to a large scale, as for the previous method of analysis.

Draw up a table (see Table III.) with the angles showing the sub-divisions which it is proposed to take in the first column. In the second column enter the measured values of the curve corresponding to these values. In column 3 put the values of the curve for the angle (360° – angle in first column). In column 4 enter the difference between the numbers in columns 2 and 3. Column 5 is obtained by dividing the figures in column 4 by two. Column 6 is the sum of the readings entered in columns 2 and 3, while column 7 is the half values of these numbers.

The coefficients of the cosine terms will be obtained from column 7, while those of the sine terms will be got from column 5.

In order to determine the coefficients for the nth harmonic proceed as follows :—

For the sine term, divide the complete period of the curve into n parts, taking the angle $\frac{\pi}{2n}$ as the starting point, since this makes $\sin\theta = 1$.

Measure the ordinates of the curve for the angles $\frac{\pi}{2n}$, $\frac{\pi}{2n} + \frac{2\pi}{n}$, $\frac{\pi}{2n} + \frac{4\pi}{n}$. Add these ordinates together and divide by n. The result is the required coefficient.

For the cosine term, divide the curve into n parts, starting from the angle 0 (making $\cos\theta = 1$).

Measure the ordinates for the angle 0, $\frac{2\pi}{n}$, $\frac{4\pi}{n}$ Add them together and divide by n.

Having obtained one harmonic, *e.g.*, the third, subtract its corresponding ordinates from the original curve, and so obtain a new curve which is the original curve—third harmonic.

The coefficients were worked out as follows:—

TABLE III.

1	2	3	4	5	6	7
θ	$f(\theta) = Y_1$	$f(360° - \theta) = Y_2$	Difference $Y_1 - Y_2$	$\frac{1}{2}$ Differ'ce $\frac{Y_1 - Y_2}{2}$	Sum $Y_1 + Y_2$	$\frac{1}{2}$ Sum $\frac{Y_1 + Y_2}{2}$
0	26·5	26·5	0	0	53·0	26·5
6	37·9	11·5	26·4	13·2	49·4	24·7
12	45·6	− 6·2	51·8	25·9	39·4	19·7
18	49·6	−25·3	74·9	37·45	24·3	12·15
24	51·0	−44·0	95·0	47·5	7·0	3·5
30	50·2	−60·2	110·4	55·2	−10·0	− 5·0
—	—	—	—	—	—	—
—	—	—	—	—	—	—
162	25·3	−49·6	74·9	37·45	−24·3	−12·15
168	6·2	−45·6	51·8	25·9	−39·4	−19·7
174	−11·5	−37·9	26·4	13·2	−49·4	−24·7

NOTE.—In the construction of this table it will be seen from the use made of the figures that not all the figures require to be filled in in all the columns. In the present example only 14 out of 30 values of θ are actually required for the harmonics plotted in Fig. 204. In testing for the higher harmonics, however, a greater number of values was required. It is best, therefore, to put down the first column complete, and to fill in the further columns as required.

Unless this curve is found on trial to be a simple sine curve, determine the next higher harmonic—say 5th—and again subtract its ordinates from the residual curve. Proceed thus until only the fundamental is left. In practice the third and fifth (and occasionally the seventh) harmonics will be the only ones to be determined.

The figures given (Table III.) are taken from the same curve (Fig. 204) as that from which Table II. was derived.

3rd Harmonic.

Cosine Term.		Sine Term.	
Angle.	$\frac{1}{2}$ Sum.	Angle.	$\frac{1}{2}$ Difference.
0°	26·5	30°	55·2
120°	17·8	150°	55·2
240°	17·8	270°	−41·4
	3)62·1		3)69·0
	$20·7 = b_3$		$23·0 = a_3$

To confirm the result a check reading was taken in each case, thus :—

$\theta = 12$ $\quad 3\theta = 36.$

Angle.	½ Sum.	Angle.	½ Difference.
12°	19·7	12°	25·9
132°	19·7	132°	63·8
252°	11·0	252°	−49·2
	3)50·4		3)40·5
	16·8 $= b_3 \cos 3\theta$		13·5 $= a_3 \sin 3\theta$
	$= b_3 \cos 36$		$= a_3 \sin 36$

Thus we get $\cos 36° = \frac{16·8}{20·7} = ·8116$

and $\sin 36° = \frac{13·5}{23} = ·587$

Now from Tables we see value of cos 36° = ·8192 and sin 36° = ·5878.

As both of these values are within 1 per cent. of being correct we may consider the result satisfactory, considering that the readings from the curve were only taken to 1st place of decimals.

5th Harmonic.

Cosine Term.		Sine Term.	
Angle.	½ Sum.	Angle.	½ Difference.
0°	26·5	18°	37·45
72°	−11·0	90°	41·4
144°	12·2	162°	37·45
216°	12·2	234°	−62·1
288°	−11·0	306°	−62·1
	5)28·9		3)− 7·9
	5·78 $= b_5$		− 1·58 $= a_5$

A check reading was again taken as before, and found to confirm these results.

First Harmonic (Fundamental) Cosine Term.

When $\theta = 0$ $\quad 3\theta = 0$ $\quad 5\theta = 0$

$\cos\theta = 1$ $\quad \cos 3\theta = 1$ $\quad \cos 5\theta = 1$

$b_1 \cos \theta + b_3 \cos 3\theta + b_5 \cos 5\theta = 26{\cdot}5$

$\therefore b_1 + b_3 + b_5 = 26{\cdot}5$

i.e., $b_1 + 20{\cdot}7 + 5{\cdot}78 = 26{\cdot}5$ and $b_1 = {\cdot}02$

This may be assumed = 0, since readings are only taken to 1st place of decimals. Hence $b_1 = 0$.

Sine Term.

When $\theta = 90$ $3\theta = 270$ $5\theta = 450$

$\sin \theta = 1$ $\sin 3\theta = -1$ $\sin 5\theta = 1$

$a_1 \sin \theta + a_3 \sin 3\theta + a_5 \sin 5\theta = 41{\cdot}4$

i.e., $a_1 - a_3 + a_5 = 41{\cdot}4$

and $a_1 - 23{\cdot}0 - 1{\cdot}58 = 41{\cdot}4$

$\therefore a_1 = 66{\cdot}0$

We have now obtained the equation to the curve in the following form :—

$$f(\theta) = 66 \sin \theta + 23{\cdot}0 \sin 3\theta + 20{\cdot}7 \cos 3\theta - 1{\cdot}58 \sin 5\theta + 5{\cdot}78 \cos 5\theta$$

This may be simplified as previously described.

Thus :—

$$a_3 \sin 3\theta + b_3 \cos 3\theta = \sqrt{a_3^2 + b_3^2} \sin \left\{ 3\theta + \tan^{-1}\left(\frac{b_3}{a_3}\right) \right\}$$

$$= 31 \sin (3\theta + 42°)$$

$$a_5 \sin 5\theta + b_5 \cos 5\theta = \sqrt{a_5^2 + b_5^2} \sin \left\{ 5\theta + \tan^{-1}\left(\frac{b_5}{a_5}\right) \right\}$$

$$= 6 \sin (5\theta + 105),$$

and our equation thus becomes

$f(\theta) = 66 \sin \theta + 31 \sin (3\theta + 42°) + 6 \sin (5\theta + 105°)$,

which agrees with the expression obtained by the method of summation (page 427).

Exactly the same operation as just described for the determination of the third and fifth harmonics was carried out for the seventh, ninth, and eleventh harmonics, by dividing the curve into 7, 9, and 11 parts. In each case the values of the coefficients was found to be zero, or practically zero, showing that these harmonics were not present.

A single check reading has been taken above to confirm the result of the calculation of each harmonic. It would be preferable in most cases to take sufficient points to enable a curve to be plotted from them. If the curve is found to be a sine curve, the readings are shown to be trustworthy. If this is not so, a mean curve should be plotted through the points obtained.

R.M.S. Value.—The R.M.S. or virtual value of the curve, whose equation has been determined, can be obtained from the maximum values of the component waves of which it is formed.

Thus, if the equation has been obtained in the form

$$e = E_1 \sin(\theta + \phi_1) + E_3 \sin(3\theta + \phi_3) + E_5 \sin(5\theta + \phi_5) + \ldots$$

where e is the instantaneous value of the quantity for any value of θ,

then E_1, E_3, E_5, &c., are the maximum values of the respective harmonics, and the R.M.S. value is given by the expression

$$e_{\text{virt.}} = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}$$

In the curve analysed above we have

$$E_1 = 66, \quad E_3 = 31, \quad E_5 = 6,$$

and the virtual voltage is consequently

$$e_{\text{virt.}} = \sqrt{\frac{66^2 + 31^2 + 6^2}{2}} = \sqrt{\frac{5353}{2}} = 51{\cdot}73 \text{ volts.}$$

As mentioned on page 428, the actual value registered by the voltmeter was 51·8 volts.

INDEX.

PAGE.

Alternating E.M.F., Curve of 8
Alternating E.M.F., Nature of 5
Alternators 191
Alternators, Action of Armature Currents on Field 219
Alternators, Armature Reaction 211, 312
Alternators, Characteristic 213, 225
Alternators, Characteristic Calculation of 231
Alternators, Compounding..... 238
Alternators, Determination of Drop with Any Power Factor 245
Alternators, Efficiency Tests... 252
Alternators, Excitation to Maintain Constant Voltage 237
Alternators, Excitation Affected by Armature Current 223
Alternators, Field Effect of Armature Currents on 219
Alternators, Impedance and Self-induction of Armature ... .51, 203
Alternators, In Parallel Diagram of 286
Alternators, Iron Losses, Measurement of 254
Alternators, Losses in 252
Alternators, Magnetic Circuit 195
Alternators, Magnetisation Curve 196, 201, 208
Alternators, Parallel Running of 257
Alternators, Periodicity of.... 194
Alternators, Phase, Difference Between Total Volts and Current 233
Alternators, Relation Between Speed and Voltage 247
Alternators, Resultant Voltage of Two 25
Alternators, Short Circuit, Characteristic 240
Alternators, Speed Curves ... 247

PAGE.

Alternators, Star and Mesh Connected 300
Alternators, Synchronising of 263
Alternators, Tracing Wave Form of 84
Alternators, Voltage of 193
Alternators, Wave Form of... 192
Ammeters, Three for Measurement of Power 80
Analysis by Summation 425
Analysis by Superposition ... 430
Analysis of Curves 420
Angle of Lag 19
Angle of Lag, Determination of 29
Apparent Resistance (see Impedance).
Apparent Watts 59
Armature, Impedance of (Stationary) 51
Armature, Reaction in Alternator 211, 312
Armature, Reaction in Induction Motor 351
Auto-transformer 179

Back E.M.F. 12

Capacity and Self-induction in Series 114
Capacity and Self-induction in Parallel 121
Capacity, Effect of, on Phase Relations 103
Capacity, in Alternating Circuit 98
Capacity, Variation of Current with 106
Cardew Voltmeter, Scale of... 92
Characteristic of Alternator 213, 225
Characteristic of Alternator, Calculation of 231
Characteristic of Alternator, Short Circuit 240
Characteristic of Rotary Converter 321, 330
Charge of Condenser 97

PAGE.

Charging Current of Condenser, Effect of Capacity on 106
Charging Current of Condenser, Effect of Frequency on 109
Charging Current of Condenser, Effect of Voltage on 111
Choking Coil 66, 68
Component E.M.F., Curves of 17
Component E.M.F. 64
Compounding Alternators 238
Compounding Rotary Converter 338
Condenser in Alternating Circuit 97
Conductor Carrying Current, Force on 4
Coulomb 97
Current and Voltage, Tracing Curves of 87
Current, Variation of Impedance with 44
Currents, Diagram of 76
Currents, Idle and Energy 73, 75, 101
Currents, Resultant of Two... 71
Curve of E.M.F. 8
Curve, Tracing by Rotating Contact 84, 190
Curves, Analysis of 420
Curves, Analysis of, by Summation 425
Curves, Analysis of, by Superposition 430

Delta Connection 299
Diagrams, Alternators in Parallel 286
Diagrams of Induction Motor.. 403
Diagrams of Transformer..... 183
Diagrams, Vector 21, 24, 31, 76, 118
Dyne, Definition 1

Eddy Current, Losses in Transformer 148
Effective Value of Alternating Current 90
Efficiency Tests of Alternator 252
Efficiency Tests of Transformer 157, 160, 161
E.M.F., Alternating, Nature of 5
E.M.F., Alternating, Curve of 8
E.M.F., Component and Resultant 18, 32
E.M.F. of Self-induction 11

PAGE.

E.M.F. of Self-induction, Value of 16
E.M.F., Triangle of 24, 31
E.M.F., Unit 2
Energy Current 74
Energy Voltage 64
Excitation of Alternator for Constant Voltage 237
Excitation of Synchronous Motors, Variation of 271

Farad, Unit of Capacity 97
Fleming's Rule 3
Force on Conductor Carrying Current 4
Form Factor 194
Frequency 9
Frequency, Effect upon Impedance 39

Graphic Calculation of Characteristic of Alternator 231
Graphic Calculation of Impedance 48
Graphic Determination of Drop in Transformer 162
Graphic Determination of Drop of Alternator at any Power Factor 246
Graphic Representation of Induction Motor 403
Graphic Representation of Transformer 183

Harmonics (Curves) 420
Henry, Definition 15
Heyland, Diagram of Induction Motor 405
Heyland, Diagram of Induction Motor, Experimental Derivation 411
Hysteresis in Magnetic Circuit of Alternator 198
Hysteresis, Losses in Transformer 148

Idle Current 75, 101
Idle Voltage 64
Impedance 34, 102
Impedance, Dependence on Capacity 106
Impedance, Dependence on Current Strength 44
Impedance, Dependence on Frequency 39, 109
Impedance, Dependence on Resistance 36
Impedance, Dependence on Self-induction 42

PAGE.

Impedance, Graphic Calculation of 48
Impedance of Armature (Rotating) 203
Impedance of Armature (Stationary) 51
Impedance of Circuit Having Inductance and Capacity ... 115
Inductance 15
Inductance and Capacity in Parallel 121
Inductance and Capacity in Series 114
Induction Motor 341
Induction Motor, Armature Reaction 351
Induction Motor, Braking and Return of Power to Line.... 368
Induction Motor, Current in Stator 384
Induction Motor, Diagrams... 403
Induction Motor, Diagram, Experimental, Derivation of 411
Induction Motor, Efficiency ... 388
Induction Motor, Efficiency by Indirect Measurement 397
Induction Motor, Flux in 351
Induction Motor, Heating of... 352
Induction Motor, Load Test... 378
Induction Motor, Losses ...388, 397
Induction Motor, Magnetic Leakage 357
Induction Motor, No-load Curves 369
Induction Motor, No-load Test 370
Induction Motor, Output..... 410
Induction Motor, Power Absorbed 388, 408
Induction Motor, Power Factor 390, 398
Induction Motor, Production of Current in Rotor 348
Induction Motor, Ratio of Stator to Rotor Currents 377, 386
Induction Motor, Resistance of Windings, Measurement of 365
Induction Motor, Rotating Field of 344
Induction Motor, Rotation of Armature 347
Induction Motor, Rotor Resistance, Effect of Varying..... 393
Induction Motor, Single-phase 399
Induction Motor, Slip ...350, 391, 398
Induction Motor, Slip and Torque, Relation Between... 391

PAGE.

Induction Motor, Slip, Measurement of 362
Induction Motor, Speed Regulation 366
Induction Motor, Stator Current 384
Induction Motor, Torque of 352, 355, 360, 391, 408
Induction, Coefficient of Self 14
Instantaneous Value 22
Instruments, Scales of 91
Insulation, Losses in Transformer, Due to Defective..... 145
Iron Losses in Alternator 254
Iron Losses in Transformer... 148

Lag of Current 19
Lag of Current, Determination of 29
Leakage, Magnetic, in Alternator 200
Leakage, Magnetic, in Induction Motor 357
Leakage, Magnetic, in Transformer 154, 187
Losses in Transformer ...148, 152

Magnetic Circuit of an Alternator 195
Magnetic Field, Action on Current 4
Magnetic Leakage in Alternator 200
Magnetic Leakage in Induction Motor 357
Magnetic Leakage in Transformer 154, 187
Magnetic Pole Unit 1
Magnetisation Curve of Alternator 196, 201, 208
Magnetisation Curve of Choking Coil 47
Magnetisation Curve of Rotary Converter 318
Magnetising Current of Transformer 139
Mesh Connection 299
Microfarad 97
Mordey, Example of Charging Current of Cable 123
Motor Induction 341
Motor, Single-phase 399
Motor, Synchronous 266

Non-inductive Resistance 49

Parallel Running of Alternators 257
Period Definition 9
Periodicity of an Alternator... 10

PAGE.

Phase, Definition 9
Phase Difference Between Current and Volts, Determination of 29
Polyphase Circuit 294
Power Factor 58, 60
Power Factor, Determination of 61
Power in Alternating Circuit 56, 95
Power in 3-phase Circuit 302
Power, Measurement of....77, 80, 112
Power, Measurement of, by Three Ammeters 80
Power, Measurement of, by Three Voltmeters 77

Rate of Change of Current... 13
Reactance 35
Regulation of Transformer 130, 135
Resistance of Windings of Induction Motor 365
Resistances, Iron Wire, Impedance of 50
Resistances, Self-induction of 49
Resonance 120
Resultant Currents 71
Resultant Voltage 18, 32
Resultant Voltage of Two Alternators 25
Resultant of Two Vectors ... 23
Retardation Method of Measuring Losses 254
R.M.S. (Root of Mean Squares) Value 90, 437
Rotating Contact for Curve Tracing 84
Rotating Field, Constancy of 344
Rotating Field, Production of 341
Rotary Converter 313
Rotary Converter, Characteristic 321, 330
Rotary Converter, Compounding 338
Rotary Converter, Effect of Varying Excitation ...327, 333
Rotary Converter, Loss of Voltage in 325
Rotary Converter, Losses in... 314
Rotary Converter Magnetisation Curve 318
Rotary Converter, Polyphase.. 315
Rotary Converter, Ratio of Transformation 314, 318
Rotor of Induction Motor..... 348
Rule, Fleming's 3

Scales of Instruments 91

PAGE.

Self-induction and Capacity in Parallel 121
Self-induction and Capacity in Series 114
Self-induction, Coefficient of... 14
Self-induction, Effect on Impedance 42
Separation of Losses in Transformer 149
Short Circuit Characteristic of Alternator 240
Single-phase Induction Motor 399
Single-phase Induction Motor Starting Devices 402
Slip of Induction Motor 350
Slip of Induction Motor, Measurement of 362
Spiral Resistances, Self-induction of 50
Split Dynamometer, Efficiency of Transformer by 166
Star Connection 299
Starting Devices for Single-phase Motors 402
Starting Resistance of Induction Motor 397
Stator of Induction Motor ... 348
Sumpner Method of Testing Transformer 170
Synchroniser 258
Synchronising Alternators, Connections for 262
Synchronous Motors 266
Synchronous Motors, Diagram of 286
Synchronous Motors, Method of Starting 268
Synchronous Motors, Variation of Excitation 271
Synchronous Motors with Varying Load 283

Three-ammeter Measurement of Power 80
Three-phase Circuit 296
Three-phase Circuit, Connections of 299
Three-phase Circuit, Power in 302
Three-phase Current 295
Three Voltmeter Measurement of Power 77
Torque of Induction Motor 355, 360
Transformer, Auto- 179
Transformer, Copper Losses ... 151
Transformer, Determination of Magnetising Current 141
Transformer, Determination of Magnetising Current at Various Frequencies 146

	PAGE.
Transformer Diagrams	183
Transformer, Effect of Lag in Secondary Circuit	156
Transformer Efficiency by Split Dynamometer	166
Transformer Efficiency by Sumpner's Method	170
Transformer Efficiency Tests	157, 160
Transformer Efficiency Tests by Double Conversion	166
Transformer Efficiency Tests Indirect Methods	161
Transformer, General Principles	125
Transformer, Graphical Determination of Drop at any Power Factor	162
Transformer, Insulation Losses due to Defective	145
Transformer, Iron Losses	148
Transformer, Iron Losses, Separation of	149
Transformer, Magnetic Leakage	154, 187
Transformer, Maker's Tests	177
Transformer, Measurement of Ratio of Transformation	137
Transformer, No-load Current	139
Transformer, Power Factor, Drop at any	162
Transformer, Ratio of Currents	127
Transformer, Test of Uniformity of Conversion Ratio	178
Transformer, Tracing Curves of	190
Transformer, Voltage Drop	129
Transformer, Voltage Drop, Determination of	130, 135
Triangle of Voltages	24, 31
Unit Pole, Definition	1
Vector Diagrams	21
Vectors, Resultant of Two	23
Values of Direct and Alternating Currents	89
Virtual Value of Current	90, 197
Volt-amperes	59
Voltage, Component and Resultant	18, 32
Voltage, Energy and Idle	64
Voltage of Transformer	126
Voltage Ratio of Rotary Converter	318
Voltage, Tracing Wave Form of	87
Voltage, Triangle of	24, 31
Voltmeters, Cardew, Scale of	91
Voltmeters, Three, for Measurement of Power	77
Wattless Current	74
Wattmeter	57
Wattmeter Connection in a 3-phase Circuit	305
Watts, Apparent	59
Watts, Curve of	58
Watts in Alternating Circuit	57
Wave Form, Tracing of	84
Wave Forms, Composition of	420

HALLIDIE LIBRARY OF THE UNIVERSITY OF CALIFORNIA

THIS BOOK IS DUE ON THE LAST DATE STAMPED BELOW

AN INITIAL FINE OF 25 CENTS

WILL BE ASSESSED FOR FAILURE TO RETURN THIS BOOK ON THE DATE DUE. THE PENALTY WILL INCREASE TO 50 CENTS ON THE FOURTH DAY AND TO $1.00 ON THE SEVENTH DAY OVERDUE.

OCT 12 1937

FEB 21 1940

MAR 6 1940

APR 20 1940

OCT 10 1940 M.

FEB 7 1941 A

2-21-41

NOV 19 1941 K

APR 1 1942

MAR 21 1943

APR 6 1944

LD 21-95m-7,'37

YC 69684

TK2711
S6
Smith
144880